POLITECNICO DI MILANO

Monte Carlo Simulation

The experimental view

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CONTENTS

- Sampling Random Numbers
- Simulation of system transport
- Simulation for reliability/availability analysis of a component
- Examples

The History of Monte Carlo Simulation

Buffon	*1707-88*
↓	
Kelvin	*1824-1907*
↓	
Gosset (Student)	*1908*
↓	
Fermi, von Neumann, Ulam	*1930-40's*
↓	
Neutron transport	*1950's*
↓	
System Transport (RAMS)	*1990's*

SAMPLING RANDOM NUMBERS

Example: Exponential Distribution

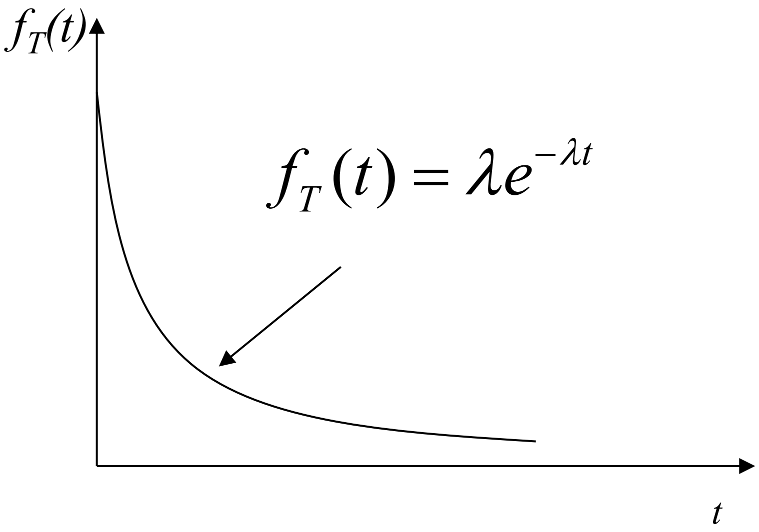
Probability density function:

$$f_T(t) = \lambda e^{-\lambda t} \qquad t \geq 0$$
$$= 0 \qquad t < 0$$

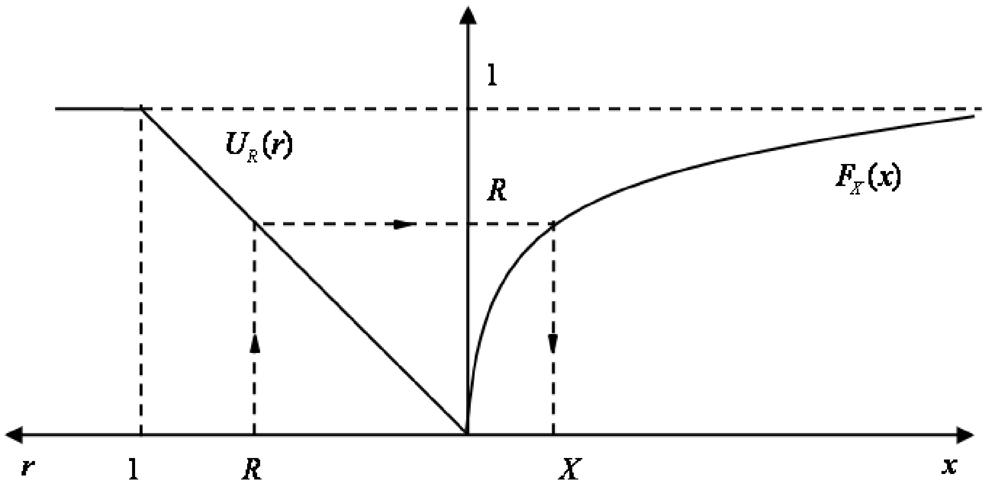
Expected value and variance:

$$E[T] = \frac{1}{\lambda}$$

$$Var[T] = \frac{1}{\lambda^2}$$



Sampling Random Numbers from $F_X(x)$



Sample R from $U_R(r)$ and find X:

$$X = F_X^{-1}(R)$$

Example: Exponential distribution

$$F_X(x) = 1 - e^{-\lambda x}$$

$$R = F_X(x) = 1 - e^{-\lambda x}$$

$$\Downarrow$$

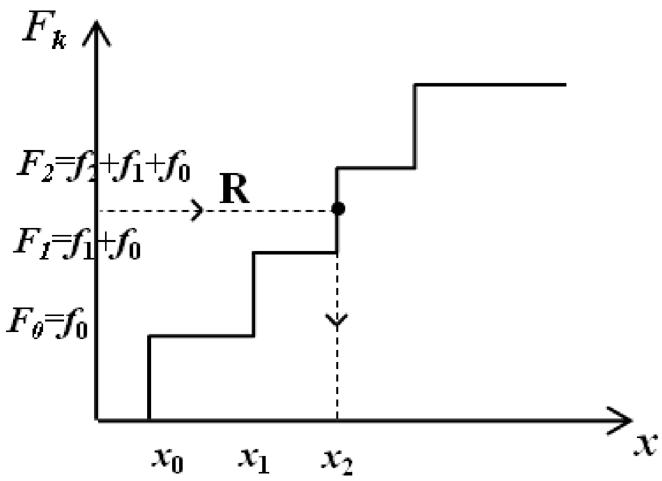
$$X = F_X^{-1}(R) = -\frac{1}{\lambda}\ln(1-R)$$

Sampling from discrete distributions

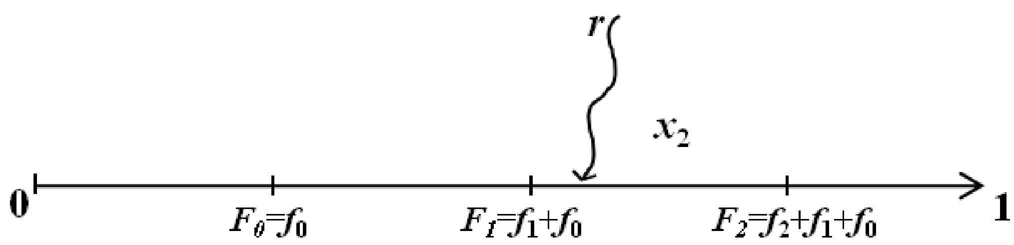
$$\Omega = \{x_0, x_1, \ldots, x_k, \ldots\}$$

$$F_k = P\{X \le x_k\} = \sum_{i=0}^{k} P[X = x_i]$$

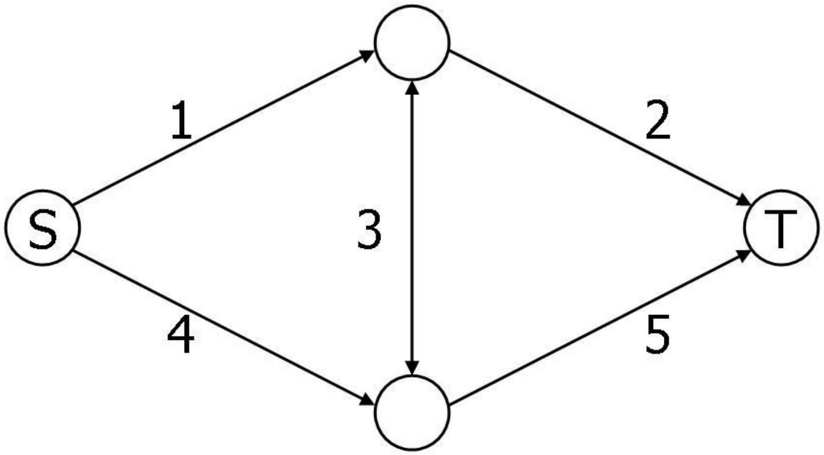
sample an $R \sim U[0,1)$



Graphically:



Failure probability estimation: example



Arc number i	*Failure probability P_i*
1	0.050
2	0.025
3	0.050
4	0.020
5	0.075

- **1- Calculate the analytic solution for the failure probability of the network, i.e., the probability of no connection between nodes S and T**

- **2- Repeat the calculation with Monte Carlo simulation**

SIMULATION OF SYSTEM TRANSPORT

Monte Carlo simulation for system reliability

PLANT = system of *Nc* suitably connected components.

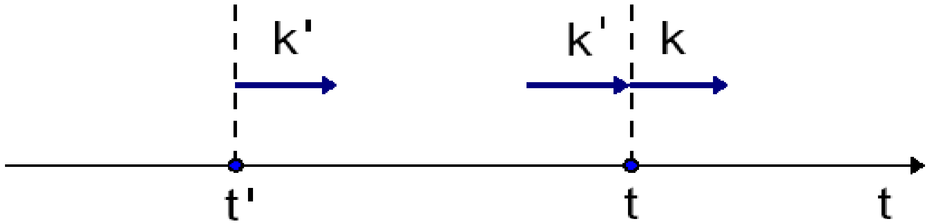
COMPONENT = a subsystem of the plant (pump, valve,...) which may stay in different exclusive (multi)states (nominal, failed, stand-by,...). Stochastic transitions from state-to-state occur at stochastic times.

STATE of the PLANT at t = the set of the states in which the *Nc* components stay at t. The states of the plant are labeled by a scalar which enumerates all the possible combinations of all the component states.

PLANT TRANSITION = when any one of the plant components performs a state transition we say that the plant has performed a transition. The time at which the plant performs the n-th transition is called t_n and the plant state thereby entered is called k_n.

PLANT LIFE = stochastic process.

Stochastic Transitions: Governing Probabilities



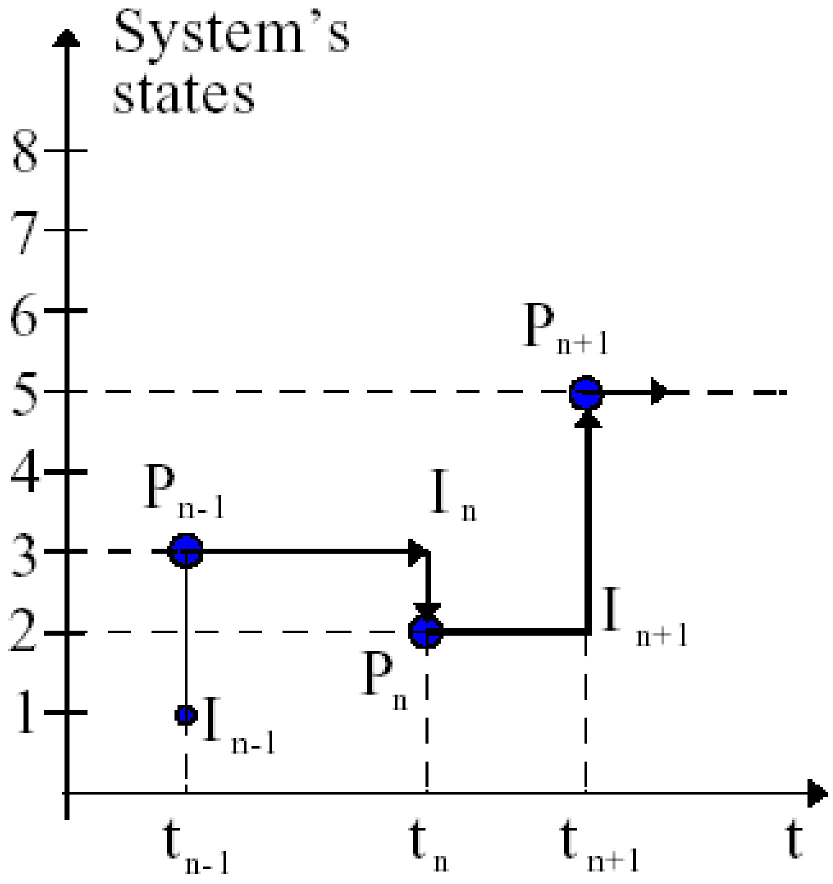
- *T(t / t'; k')dt* = conditional probability of a transition at t∈dt, given that the preceding transition occurred at *t'* and that the state thereby entered was *k'*.
- *C(k / k'; t)* = conditional probability that the plant enters state *k*, given that a transition occurred at time *t* when the system was in state *k'*. Both these probabilities form the "trasport kernel":

$$K(t; k \;/\; t'; k')dt = T(t \;/\; t'; k')dt \; C(k \;/\; k'; t)$$

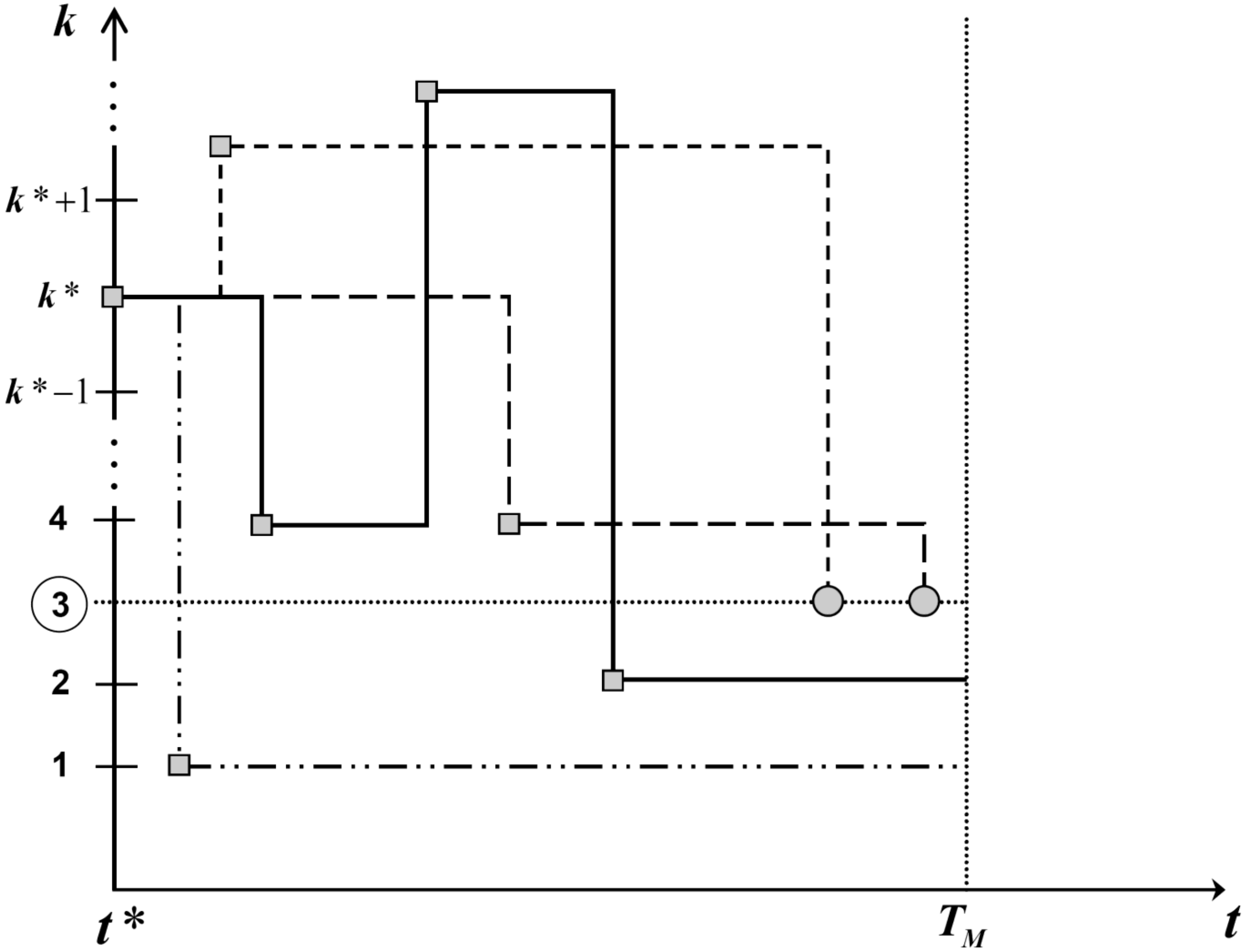
- $\psi(t; k)$ = ingoing transition density or probability density function (pdf) of a system transition at *t*, resulting in the entrance in state *k*

Plant life: random walk

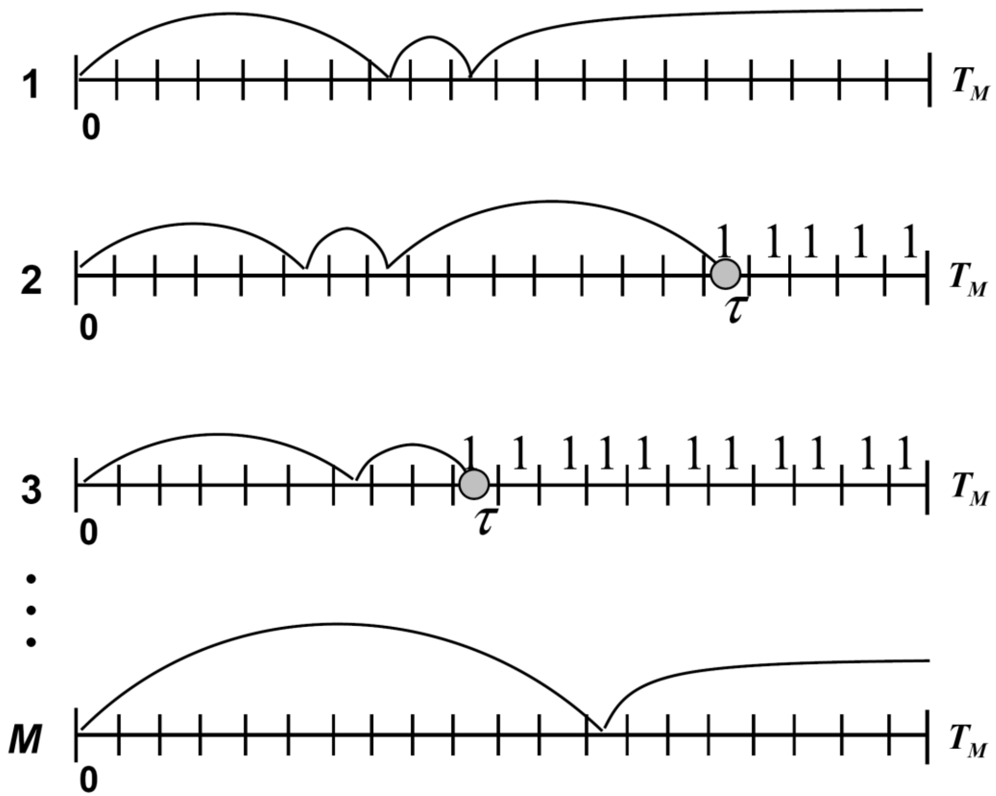
Random walk = realization of the system life generated by the underlying state-transition stochastic process.



Phase Space



Example: System Reliability Estimation



$$C^R(t) = C^R(t) \quad t \in [0, T_M]$$

$$C^R(t) = C^R(t) + 1 \quad t \in [\tau, T_M]$$

$$C^R(t) = C^R(t) + 1 \quad t \in [\tau, T_M]$$

$$C^R(t) = C^R(t) \quad t \in [0, T_M]$$

$$\hat{F}_T(t) = \frac{C^R(t)}{M}$$

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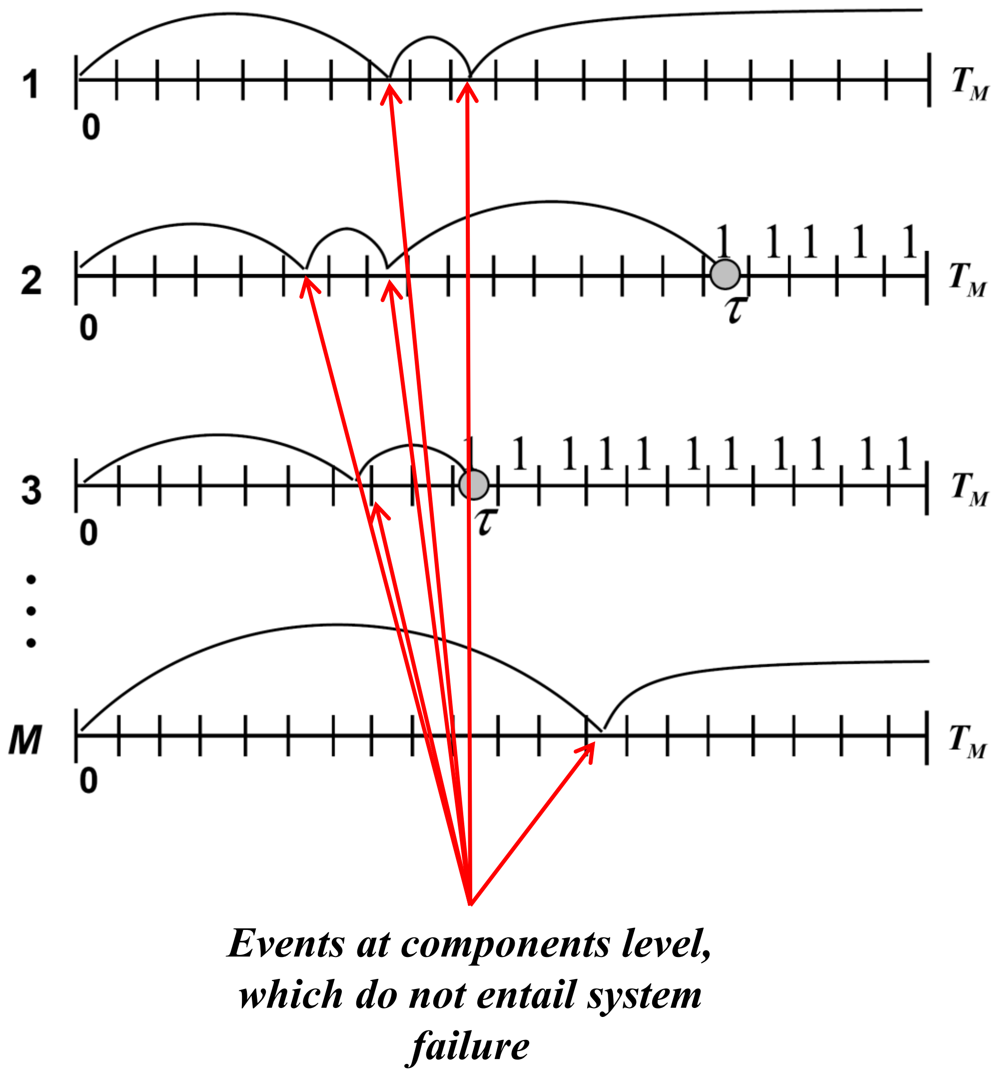

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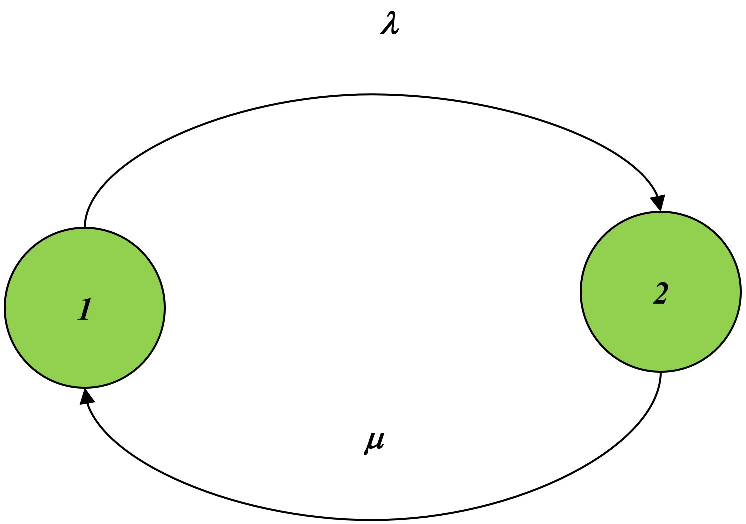
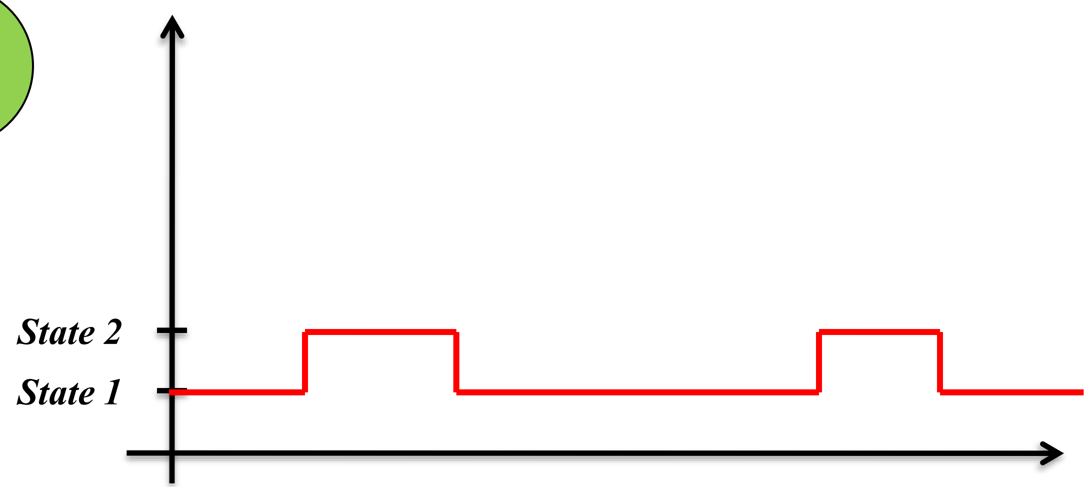
SIMULATION OF COMPONENT STOCHASTIC STATE TRANSITION PROCESS FOR AVAILABILITY / RELIABILITY ESTIMATION

One component with exponential distribution of the failure time

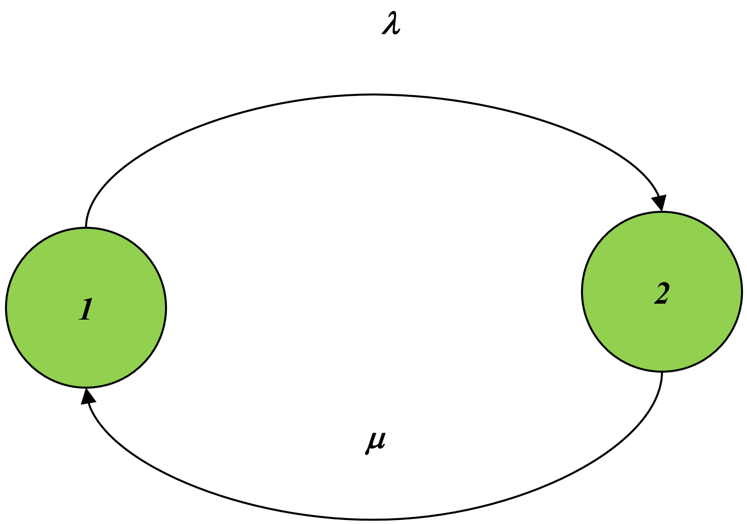


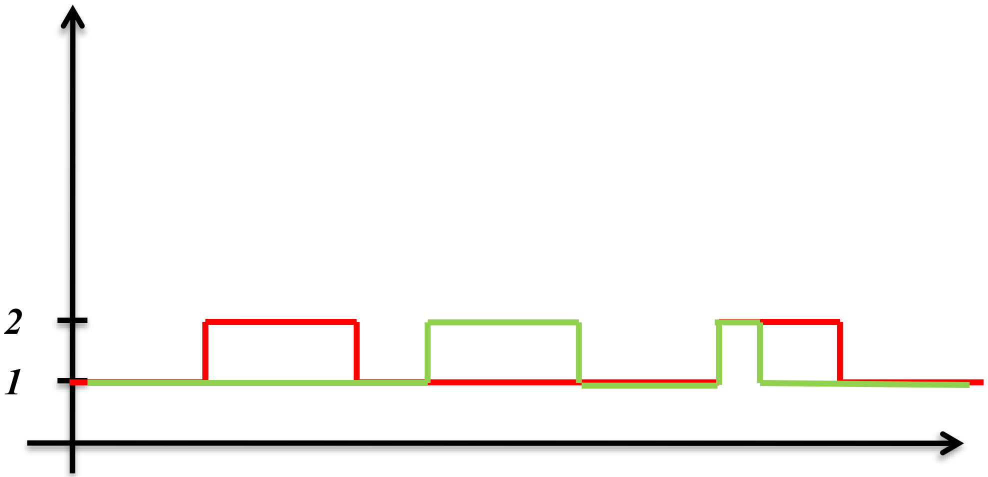


State X=1 → ON

State X=2 → OFF

One component with exponential distribution of the failure time

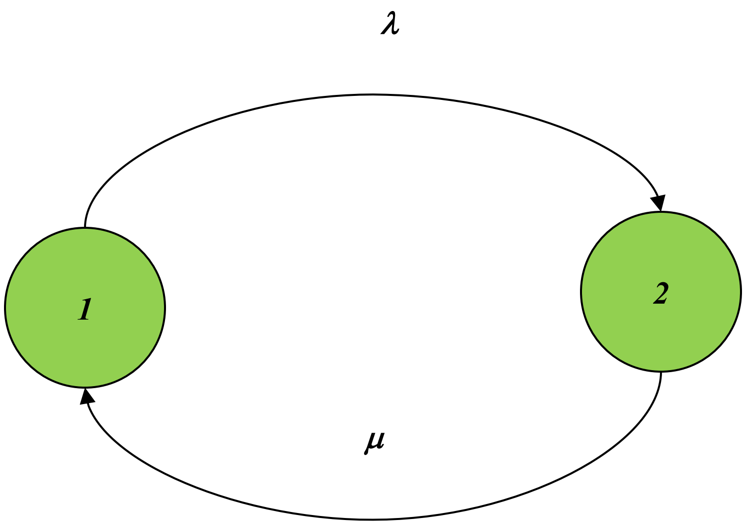


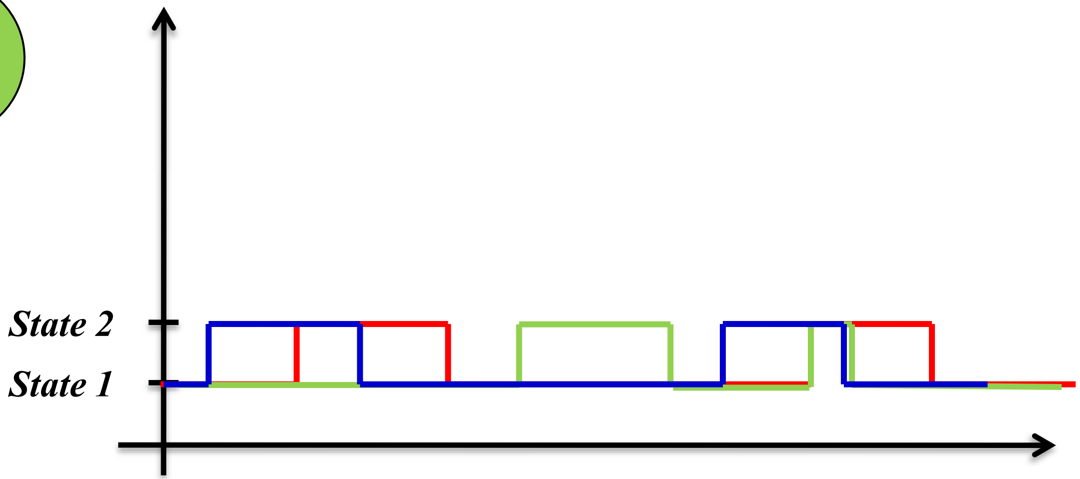


State X=1 → ON

State X=2 → OFF

One component with exponential distribution of the failure time

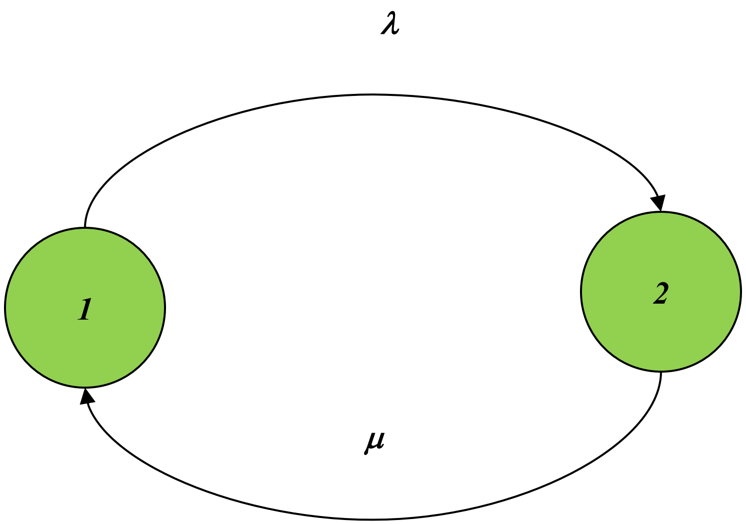




State X=1 → ON

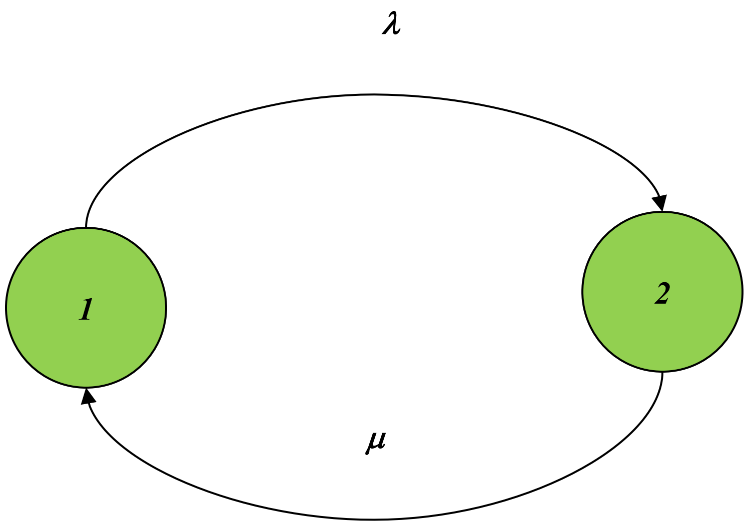
State X=2 → OFF

One component with exponential distribution of the failure time



	values
λ	$3 \cdot 10^{-3}\ h^{-1}$
μ	$25 \cdot 10^{-3}\ h^{-1}$

One component with exponential distribution of the failure time



	values
λ	$3 \cdot 10^{-3}\ h^{-1}$
μ	$25 \cdot 10^{-3}\ h^{-1}$

State X=1 → ON

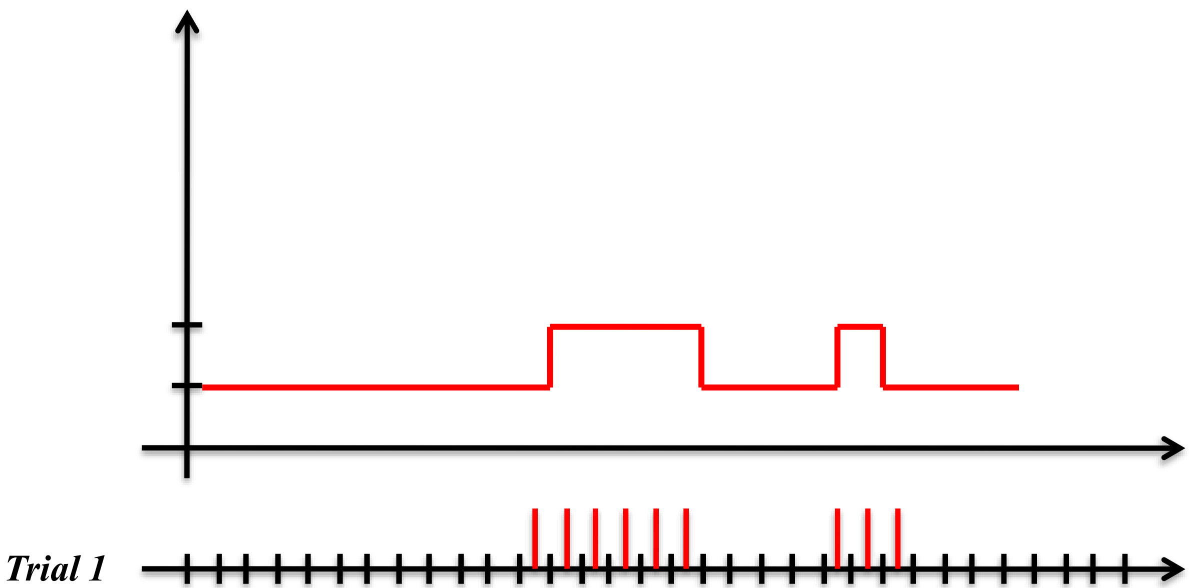
State X=2 → OFF

Limit unavailability:

$$U = \frac{1/\mu}{1/\mu + 1/\lambda} = 0.1071$$

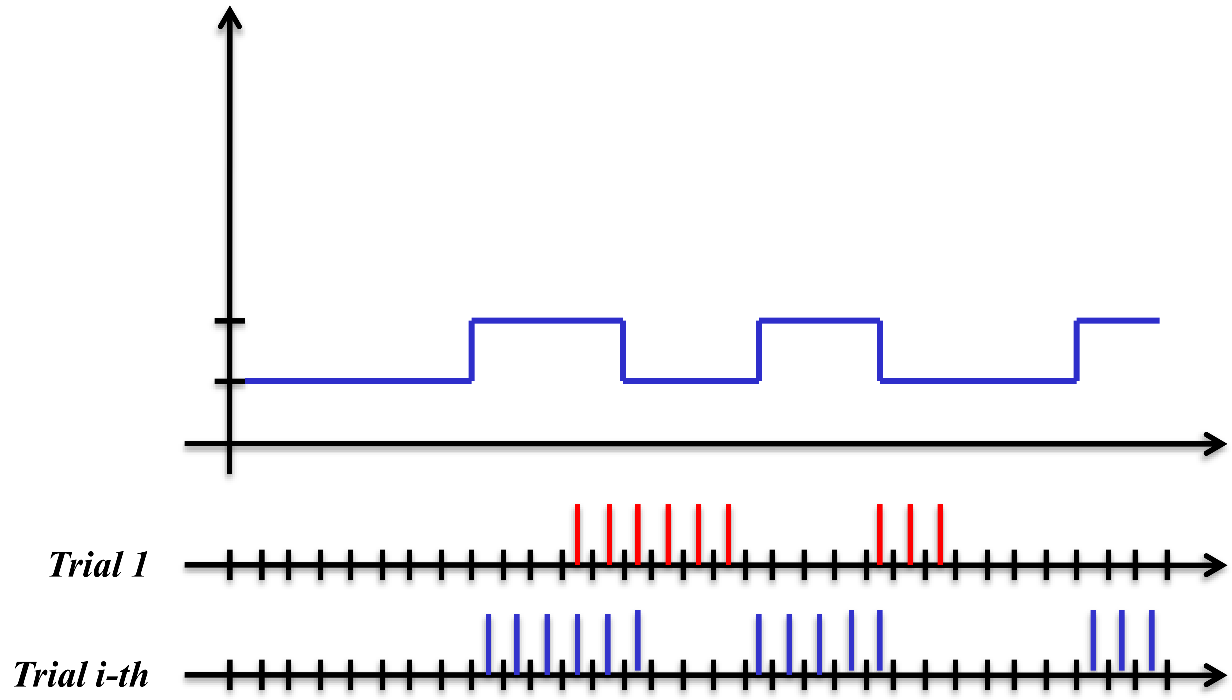
Monte Carlo simulation for estimating the system availability at time *t*

- N_t time intervals Δt
- If the component fails in t+Δt, *the counter increases* $c^A(t_j) = c^A(t_j) + 1$; otherwise, $c^A(t_j) = c^A(t_j)$



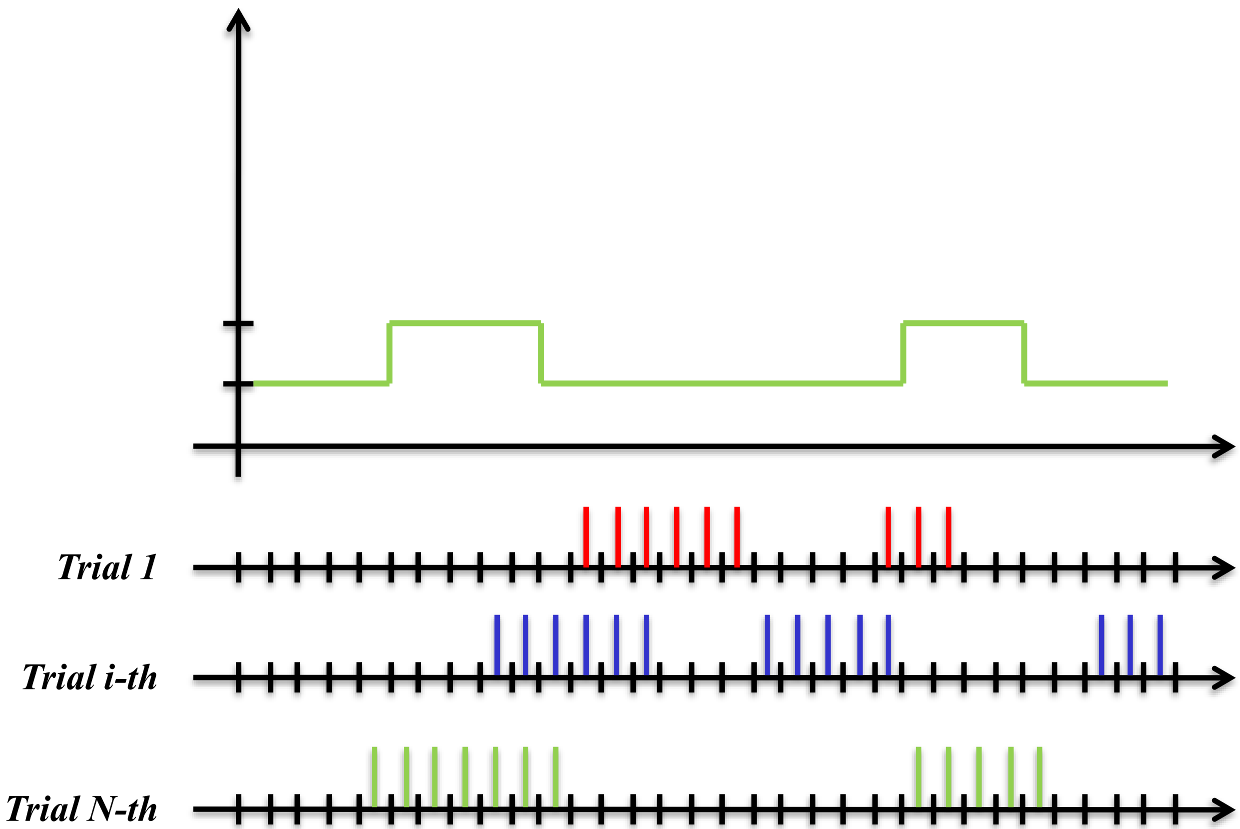
Monte Carlo simulation for estimating the system availability at time *t*

- … another trial



Monte Carlo simulation for estimating the system availability at time *t*

- … another trial

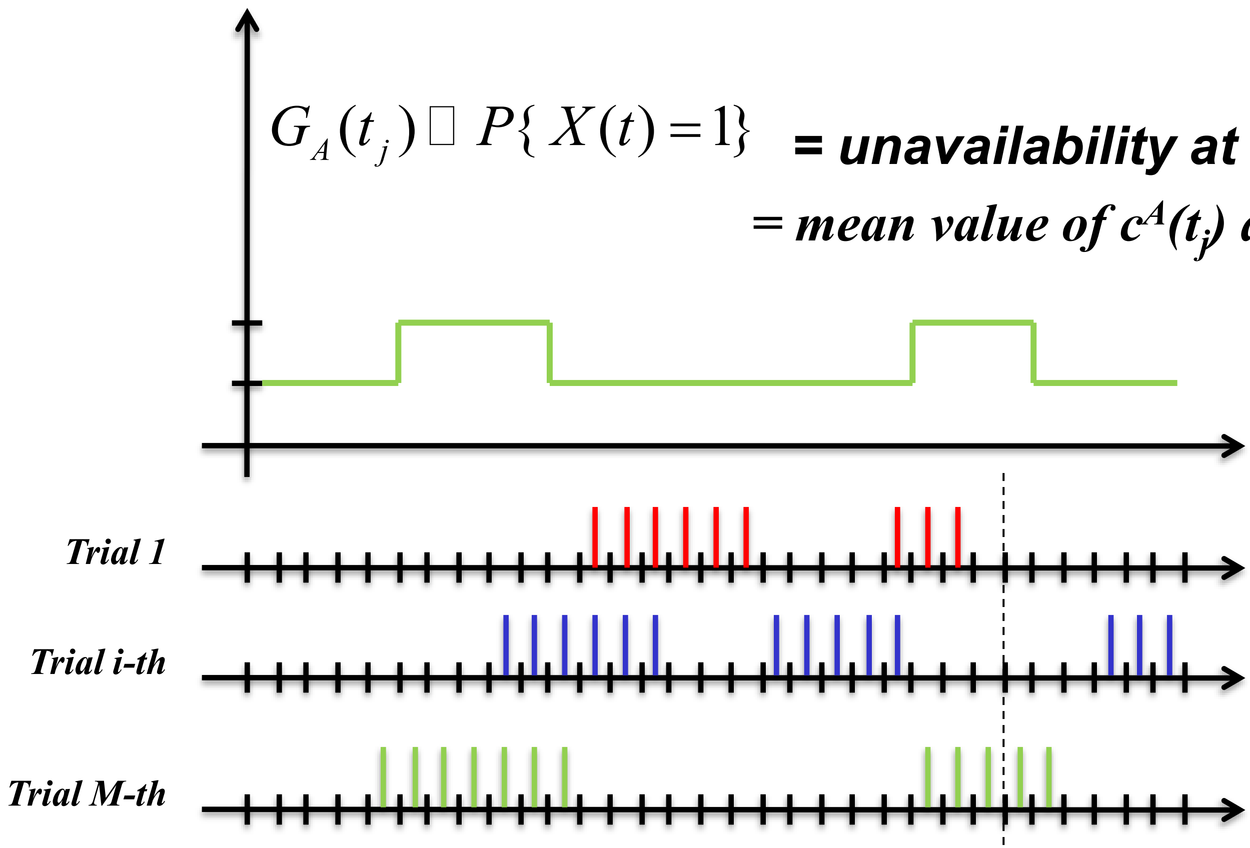


Monte Carlo simulation for estimating the system availability at time *t*

The counter $c^A(t_j)$ adds 1 until M trials have been sampled

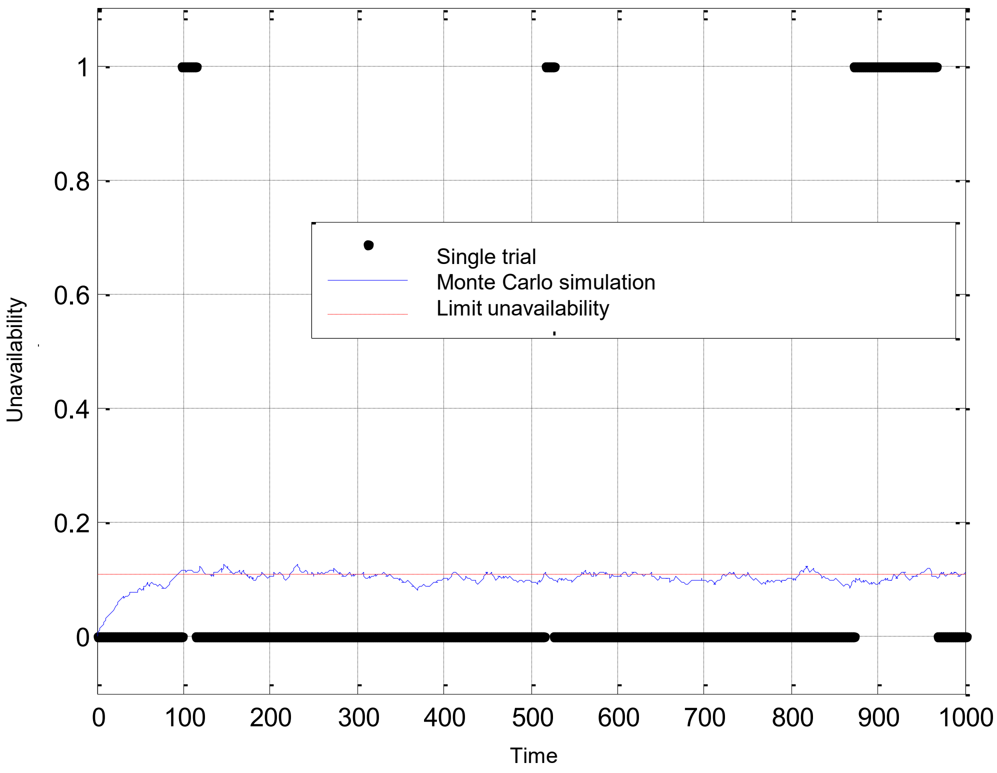
$G_A(t_j) \equiv P\{X(t)=1\}$ **= *unavailability at time t_j***

= mean value of $c^A(t_j)$ at time t_j



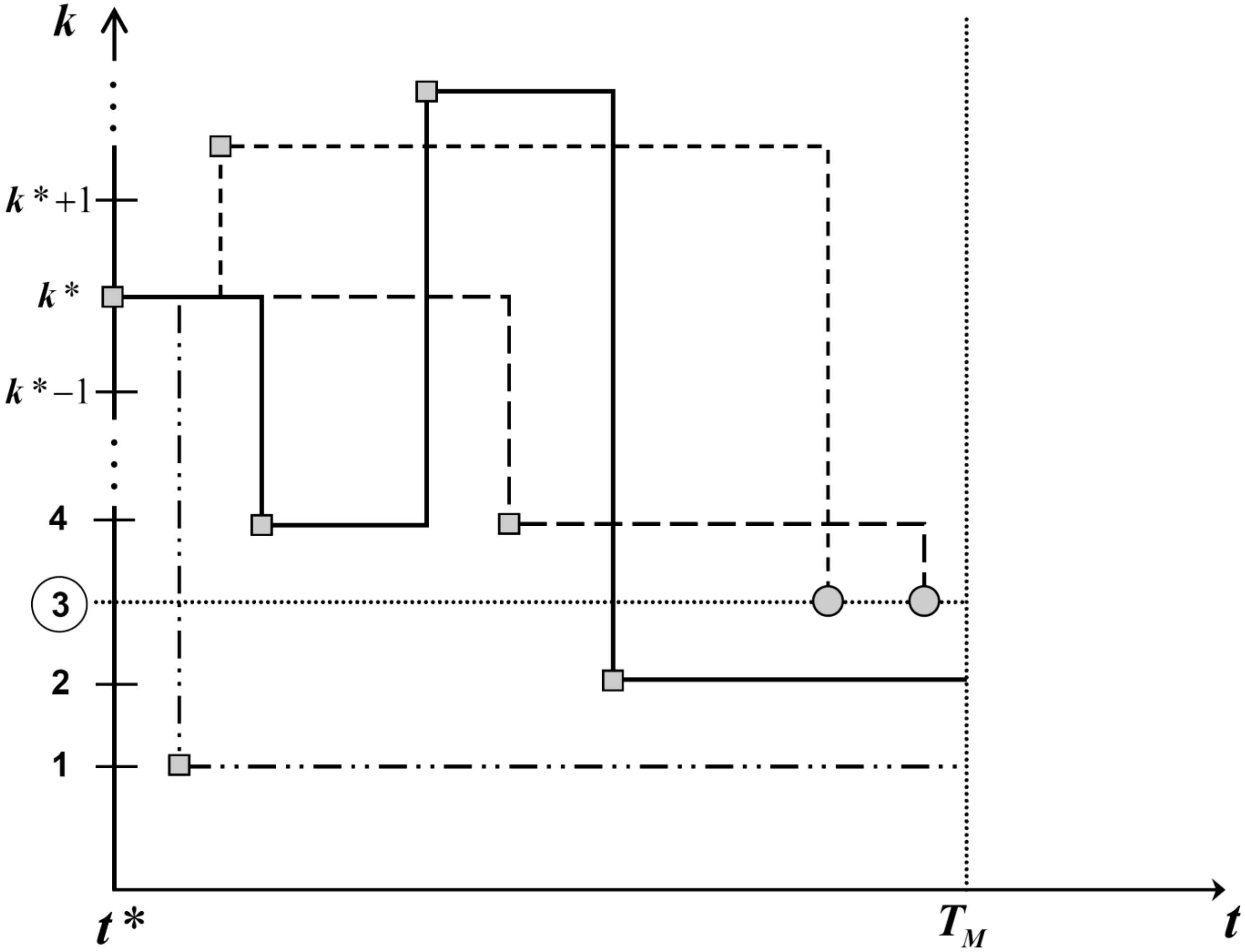
Unavailability at time t_j is equal to the mean value of $c^A(t_j)$ at time t_j

$$G_A(t_j) = c^A(t_j) / M$$

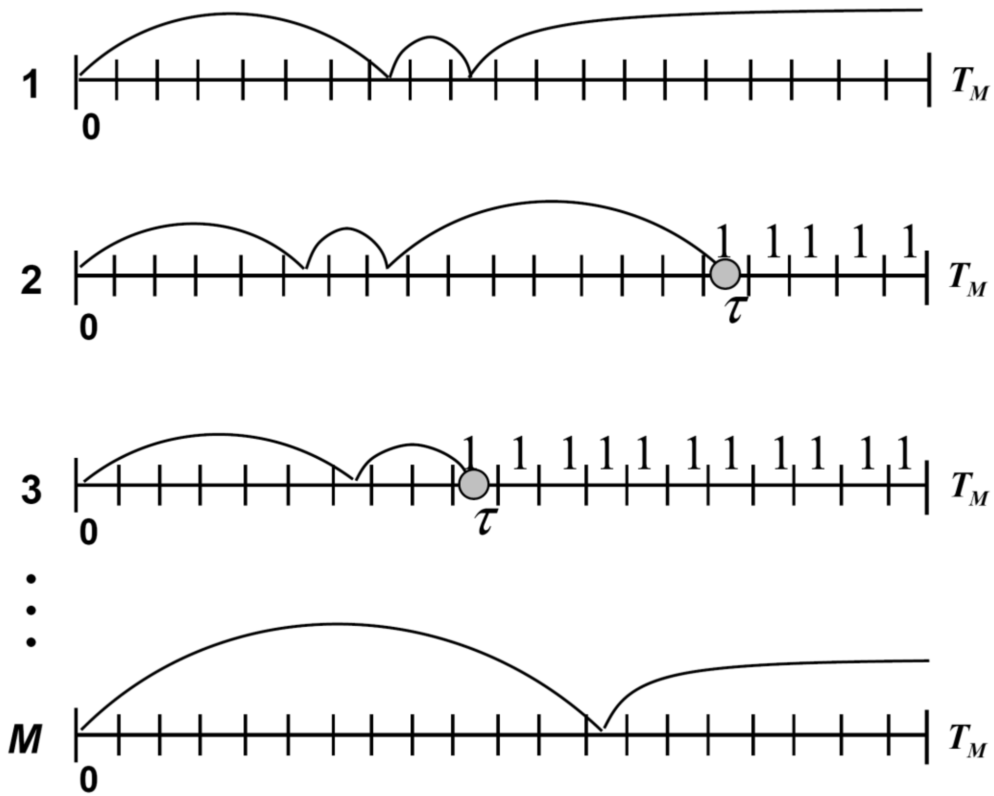

Single trial
Monte Carlo simulation
Limit unavailability
Unavailability
Time
0
0.2
0.4
0.6
0.8
1
0
100
200
300
400
500
600
700
800
900
1000

SIMULATION OF SYSTEM STOCHASTIC STATE TRANSITION PROCESS FOR AVAILABILITY / RELIABILITY ESTIMATION

Phase Space



Example: System Reliability Estimation



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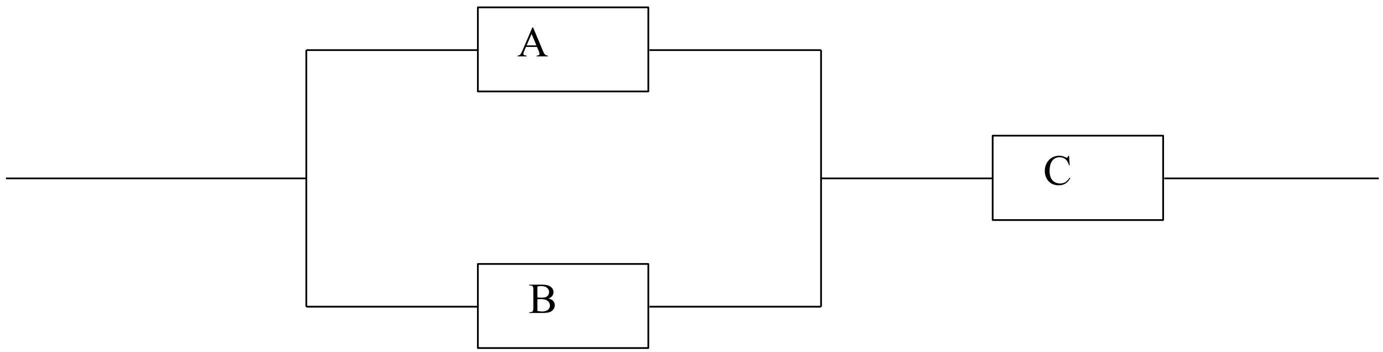
$$C^R(t) = C^R(t) \quad t \in [0, T_M]$$

$$\hat{F}_T(t) = \frac{C^R(t)}{M}$$

Indirect Monte Carlo: Example (1)



Components' times of transition between states are exponentially distributed

($\lambda^{i}_{j_i \to m_1}$ = rate of transition of component *i* going from its state j_i to the state m_i)

Initial \ Arrival	1	2	3
1	-	$\lambda^{A(B)}_{1\to 2}$	$\lambda^{A(B)}_{1\to 3}$
2	$\lambda^{A(B)}_{2\to 1}$	-	$\lambda^{A(B)}_{2\to 3}$
3	$\lambda^{A(B)}_{3\to 1}$	$\lambda^{A(B)}_{3\to 2}$	-

Indirect Monte Carlo: Example (2)

Arrival (columns) / **Initial** (rows)

Initial \ Arrival	1	2	3	4
1	-	$\lambda^C_{1\to 2}$	$\lambda^C_{1\to 3}$	$\lambda^C_{1\to 4}$
2	$\lambda^C_{2\to 1}$	-	$\lambda^C_{2\to 3}$	$\lambda^C_{2\to 4}$
3	$\lambda^C_{3\to 1}$	$\lambda^C_{3\to 2}$	-	$\lambda^C_{3\to 4}$
4	$\lambda^C_{4\to 1}$	$\lambda^C_{4\to 2}$	$\lambda^C_{4\to 3}$	-

- The components are initially (t=0) in their nominal states (1,1,1)
- One minimal cut set of order 1 (C in state 4:(*,*,4)) and one minimal cut set of order 2 (A and B in 3: (3,3,*)).

Analog Monte Carlo Trial

SAMPLING THE TIME OF TRANSITION

The rate of transition of component A(B) out of its nominal state 1 is:

$$\lambda_1^{A(B)} = \lambda_{1\to 2}^{A(B)} + \lambda_{1\to 3}^{A(B)}$$

- The rate of transition of component C out of its nominal state 1 is:

$$\lambda_1^{C} = \lambda_{1\to 2}^{C} + \lambda_{1\to 3}^{C} + \lambda_{1\to 4}^{C}$$

- The rate of transition of the system out of its current configuration (1, 1, 1) is:

$$\lambda^{(1,1,1)} = \lambda_1^{A} + \lambda_1^{B} + \lambda_1^{C}$$

- We are now in the position of sampling the first system transition time t_1, by applying the inverse transform method:

$$t_1 = t_0 - \frac{1}{\lambda^{(1,1,1)}} \ln(1 - R_t)$$

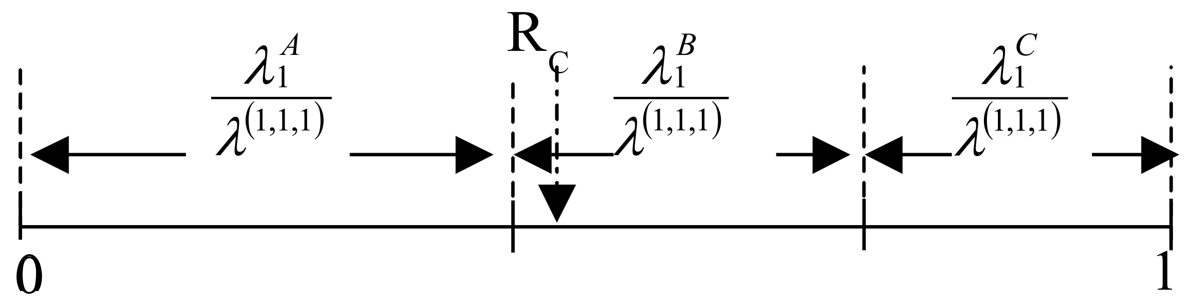
where $R_t \sim U[0,1)$

Sampling the Kind of Transition (1)

- Assuming that $t_1 < T_M$ (otherwise we would proceed to the successive trial), we now need to determine which transition has occurred, i.e. which component has undergone the transition and to which arrival state.
- The probabilities of components A, B, C undergoing a transition out of their initial nominal states 1, given that a transition occurs at time t_1, are:

$$\frac{\lambda_1^A}{\lambda^{(1,1,1)}}, \quad \frac{\lambda_1^B}{\lambda^{(1,1,1)}}, \quad \frac{\lambda_1^C}{\lambda^{(1,1,1)}}$$

- Thus, we can apply the inverse transform method to the discrete distribution

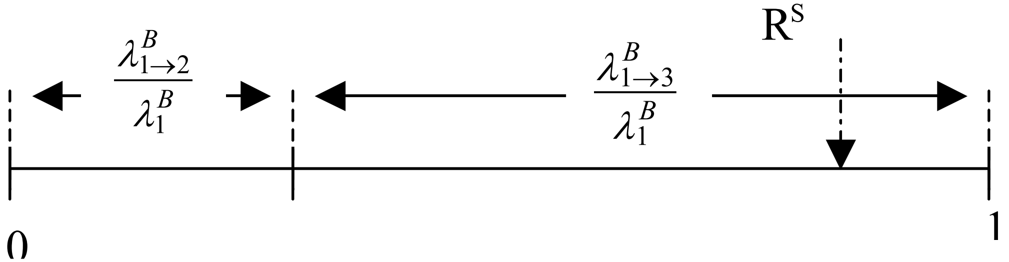


Sampling the Kind of Transition (2)

- Given that at t_1 component B undergoes a transition, its arrival state can be sampled by applying the inverse transform method to the set of discrete probabilities

$$\left\{ \frac{\lambda^B_{1\to2}}{\lambda^B_1}, \frac{\lambda^B_{1\to3}}{\lambda^B_1} \right\}$$

of the mutually exclusive and exhaustive arrival states



$R_S \sim U[0,1)$

- As a result of this first transition, at t_1 the system is operating in configuration (1,3,1).
- The simulation now proceeds to sampling the next transition time t_2 with the updated transition rate

$$\lambda^{(1,3,1)} = \lambda^A_1 + \lambda^B_3 + \lambda^C_1$$

Sampling the Next Transition

- The next transition, then, occurs at

$$t_2 = t_1 - \frac{1}{\lambda^{(1,3,1)}} \ln(1 - R_t)$$

where $R_t \sim U[0,1)$.

- Assuming again that $t_2 < T_M$, the component undergoing the transition and its final state are sampled as before by application of the inverse trasform method to the appropriate discrete probabilities.

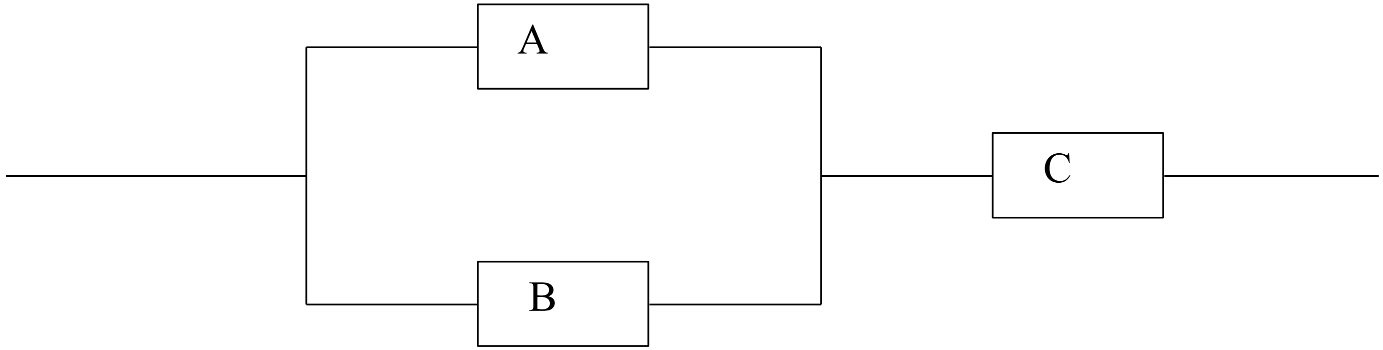
- The trial simulation then proceeds through the various transitions from one system configuration to another up to the mission time T_M.

Unreliability and Unavailability Estimation

- When the system enters a failed configuration (*,*,4) or (3,3,*), where the * denotes any state of the component, tallies are appropriately collected for the unreliability and instantaneous unavailability estimates (at discrete times $t_j \in [0, T_M]$);

- After performing a large number of trials M, we can obtain estimates of the system unreliability and instantaneous unavailability by simply dividing by M, the accumulated contents of $C^R(t_j)$ and $C_A(t_j)$, $t_j \in [0, T_M]$

Direct Monte Carlo: Example (1)



For any arbitrary trial, starting at t=0 with the system in nominal configuration (1,1,1) we would sample all the transition times:

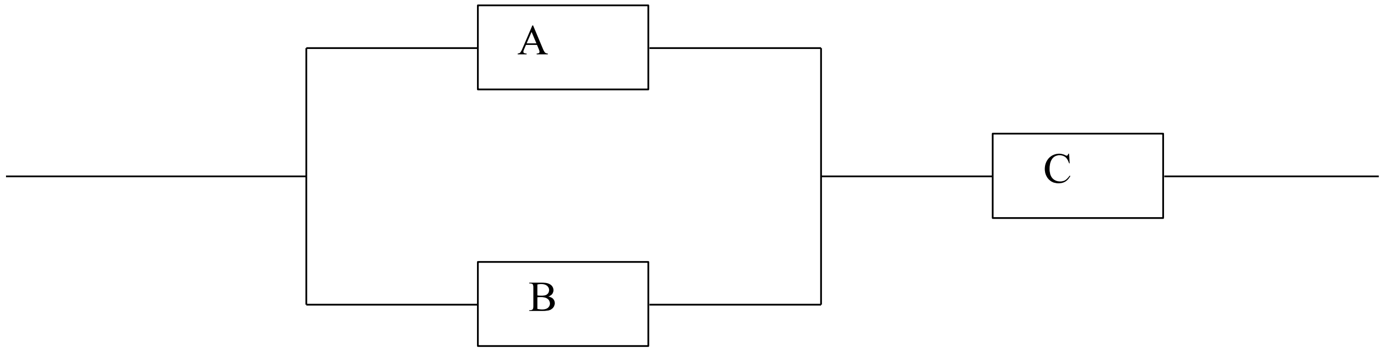
$$t^{i}_{1\to m_i} = t_0 - \frac{1}{\lambda^{i}_{1\to m_i}} \ln(1 - R^{i}_{t,1\to m_i})$$

$$\left.\begin{array}{lr} i = A, B, C & \\ m_i = 2,3 & \text{for } i = A, B \\ m_i = 2,3,4 & \text{for } \mathrm{i} = C \end{array}\right\}$$

where $R^{i}_{t,1\to m_i} \sim U[0,1)$

These transition times would then be ordered in ascending order from t_{min} to $t_{max} \leq T_M$. Let us assume that t_{min} corresponds to the transition of component A to state 3 of failure. The current time is moved to $t_1 = t_{min}$ in correspondence of which the system configuration changes, due to the occurring transition, to (3,1,1) still operational.

Direct Monte Carlo: Example (2)



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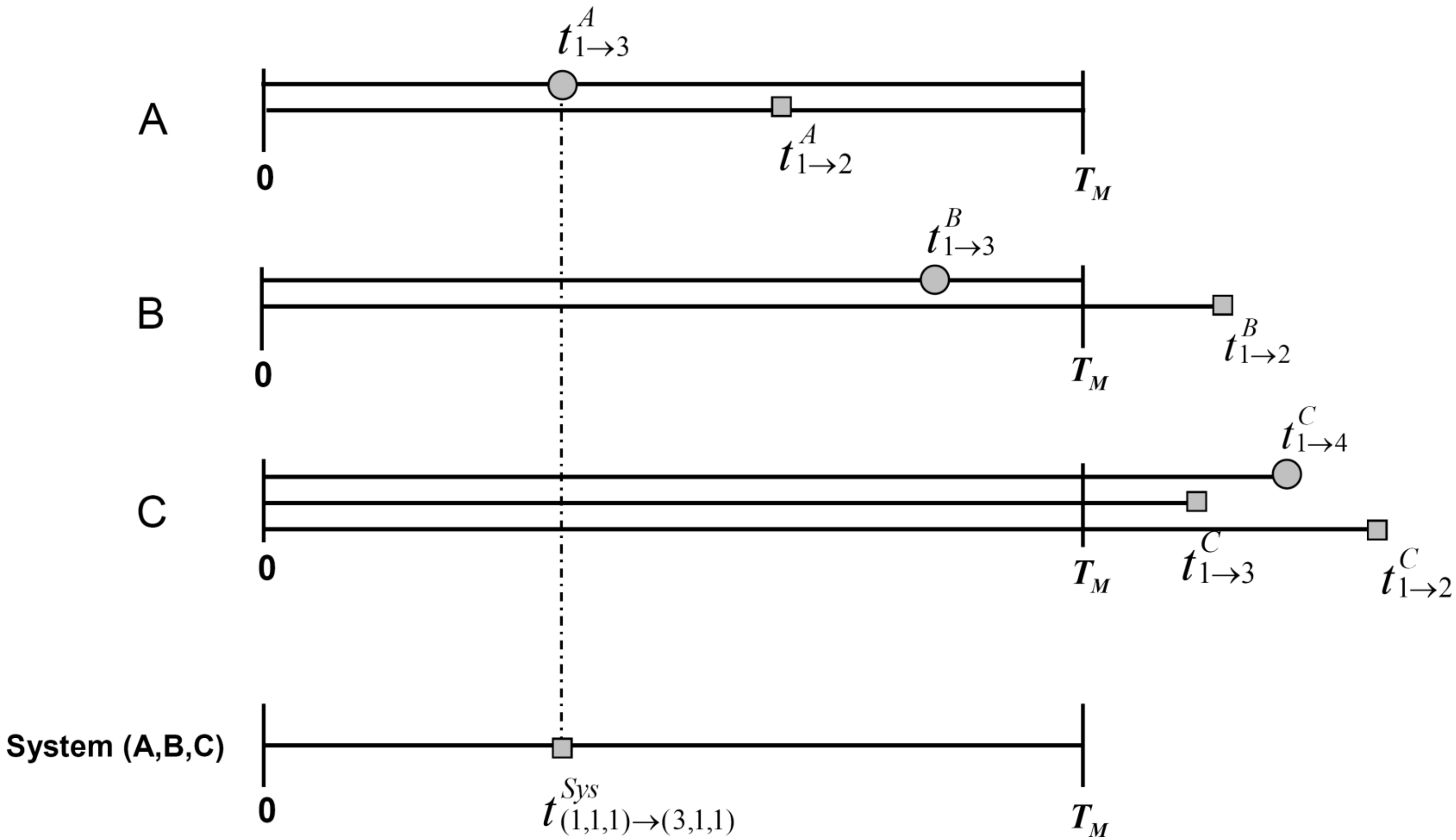
Example (1)


A
B
C
System (A,B,C)
0
T_M
$t^{A}_{1\to 3}$
$t^{A}_{1\to 2}$
$t^{B}_{1\to 3}$
$t^{B}_{1\to 2}$
$t^{C}_{1\to 4}$
$t^{C}_{1\to 3}$
$t^{C}_{1\to 2}$
$t^{Sys}_{(1,1,1)\to(3,1,1)}$

Example (2)

The new transition times of component A are then sampled

$$t^A_{3\to m_A} = t_1 - \frac{1}{\lambda^A_{3\to m_A}} \ln(1 - R^A_{t,3\to m_A}) \qquad k = 1,2 \qquad R^A_{t,3\to m_A} \sim U[0,1)$$

and placed at the proper position in the timeline of the succession of occurring transitions

- The simulation then proceeds to the successive times in the list, in correspondence of which a system transition occurs.
- After each transition, the timeline is updated with the times of the transitions that the component which has undergone the last transition can do from its new state.
- During the trial, each time the system enters a failed configuration, tallies are collected and in the end, after M trials, the unreliability and unavailability estimates are computed.

POLITECNICO DI MILANO

Availability Assessment of Oil and Gas Processing Plants Operating under Dynamic Arctic Weather Conditions

A real example of Direct MC Simulation

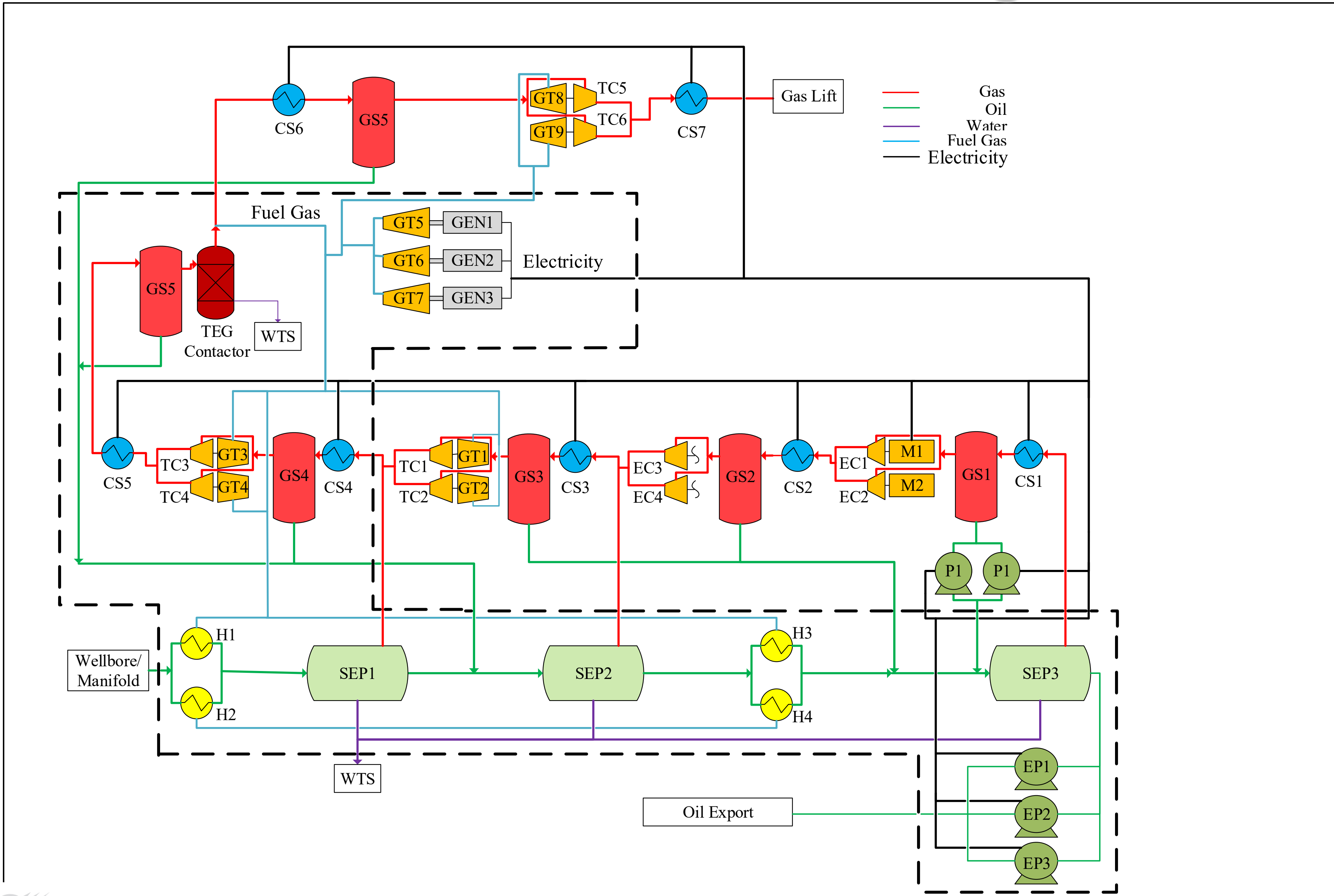
CENTRALE PARIS

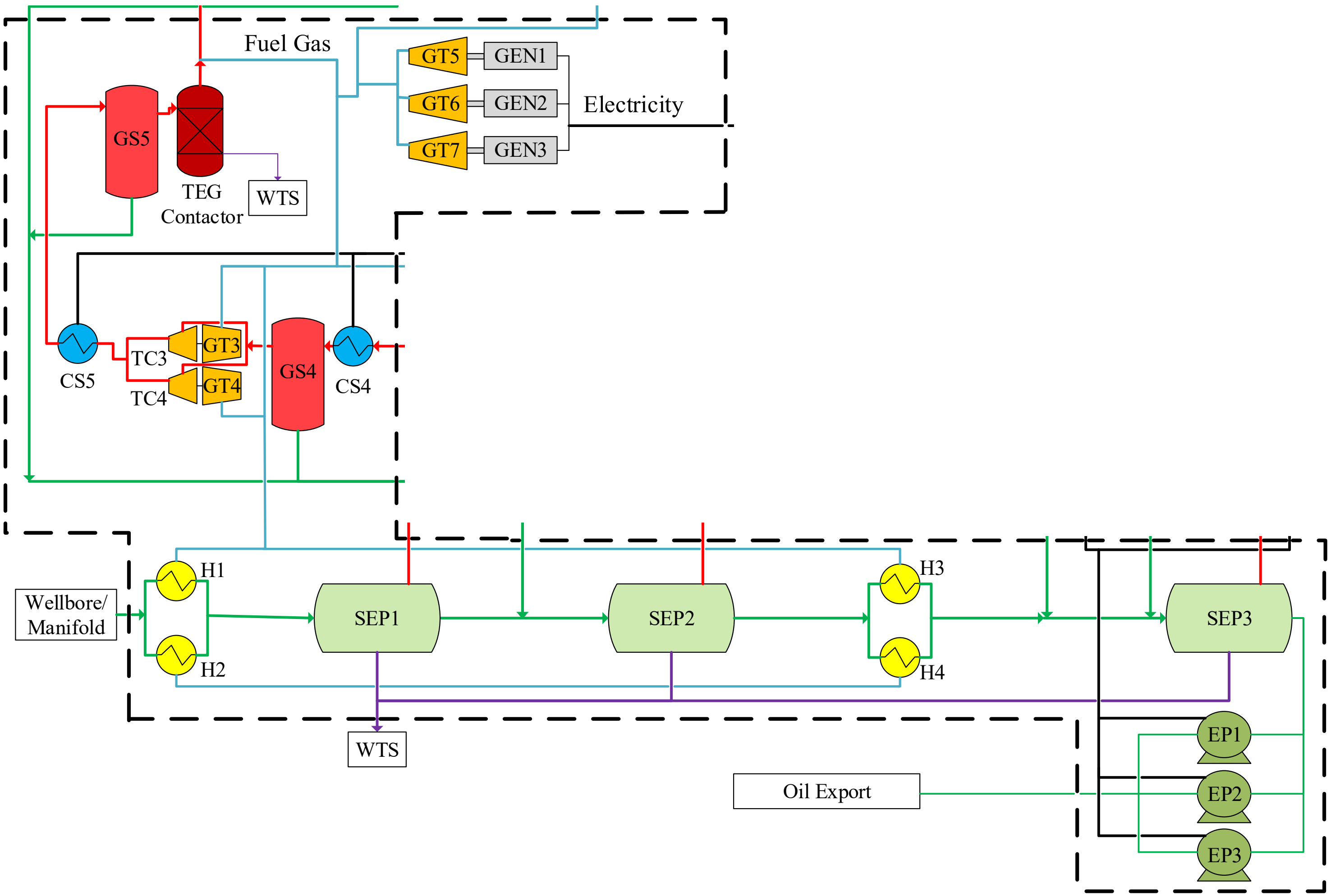
Supélec


МЛСП «ПРИРАЗЛОМНАЯ»

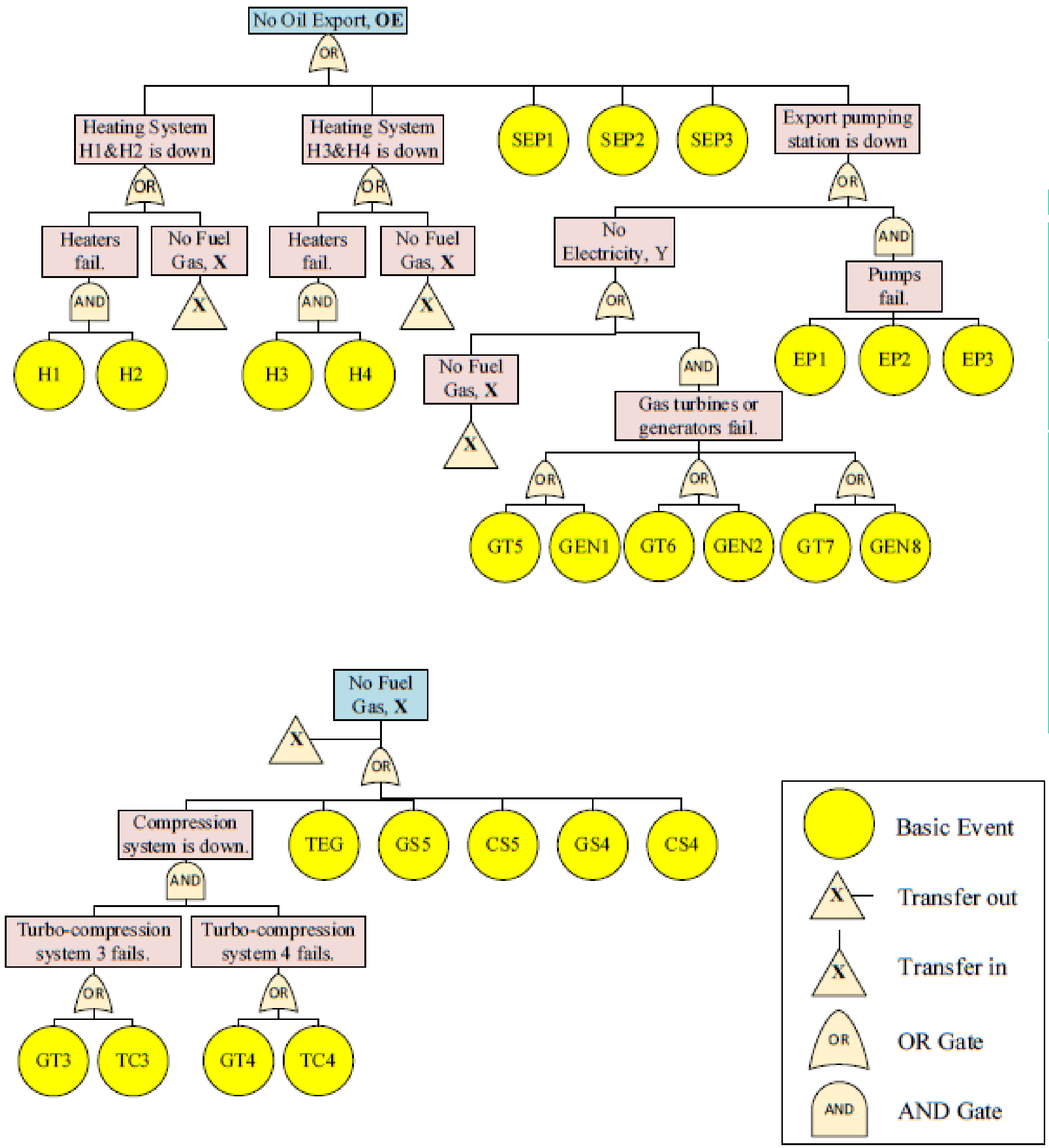

Hopen Island
Barents Sea
Norwegian Sea
North Sea, Ekofisk B
Sweden
Finland
Norway
Denmark
United Kingdom
Belarus

An Offshore Oil and Gas Processing Plant




Fuel Gas
GT5
GEN1
GT6
GEN2
Electricity
GT7
GEN3
GS5
TEG
Contactor
WTS
CS5
TC3
GT3
GT4
TC4
GS4
CS4
H1
Wellbore/
Manifold
H2
SEP1
SEP2
H3
H4
SEP3
WTS
Oil Export
EP1
EP2
EP3

System Failures and Minimal Cut Sets

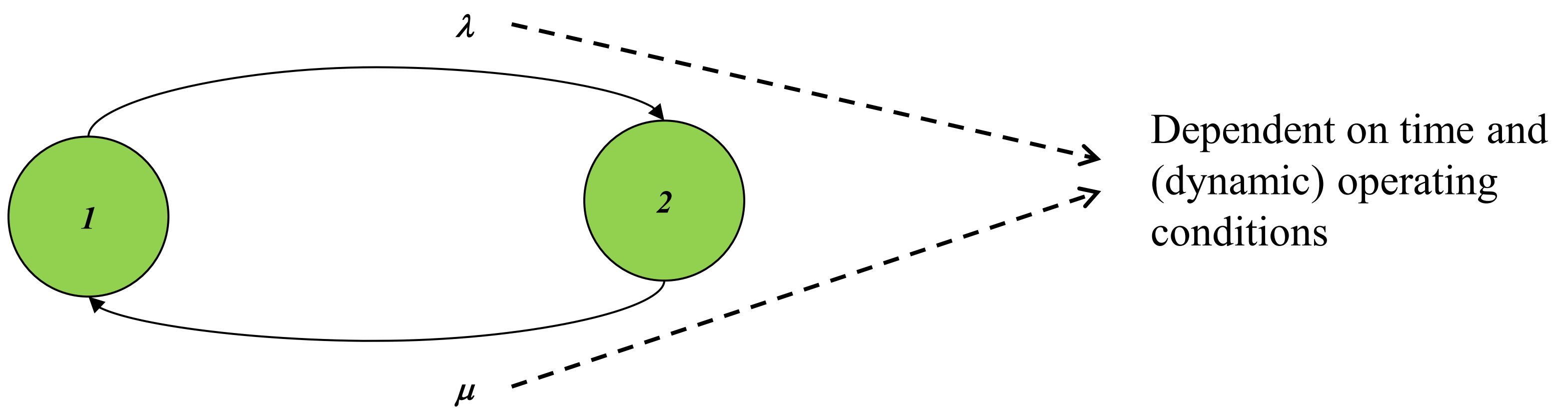


System Minimal Cut Sets

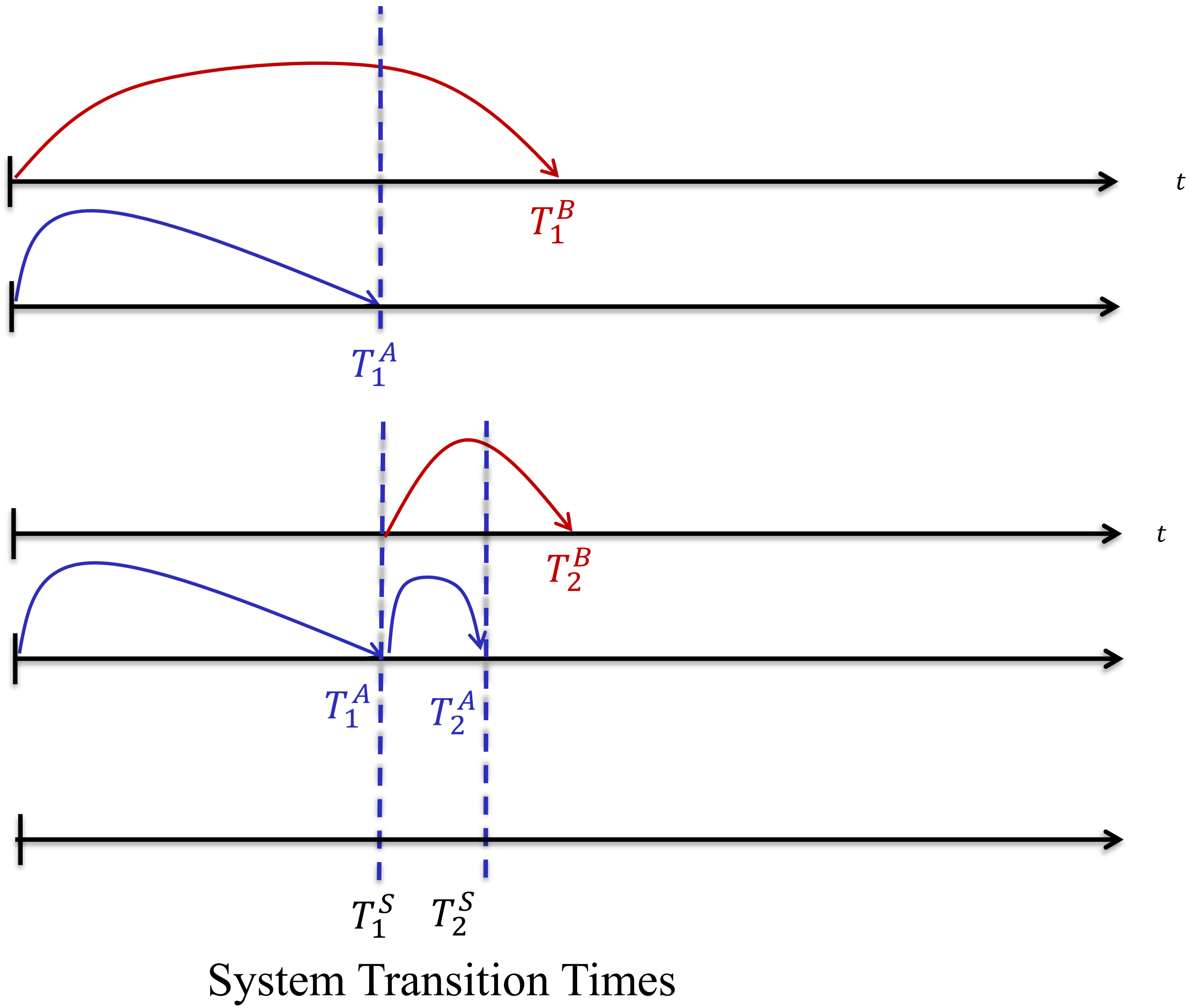
Order	Minimal cut sets	
1	SEP1	SEP2
	SEP3	TEG
	GS4	GS5
	CS4	CS5
2	H1 & H2	H3 & H4
	GT3 & GT4	GT3 & TC4
	TC3 & GT4	TC3 & TC4
3	EP1 & EP2 & EP3	GT5 & GT6 & GT7
	GT5 & GT6 & GEN3	GT5 & GEN2 & GT7
	GT5 & GEN2 & GEN3	GEN1 & GT6 & GT7
	GEN1 & GT6 & GEN3	GEN1 & GEN2 & GT7
	GEN1 & GEN2 & GEN3	

Component Reliability and Repair Data

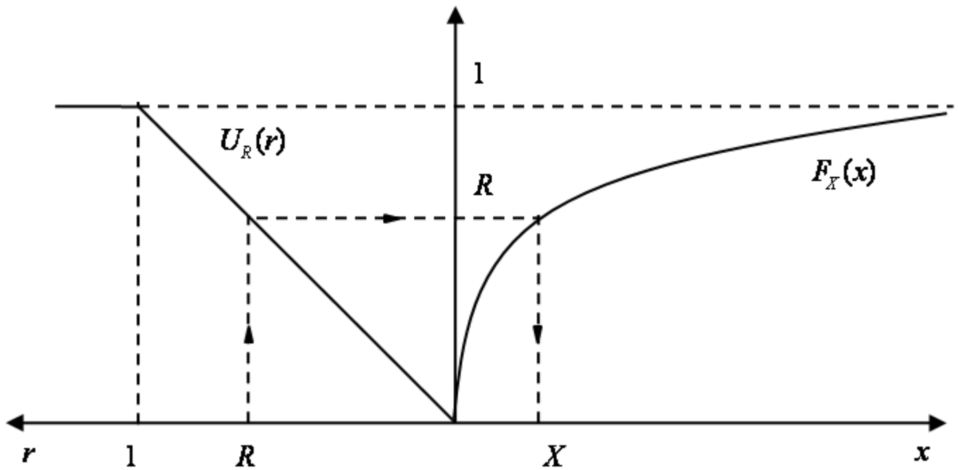
Component	ID	β_{i,L^0}	η_{i,L^0}, hr	Mean active repair time, hr
Separator	SEP1, SEP2, SEP3	0.7621	22620	5.1
Gas scrubber	GS4, GS5	0.8685	31837	5.1
Triethylene glycol contactor	TEG	1.2348	13082	13
Export pump	EP1, EP2, EP3	1.1722	5182	14
Crude oil heater	H1, H2, H3, H4	1.039	10557	2.8
Cooling system	CS4, CS5	1.2963	55535	4.2
Turbine-driven generator	GEN1, GEN2, GEN3	0.8901	15735	20
Gas turbine	GT3, GT4, GT5, GT6, GT7	1.4841	2615	26
Turbo-compressor	TC3, TC4	1.0786	9126	5.2



Direct Monte Carlo Sampling



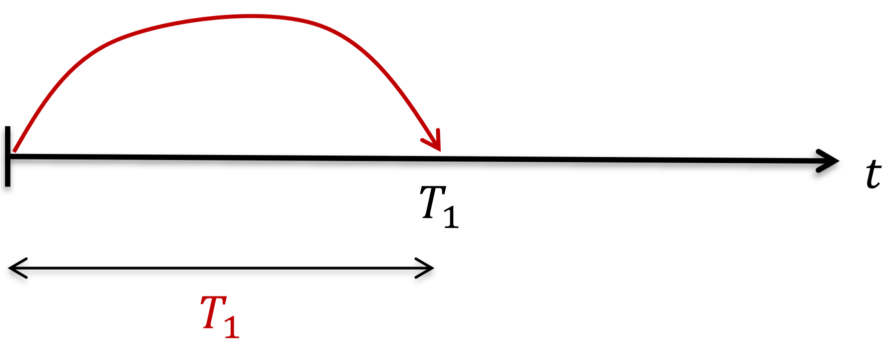
Sampling Random Numbers from $F_X(x)$



Weibull distribution:

$$F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

$$T = F_T^{-1}(R) = \eta[-\ln(1-R)]^{\frac{1}{\beta}}$$

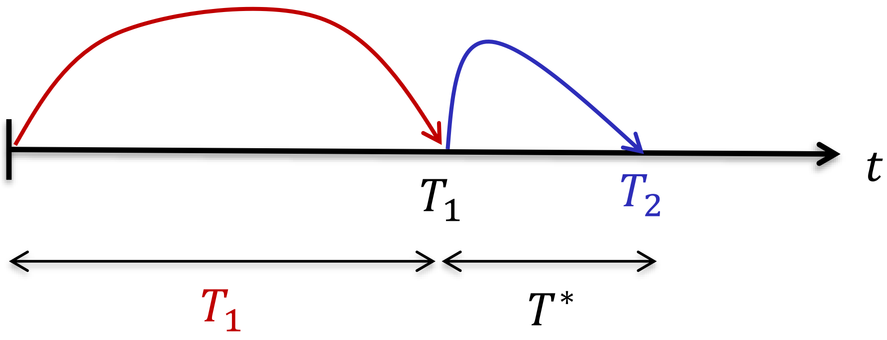


$$T_1 = \eta[-\ln(1-R)]^{\frac{1}{\beta}}$$

Sampling Random Numbers from $F_X(x)$

Suppose that the component has **survived until** $\boldsymbol{T_1}$**:** Sampling T^*

Assuming Weibull-distruibuted TTFs



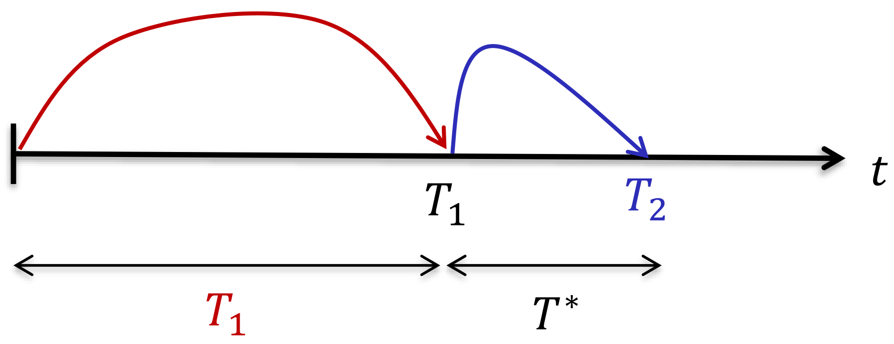
Sampling Random Numbers from $F_X(x)$

Suppose that the component has **survived until** T_1**:** Sampling T^*

Assuming Weibull-distruibuted TTFs

$$F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

$$R(t) = P(T > t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$



$$P(T_1 < T \leq T_1 + T^* | T > T_1) = \frac{F(T_1 + T^*) - F(T_1)}{R(T_1)}$$

$$P(T_1 < T \leq T_1 + T^* | T > T_1) = 1 - \exp\left(-\left[\left(\frac{T_1 + T^*}{\eta}\right)^{\beta} - \left(\frac{T_1}{\eta}\right)^{\beta}\right]\right)$$

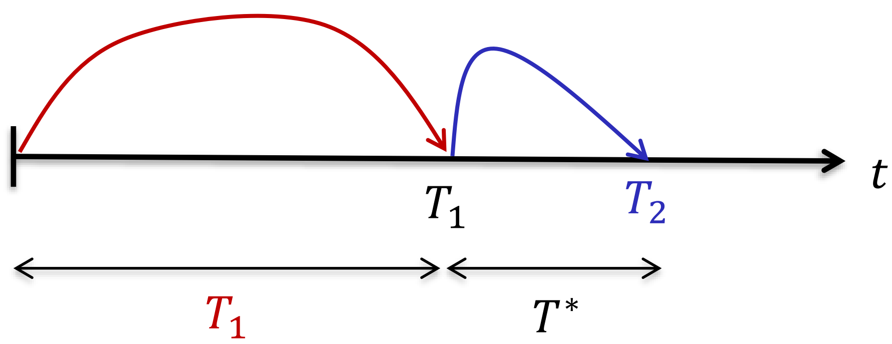
$$T^* = \eta\left[-\ln(1 - R) + \left(\frac{T_1}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} - T_1$$

$$T_2 = T^* + T_1$$

Sampling Random Numbers from $F_X(x)$

Suppose that the component has **survived until T_1:** sampling T^*

Assuming Exponentially-distributed TTFs (i.e., memoryless property)



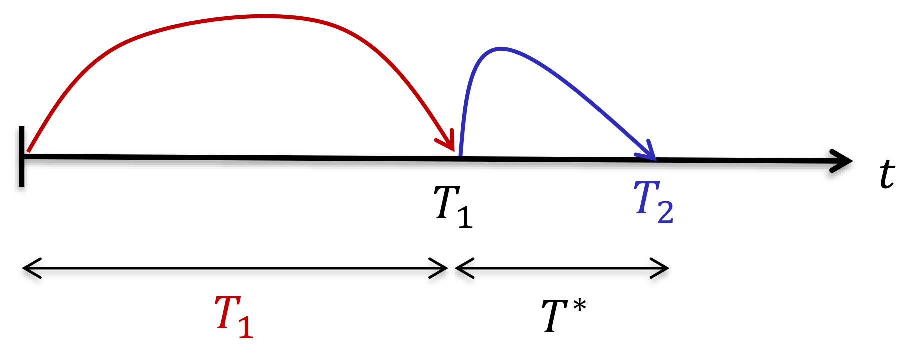
Sampling Random Numbers from $F_X(x)$

Suppose that the component has **survived until T_1:** sampling T^*

Assuming Exponentially-distributed TTFs (i.e., memoryless property)

$$F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

$$\beta = 1 \rightarrow Exponential$$



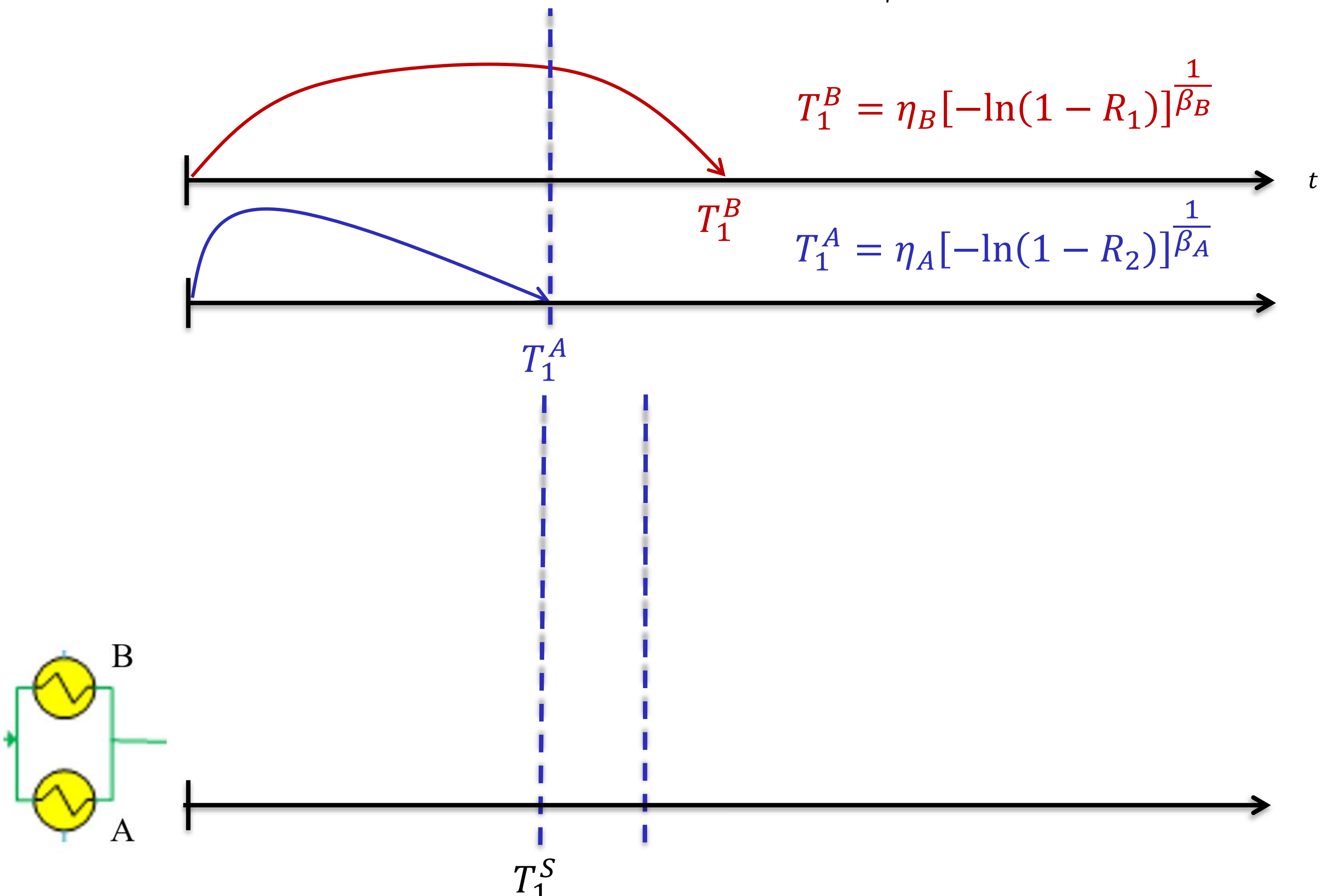
$$T^* = \eta\left[-\ln(1-R) + \left(\frac{T_1}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}} - T_1$$

$$\beta = 1$$

$$T^* = -\eta \ln(1-R)$$

Direct Monte Carlo Sampling

Weibull distribution: $F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$ ⇨ $T = F_T^{-1}(R) = \eta[-\ln(1-R)]^{\frac{1}{\beta}}$



Direct Monte Carlo Sampling

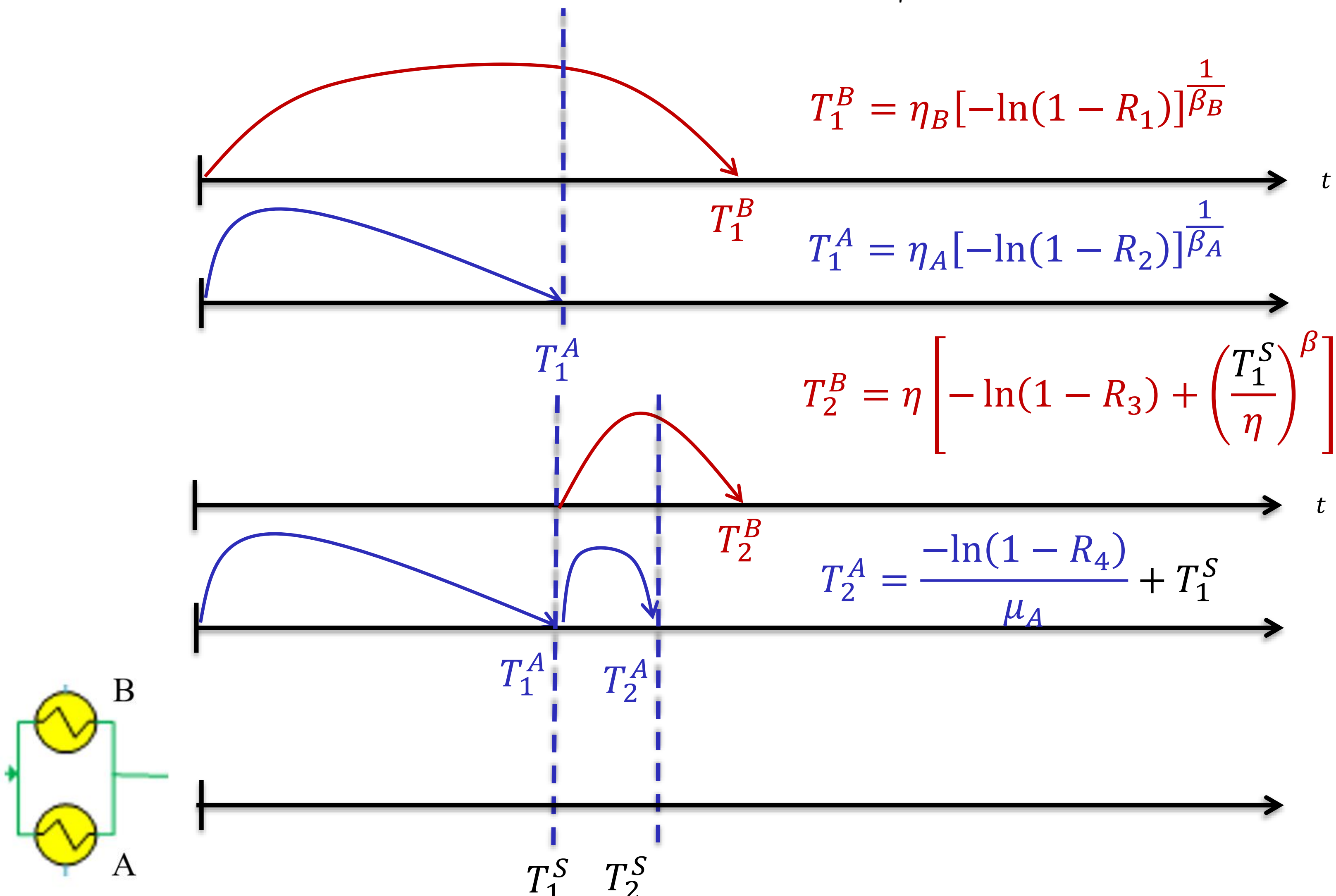
Weibull distribution: $F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$ ⇨ $T = F_T^{-1}(R) = \eta[-\ln(1-R)]^{\frac{1}{\beta}}$

$T_1^B = \eta_B[-\ln(1-R_1)]^{\frac{1}{\beta_B}}$

$T_1^A = \eta_A[-\ln(1-R_2)]^{\frac{1}{\beta_A}}$

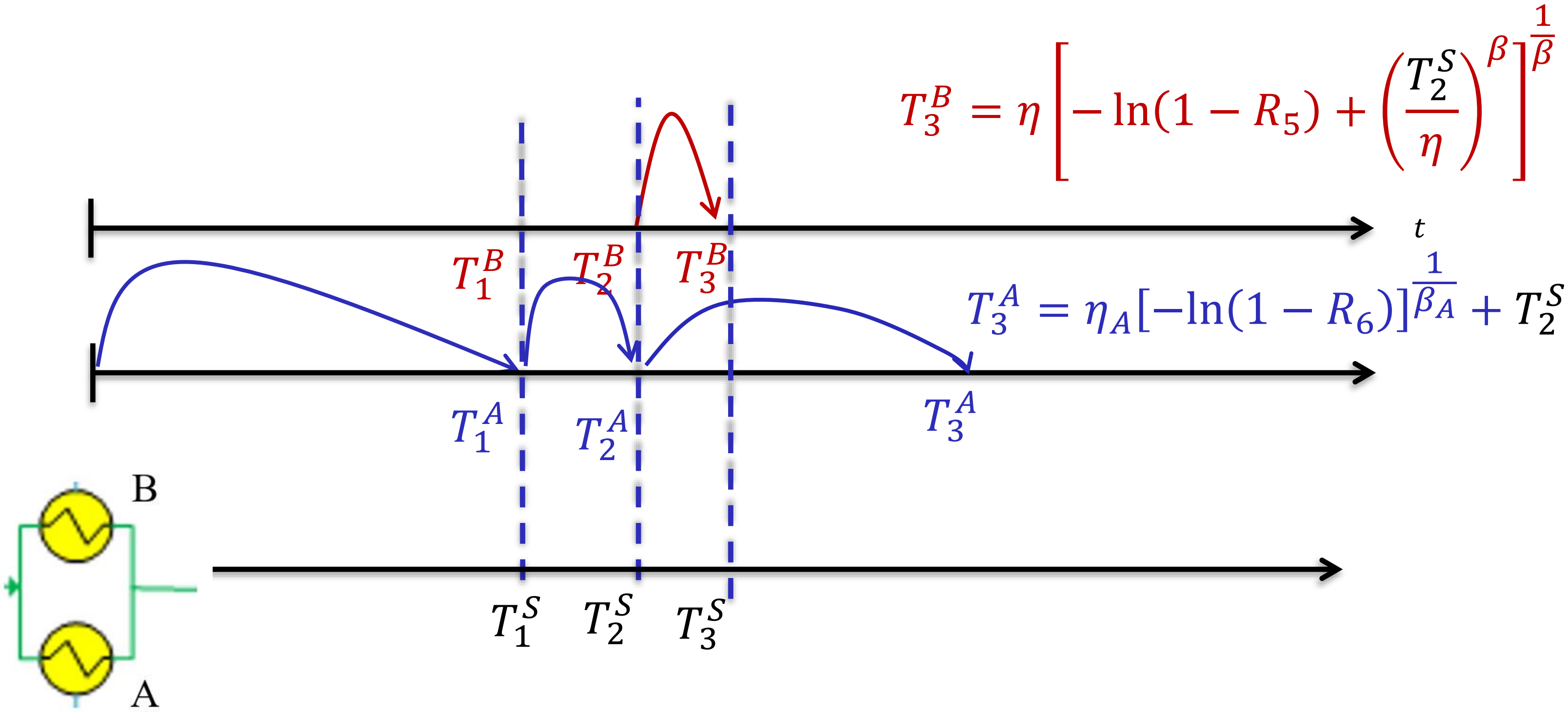
$T_2^B = \eta\left[-\ln(1-R_3) + \left(\frac{T_1^S}{\eta}\right)^{\beta}\right]^{\frac{1}{\beta}}$

$T_2^A = \frac{-\ln(1-R_4)}{\mu_A} + T_1^S$

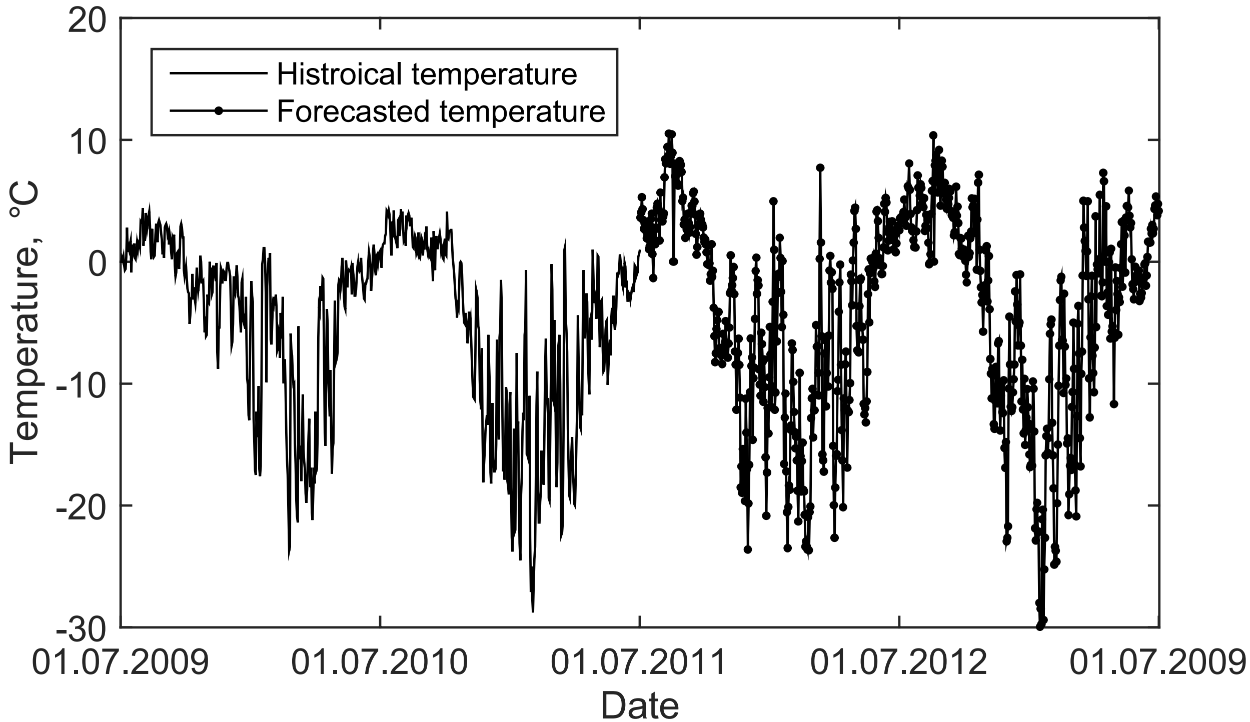


Sampling Random Numbers from $F_X(x)$

Weibull distribution: $F_T(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$ ⇒ $T = F_T^{-1}(R) = \eta[-\ln(1-R)]^{\frac{1}{\beta}}$



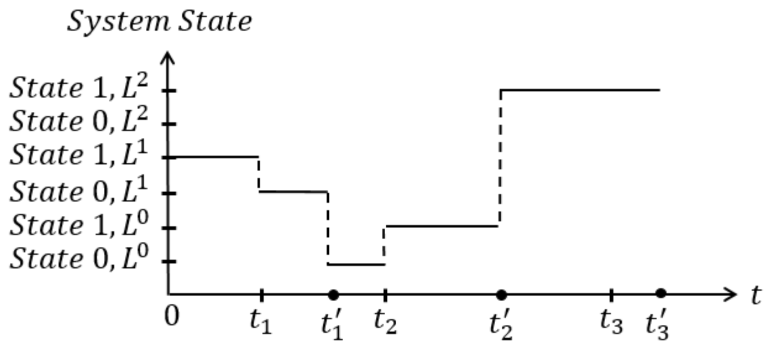
Dynamic operating conditions



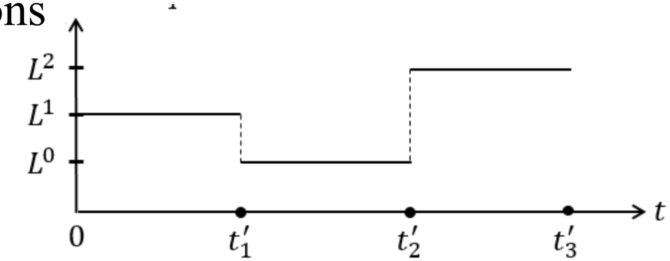
Historical Data → Forecasting Model → Future prediction + Random Noise

Dynamic operating conditions

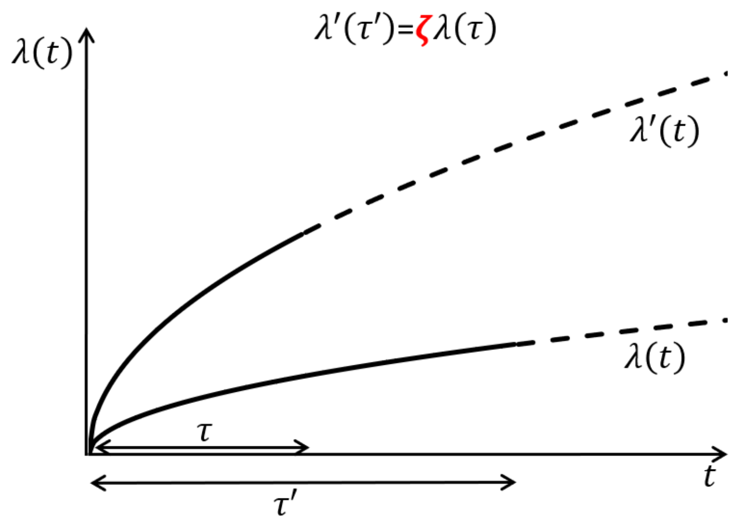
Equivalent Age: component operates under dynamic conditions



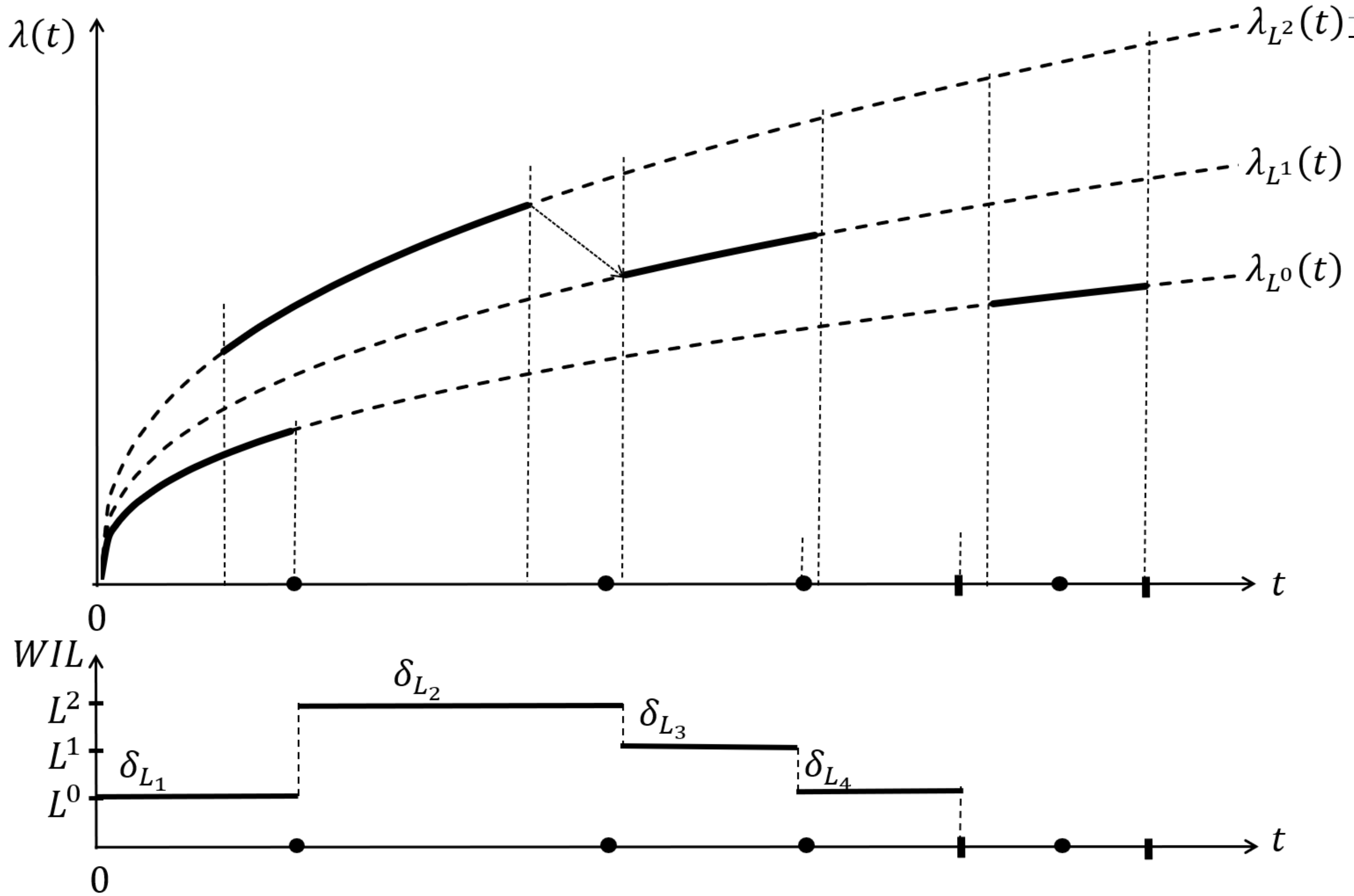
Operating conditions



Equivalent Age



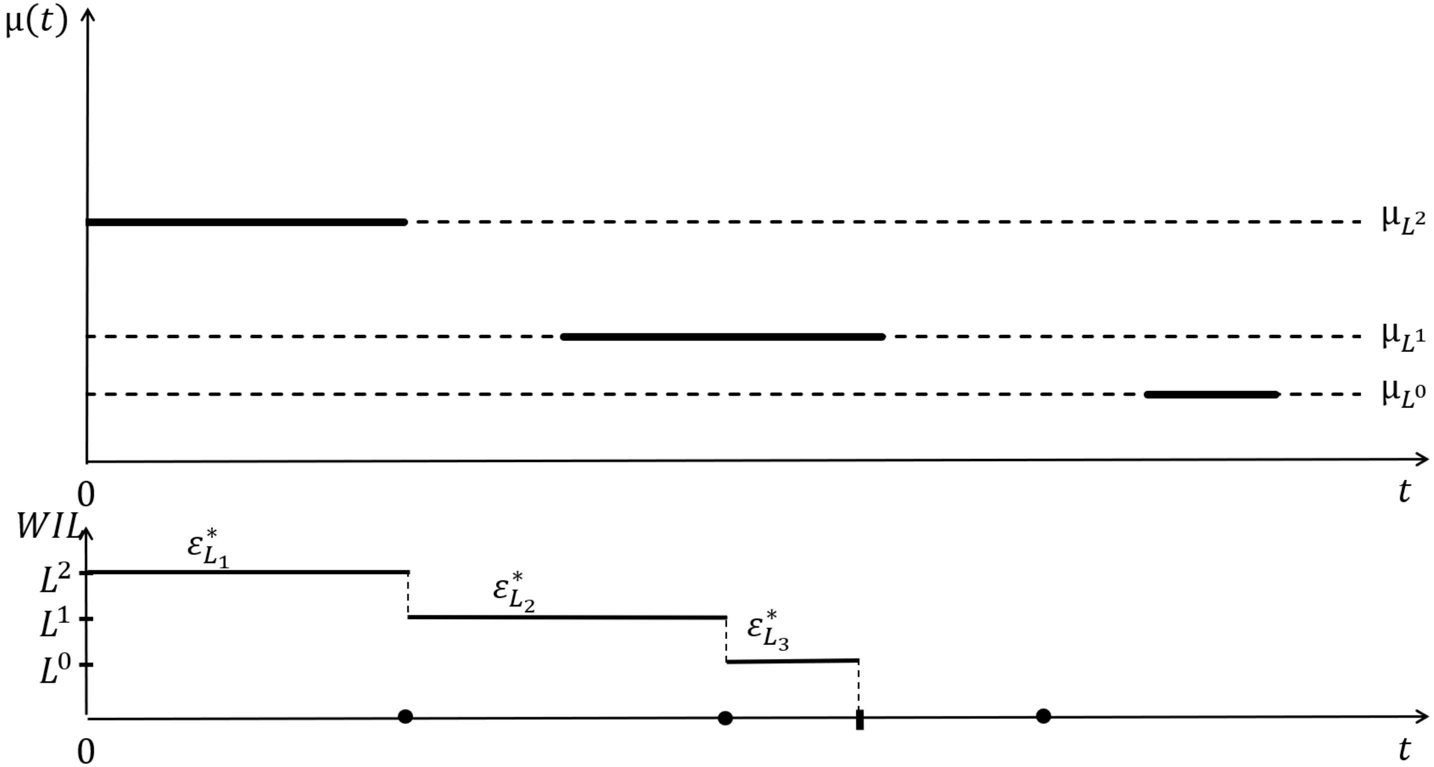
$$TTF_{L^j} = \delta_{L^j} TTF_{L^0}$$



Piecewise Weibull hazard rate under dynamic weather conditions

$$TTF_{L^j} = \delta_{L^j} TTF_{L^0}$$

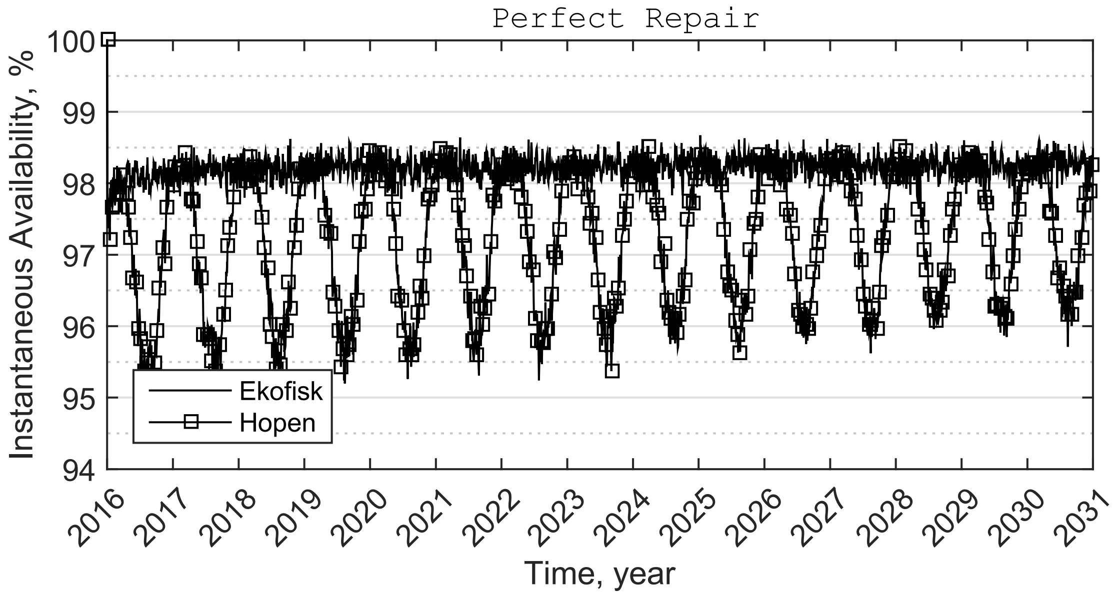
$$\delta_{L_K,i} = \begin{cases} \delta_{L^0,i} & if \quad 1°C \leq T_K \\ \delta_{L^1,i} & if \quad -5°C \leq T_K < 1 \\ \delta_{L^2,i} & if \quad -10°C \leq T_K < -5°C \\ \delta_{L^3,i} & if \quad -20°C \leq T_K < -10°C \\ \delta_{L^4,i} & if \quad T_K < -20°C \end{cases}; \quad \begin{cases} K = 1,2,\dots,5475 \\ i = 1,2,\dots,25 \end{cases}$$



Piecewise constant hazard rate under dynamic weather conditions (Wind Chill Index)

$$TTR_{L^j} = \varepsilon_{L^j} TTR_{L^0} \qquad \varepsilon_{L_K,i} = \begin{cases} \varepsilon_{L^0,i} & if \quad -7 \leq WCT_K \\ \varepsilon_{L^1,i} & if \quad -15 \leq WCT_K < -7 \\ \varepsilon_{L^2,i} & if \quad -25 \leq WCT_K < -15 \\ \varepsilon_{L^3,i} & if \quad WCT_K < -25 \end{cases}; \quad \begin{cases} K = 1,2,\ldots,5475 \\ i = 1,2,\ldots,25 \end{cases}$$

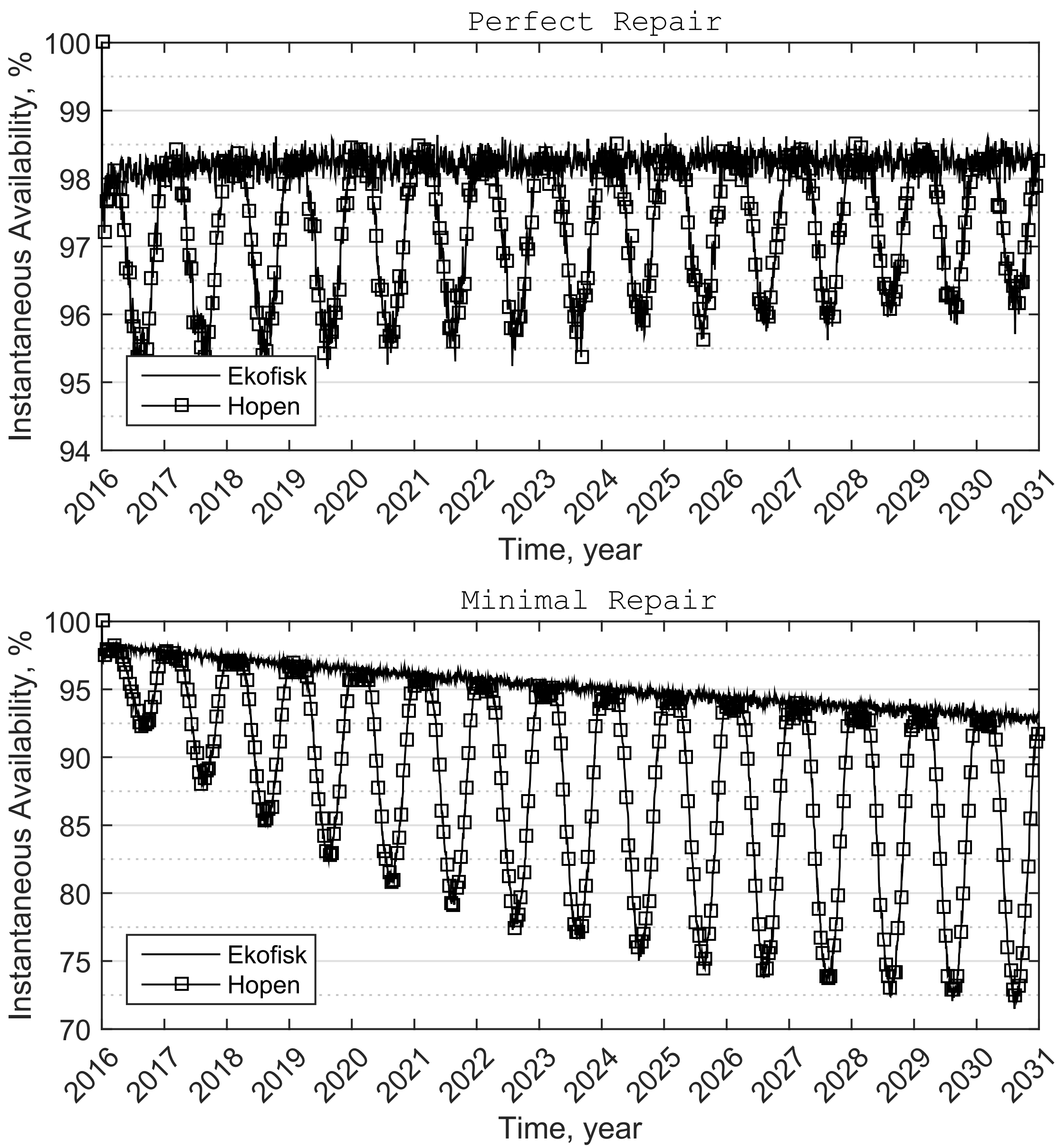
System Availability



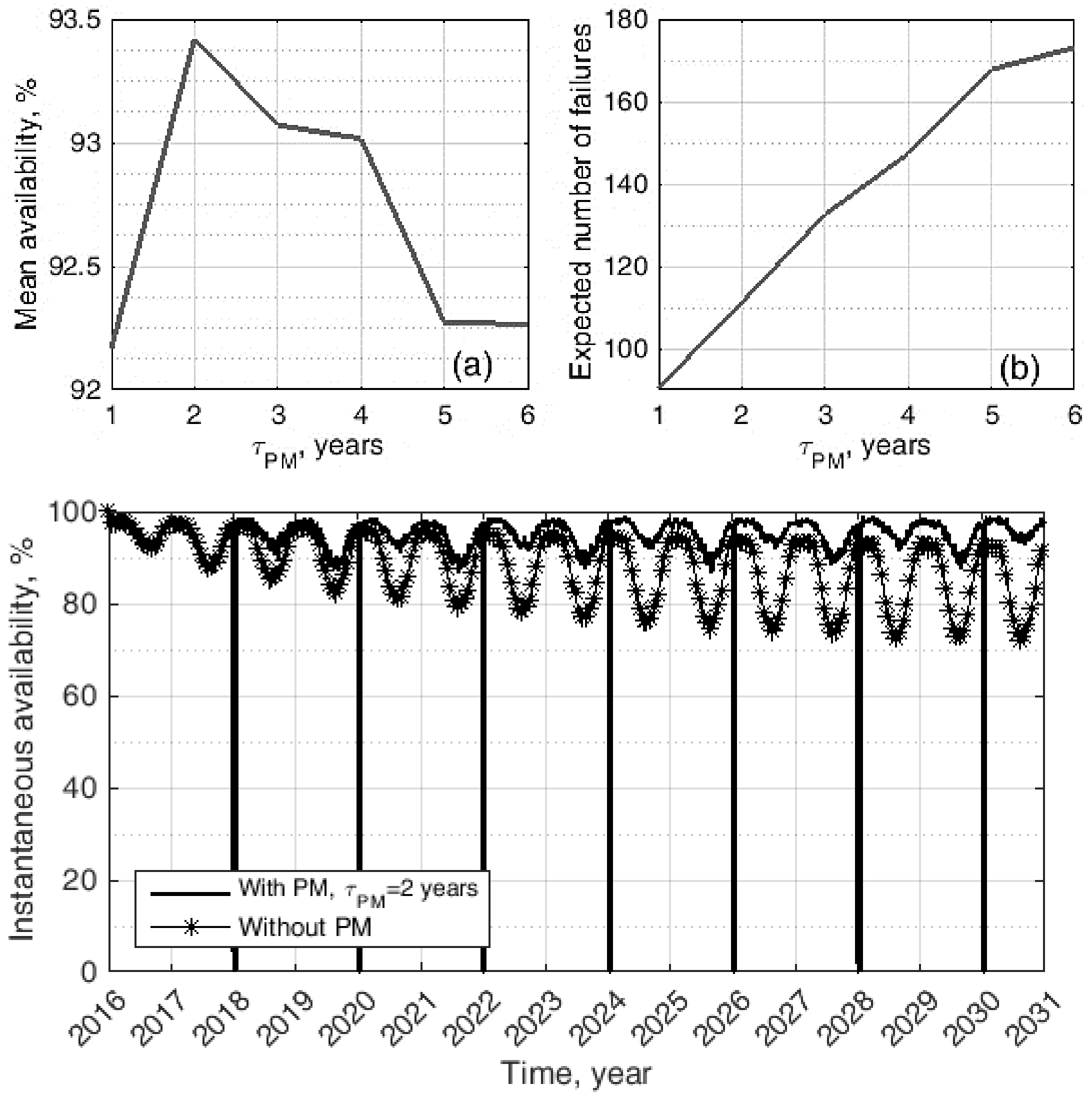
15 years, starting from 01.07.2016

Plant location	Perfect repair		Minimal repair	
	Mean Availability, %	Expected No. of Failure	Mean Availability, %	Expected No. of Failure
Hopen	97.272 ± 0.005	59.89	88.407 ± 0.009	288.78
Ekofisk	98.229 ± 0.004	43.85	95.289 ± 0.005	151.76

System Availability

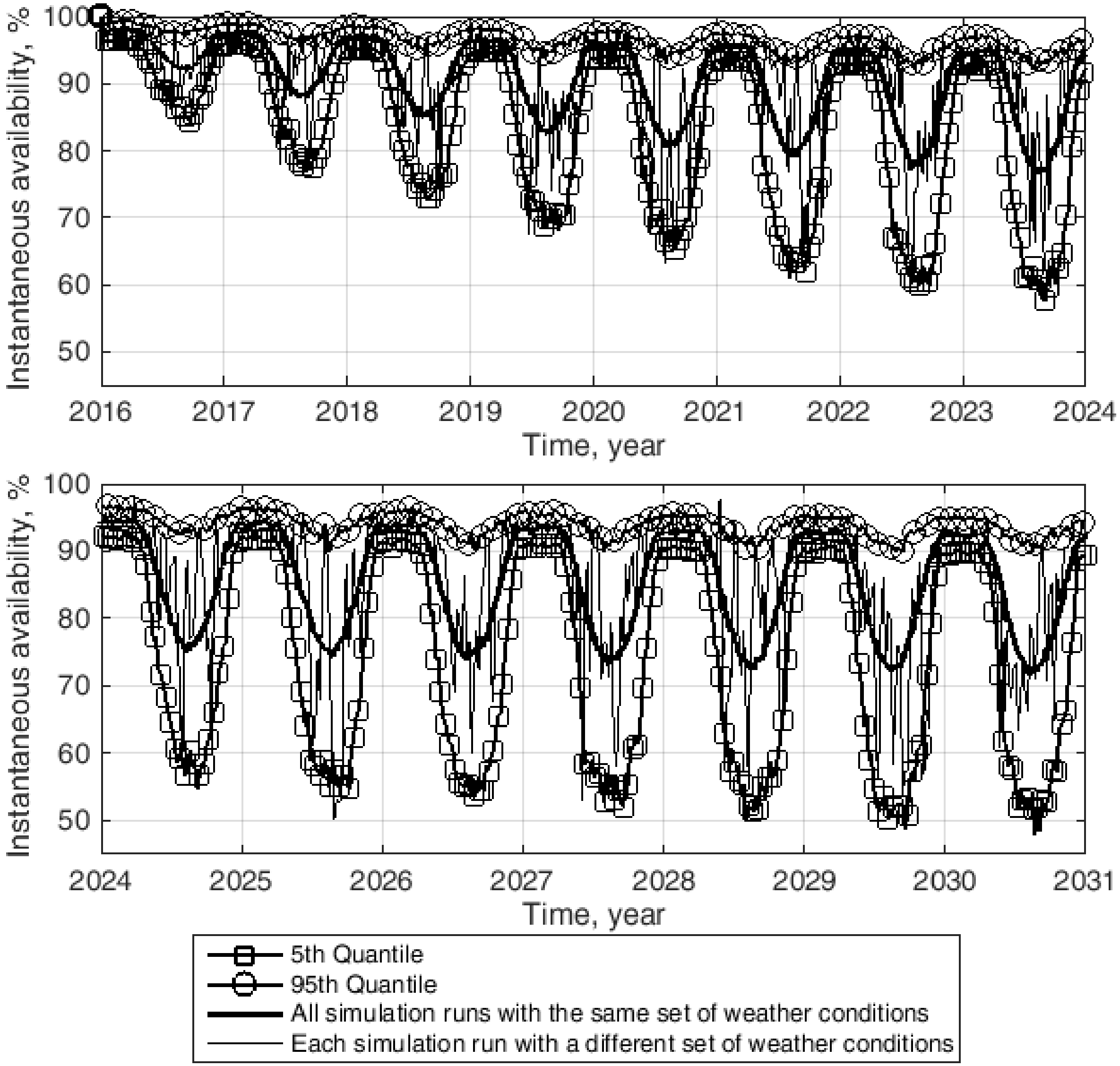


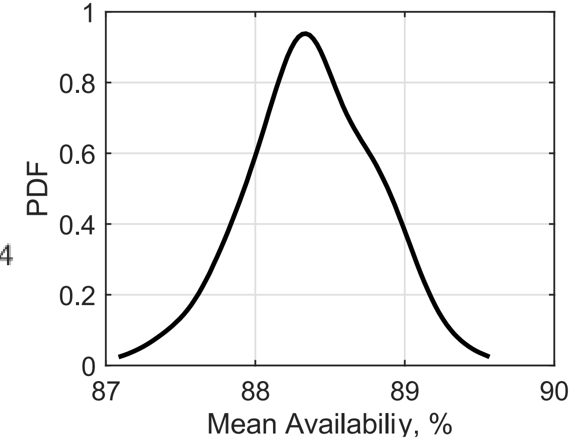
Plant Overhaul



Plant instantaneous availability with and without PM

Operating Conditions Uncertainty





Plant mean availability in the presence of uncertainties associated with weather condition forecasts

Conclusions

- Complex multi-state system
- Repair and failure rates dependent on dynamic operating conditions

↓

MC simulation

- *Stochastic evolution of environmental and operating conditions (Weather condition forecast)*
- *System state evolution*

- Minimal and perfect repair
- Overhaul and PM

POLITECNICO DI MILANO

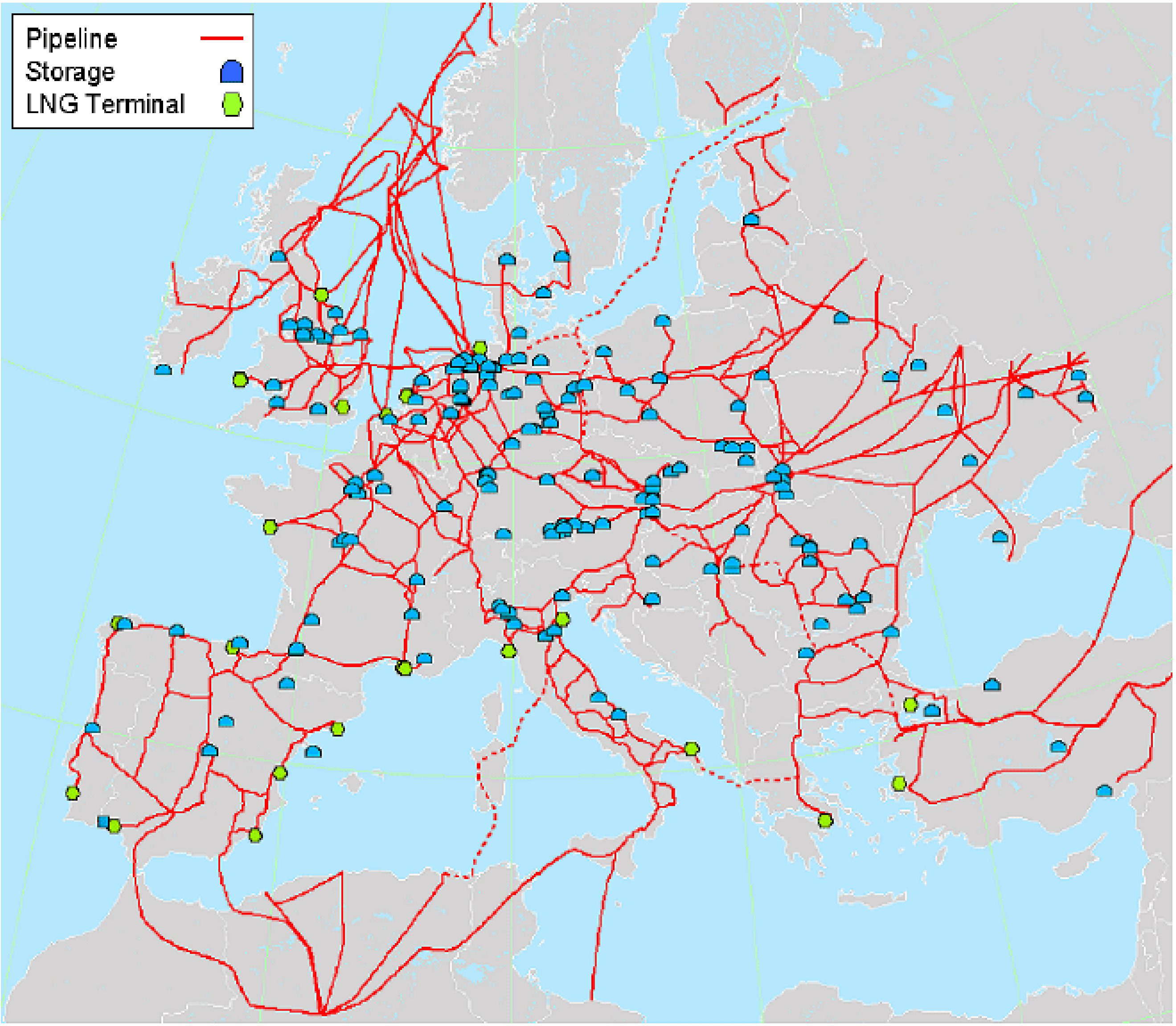
Supply Availability of a Natural Gas Transmission Pipeline Network

A real example of Direct MC Simulation

CENTRALE PARIS

Supélec

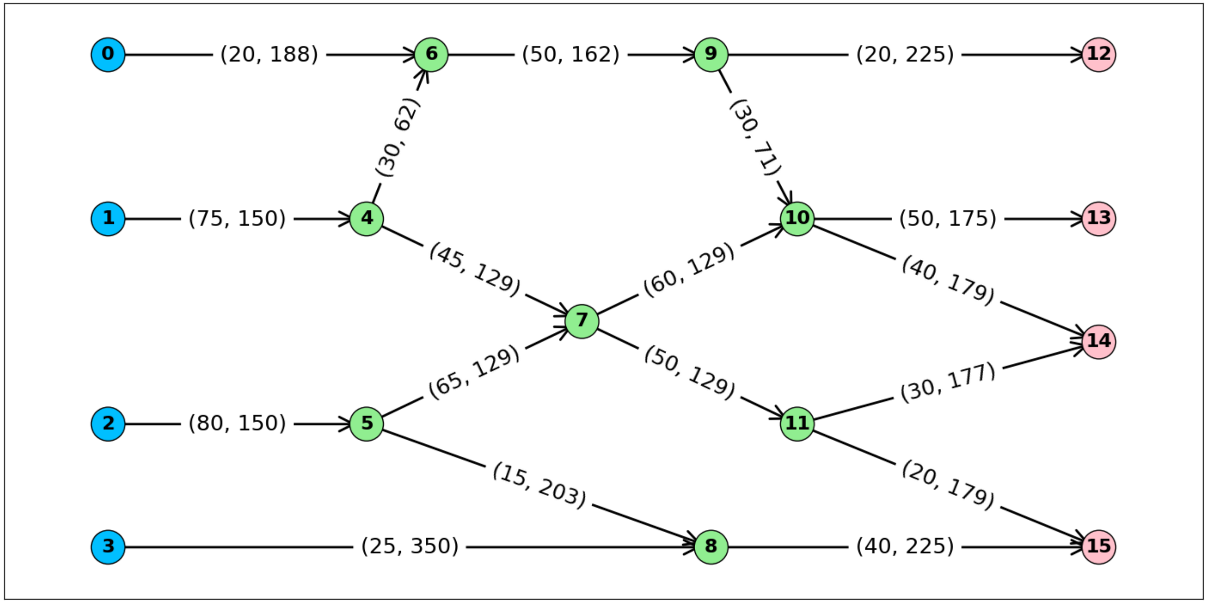
European Gas Pipeline Network



A gas transmission pipeline network

Gas-receiving terminals	Demand MMCMD
12	20
13	50
14	70
15	60

Production Facilities	Production MMCMD
12	20
13	75
14	80
15	25



- In system reliability and availability: R(t) and A(t)
- Supply availability: Often expressed as the amount of the average supply/production at a given time (e.g., flow Q(t))

Failure modes and transition rates

✓ 18 components (pipelines), each with 5 states

Pipeline states:

- Operational
- Small leakage
- Large leakage
- Rupture
- Other failures

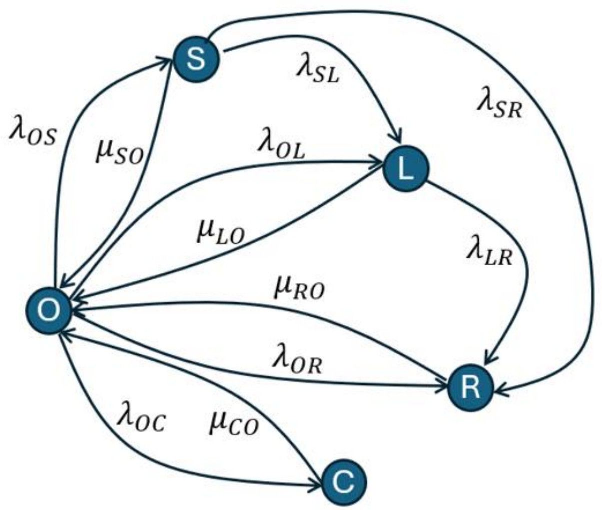
Pipeline capacity depends on its health state

$$c_{i,j}^{t} = \delta\left(s_{i,j}^{t}\right)c_{i,j}$$

$$r_{i,j} = \begin{array}{c} \\ \\ \end{array}\begin{matrix} & O & S & L & R & Ot & \\ & & & & & & \end{matrix}$$

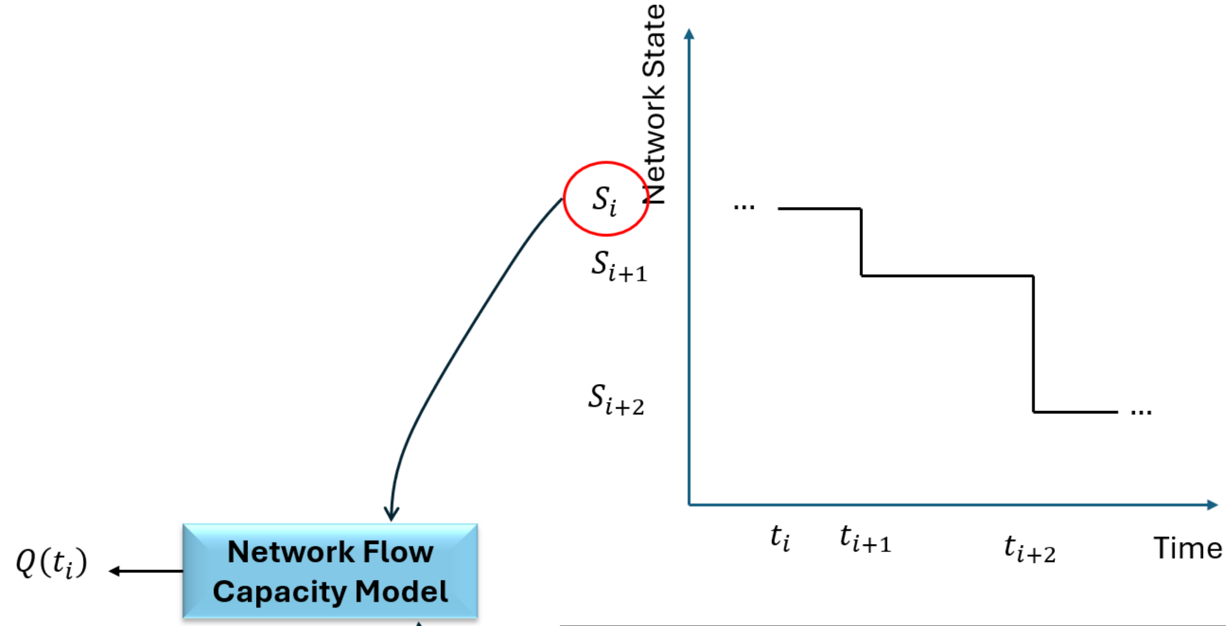
$$r_{i,j} = \begin{bmatrix} - & \lambda_{OS}^{i,j} & \lambda_{OL}^{i,j} & \lambda_{OR}^{i,j} & \lambda_{OC}^{i,j} \\ \mu_{SO}^{i,j} & - & \lambda_{SL}^{i,j} & \lambda_{SR}^{i,j} & 0 \\ \mu_{LO}^{i,j} & 0 & - & \lambda_{LR}^{i,j} & 0 \\ \mu_{RO}^{i,j} & 0 & 0 & - & 0 \\ \mu_{CO}^{i,j} & 0 & 0 & 0 & - \end{bmatrix} \begin{matrix} O \\ S \\ L \\ R \\ Ot \end{matrix}$$

(columns: O, S, L, R, Ot)



$s \rightarrow s'$	(# failures/h)	$s \rightarrow s'$	(# failures/h)
$\lambda_{OS}^{i,j}$	1.9250E-08$L_{i,j}$	λ_{SL}	3.7202E-04
$\lambda_{OL}^{i,j}$	1.1000E-08$L_{i,j}$	λ_{SR}	3.7202E-04
$\lambda_{OR}^{i,j}$	5.500E-08$L_{i,j}$	λ_{LR}	1.3228E-04
$\lambda_{OC}^{i,j}$	1.9250E-08$L_{i,j}$	μ_{SO}	2.9762E-3
μ_{LO}	1.1905E-3	μ_{RO}	9.9206E-4
μ_{CO}	9.9206E-4		

Network Flow Capacity



- Network functional and physical characteristics
- Production and demand rates
- Cost parameters
- Other requirements (e.g., user importance)

Simulation # 1: $\{Q_1(t_0), Q_1(t_1), \ldots, Q_1(t_N)\}$

$\vdots \qquad \vdots$

Simulation # M: $\{Q_M(t'_0), Q_M(t'_1), \ldots, Q_M(t'_0)\}$

$$Q(\tau) = \frac{\sum_{m=1}^{M} Q_m(\tau)}{M}$$

Network Flow Capacity

Network Flow Capacity Model

Transportation Cost — *Unmet Demand Penalty* — *Dummy Cost*

$$\min_{\overrightarrow{f^t}} g\left(\overrightarrow{f^t}\right);\ g\left(\overrightarrow{f^t}\right) = \sum_{(i,j)\in E} c_{i,j}^{T} L_{i,j} f_{i,j}^{t} + \sum_{j\in V^D} c_j^{P_D}\left(d_j^t - \sum_{i:(i,j)\in E} f_{i,j}^t\right) + c^{P_S}\sum_{i\in V^S}\left(s_i^t - \sum_{j:(i,j)\in E} f_{i,j}^t\right)$$

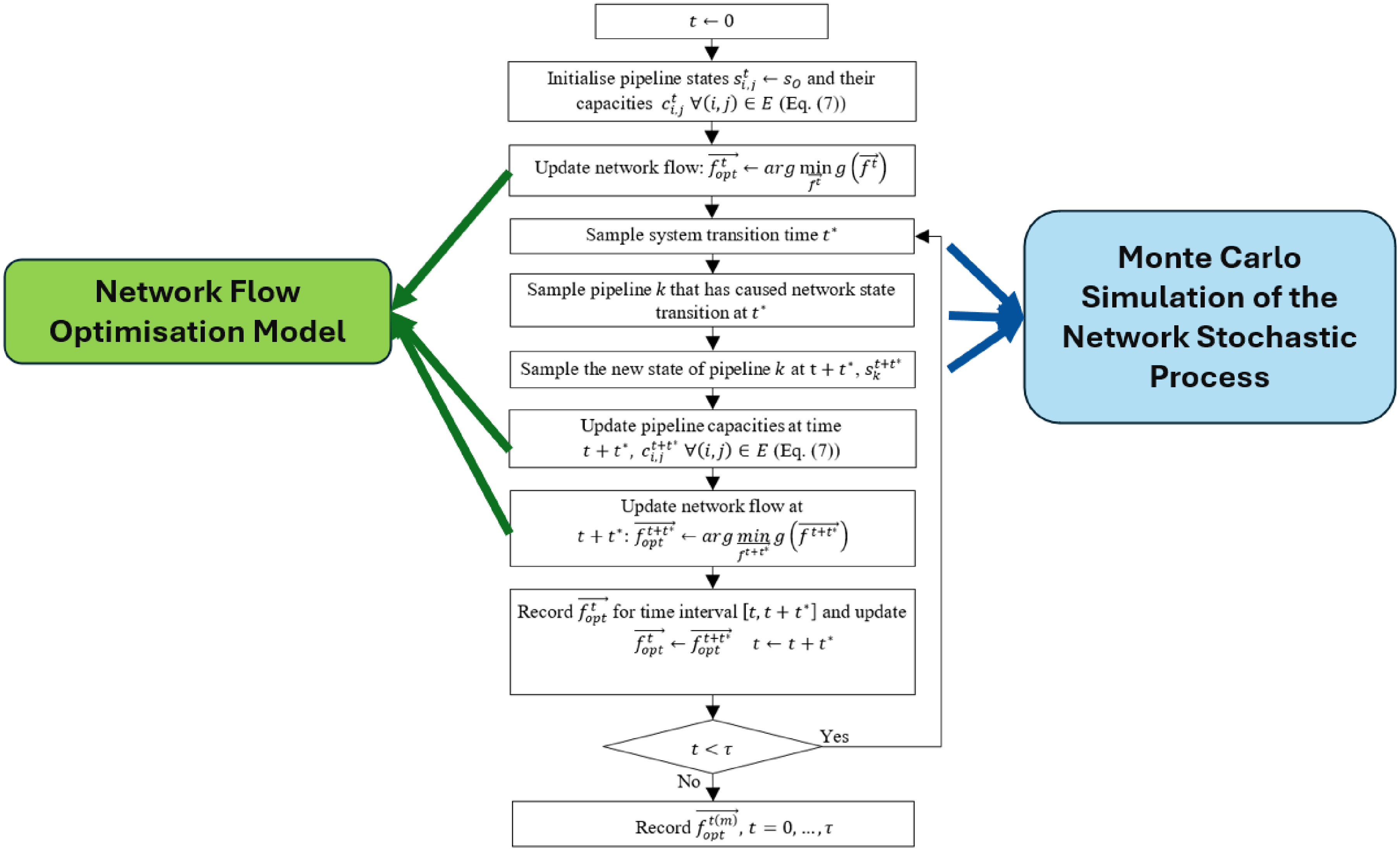
$$\sum_{j\in \mathrm{Pred}(i)} f_{j,i}^t = \sum_{k\in \mathrm{Succ}(i)} f_{i,k}^t + d_i^t, \qquad \forall i \in V\backslash V^S$$

$$0 \le f_{i,j}^t \le c_{i,j}^t \qquad \forall (i,j)\in E$$

$$\sum_{k\in \mathrm{Succ}(i)} f_{i,k}^t = s_i^t - \bar{s}_i^t, \qquad \forall i\in V^S$$

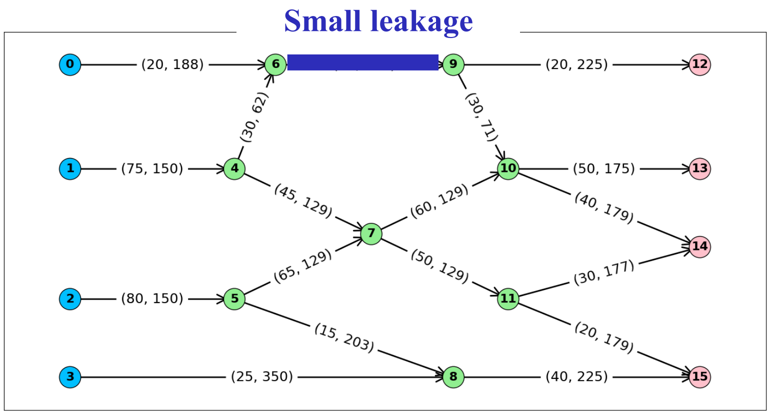
$$0 \le \bar{s}_i^t \le s_i^t \qquad \forall i \in V^S$$

Network Supply Availability Simulation



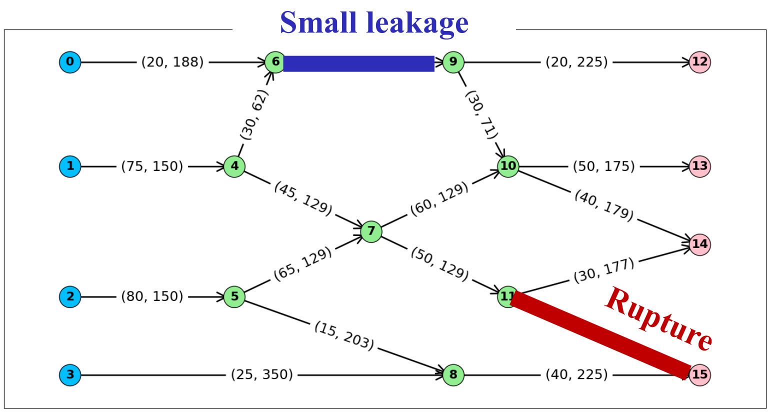
Example

Simulation # 1



$$\min_{\overrightarrow{f^t}} g\left(\overrightarrow{f^t}\right)$$

$Q^0_{Network}$ for $[0, T_0]$



$$\min_{\overrightarrow{f^t}} g\left(\overrightarrow{f^t}\right)$$

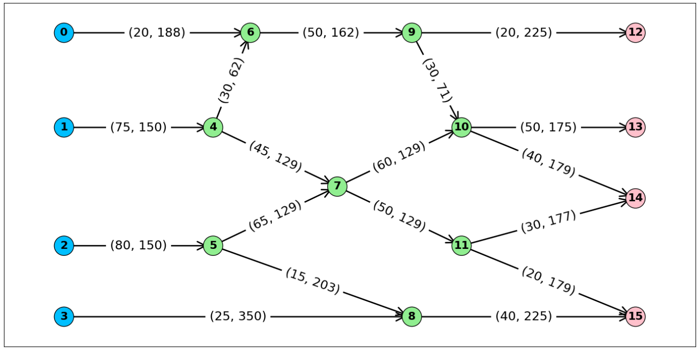
$Q^1_{Network}$ for $[T_0, T_1]$

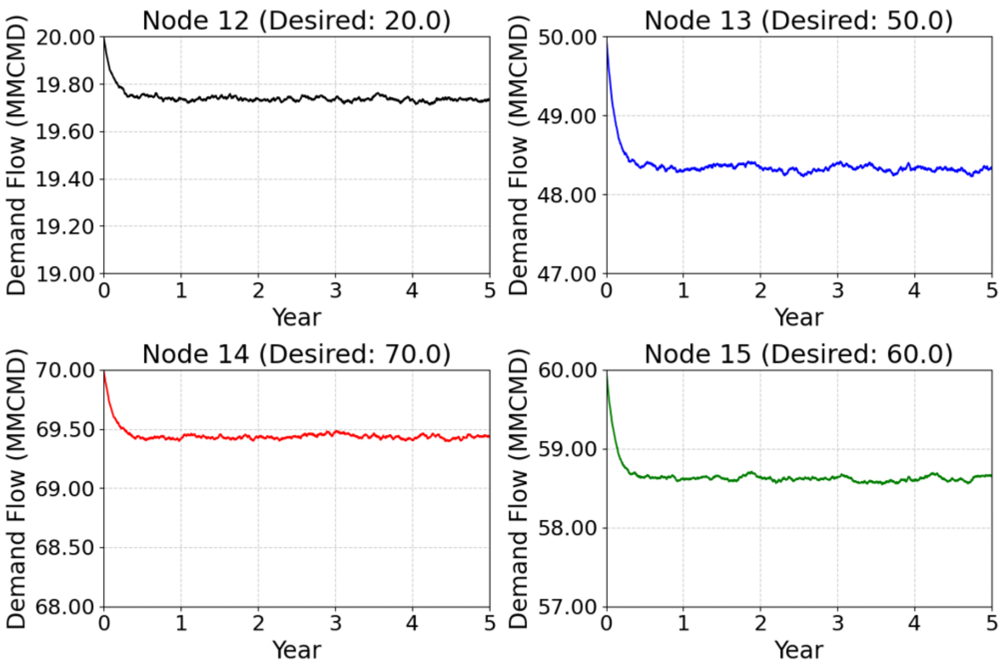
⋮

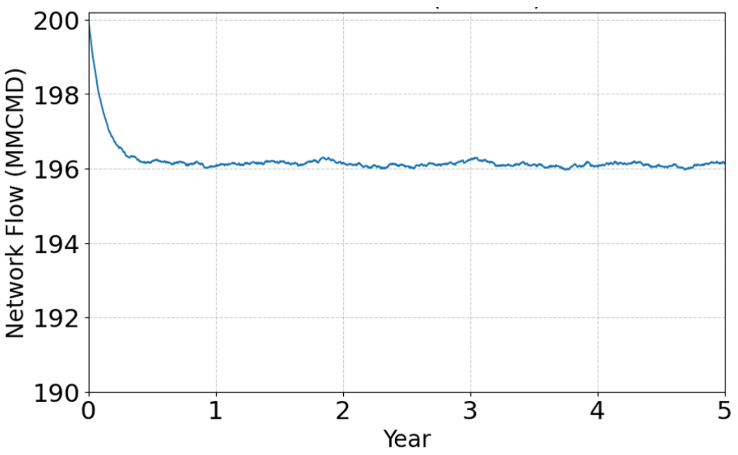
⋮

Results

- Maximum capacity of the network in healthy state: 200 MMCMD.
- The average network flow: 196.11374 (±0.06302) MMCMD







Results

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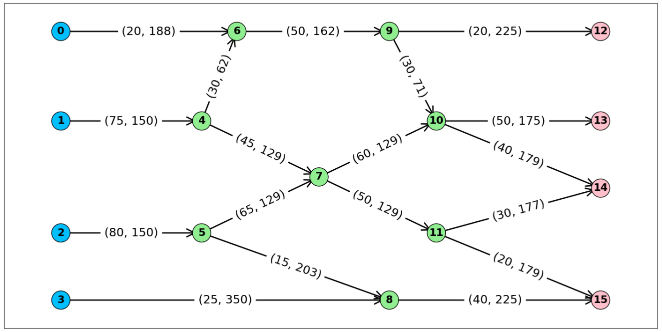

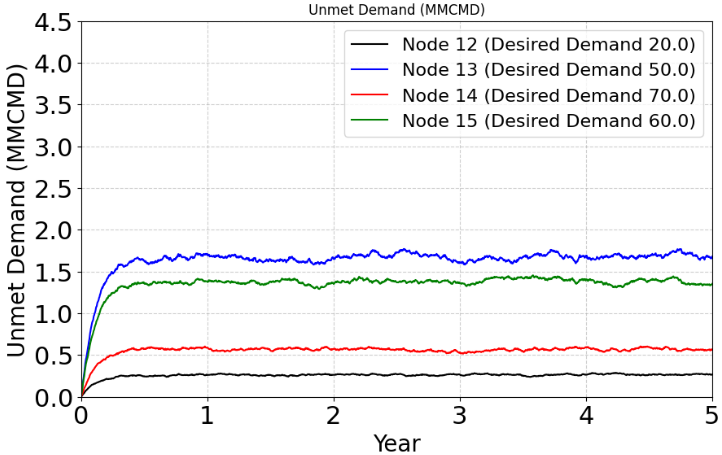

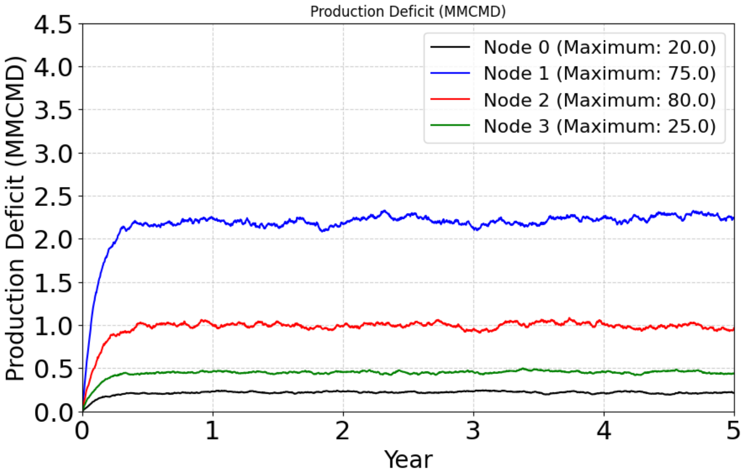

	Scenario 1	
Total Network Flow*	Mean	STD
	196.1137	0.0630
Transportation Cost**	Mean	STD
	4.61104	0.0015
Penalty Cost**	Mean	STD
	2.11986	0.0331
Node	Unmet Demand	
	Mean	STD
12	0.2646	0.0085
13	1.6723	0.0396
14	0.5664	0.0167
15	1.383	0.0319
Node	Production Deficit	
	Mean	STD
0	0.2206	0.0104
1	2.2175	0.0490
2	0.9943	0.0323
3	0.4539	0.0143

Case studies

- *M. Naseri, P. Baraldi, M. Compare, E. Zio, 2016. "Availability assessment of oil and gas processing plants operating under dynamic Arctic weather conditions", Reliability Engineering & System Safety Volume 152, August 2016, Pages 66-82. https://doi.org/10.1016/j.ress.2016.03.004*

- *M. Naseri, E. Zio, 2025. "A Modelling and Computational Framework for the Assessment of the Supply Resilience of Gas Transmission Pipeline Networks", Proceedings of the 35th European Safety and Reliability& the 33rd Society for Risk Analysis Europe Conference, Stavanger, 15-19 June. https://re.public.polimi.it/handle/11311/1311452*

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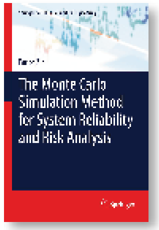

2013, 2013, XIV, 198 p. 69 illus., 24 in color.

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E. Zio, Ecole Centrale Paris, Chatenay-Malabry, France

The Monte Carlo Simulation Method for System Reliability and Risk Analysis

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- Explains the merits of pursuing the application of Monte Carlo sampling and simulation methods when realistic modeling is required so that readers may exploit these in their own applications
- Includes a range of simple academic examples in support to the explanation of the theoretical foundations as well as case studies provide the practical value of the most advanced techniques so that the techniques are accessible

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Whilst many of the topics rely on a high-level understanding of calculus, probability and statistics, simple academic examples will be provided in support to the explanation of the theoretical foundations to facilitate comprehension of the subject matter. Case studies will be introduced to provide the practical value of the most advanced techniques.

This detailed approach makes The Monte Carlo Simulation Method for System Reliability and Risk Analysis a key reference for senior undergraduate and graduate students as well as researchers and practitioners. It provides a powerful tool for all those involved in system analysis for reliability, maintenance and risk evaluations.