Monte Carlo Simulations: Exercise Session

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EXERCISE 1

Exercise 1

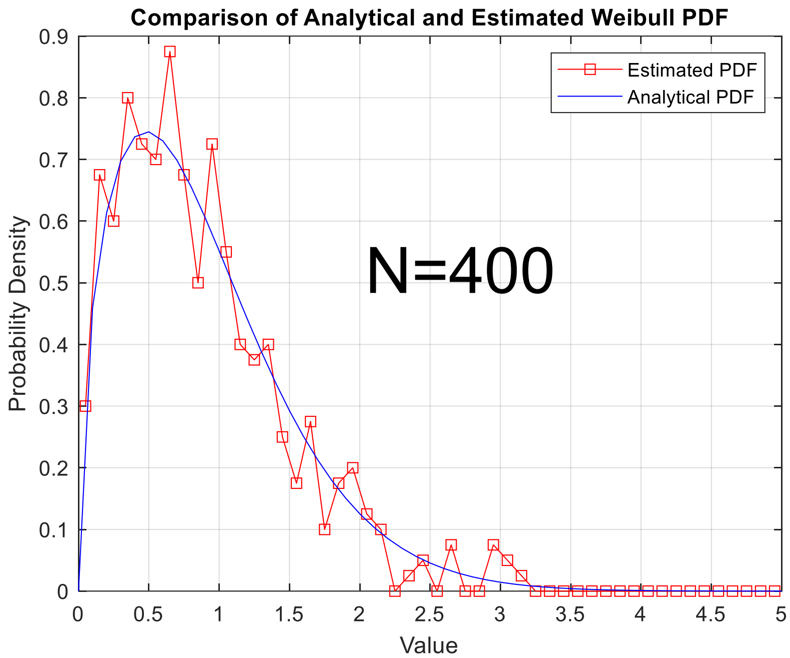
Consider the Weibull distribution:

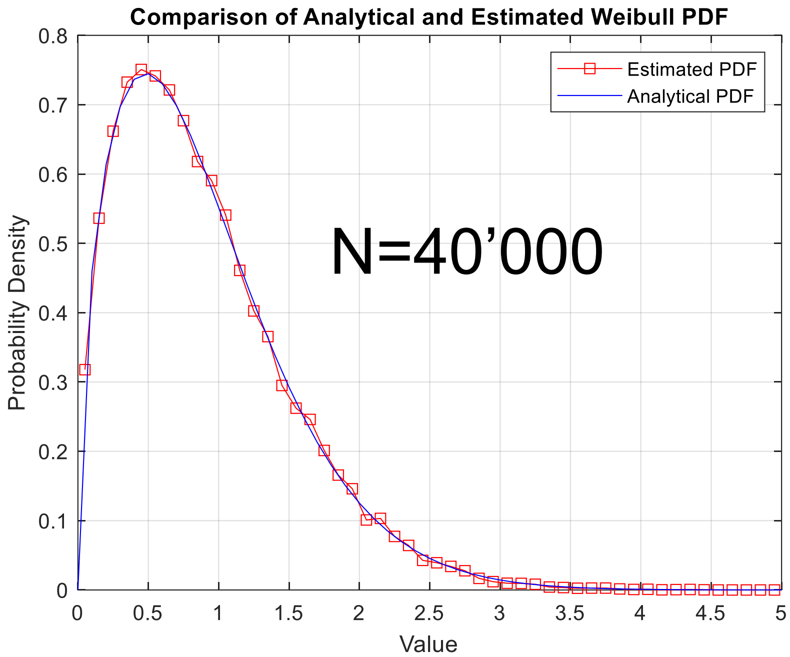
$$f_T(t) = \frac{\beta}{\tau}\left(\frac{t}{\tau}\right)^{\beta-1} e^{-\left(\frac{t}{\tau}\right)^{\beta}} \qquad F_T\,(t) = 1 - e^{-\left(\frac{t}{\tau}\right)^{\beta}}$$

with $\beta = 1{,}5$ and $\tau = 1{,}0$

1. Sample *N*=400 values from $f_T(t)$
2. Verify whether the obtained distribution provides a good approximation of the Weibull distribution. To this aim, you are required to:
   A. find the empirical probability density function (pdf) of the sampled values in 1
   B. compare the empirical pdf found in 2A. with the analytical Weibull distribution.

Exercise 1: Results (Plot)



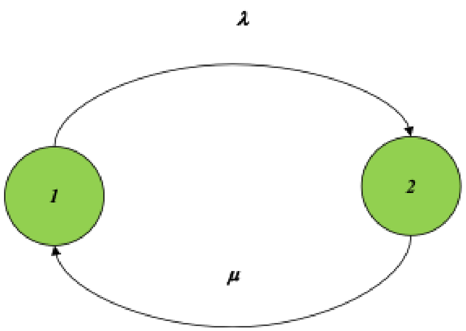


EXERCISE 2

Exercise 2

Consider a continuously monitored component with constant failure (λ) and repair (μ) rates in the table. Assuming a mission time $T = 1000\ hours$, write the MC code for the estimation of:

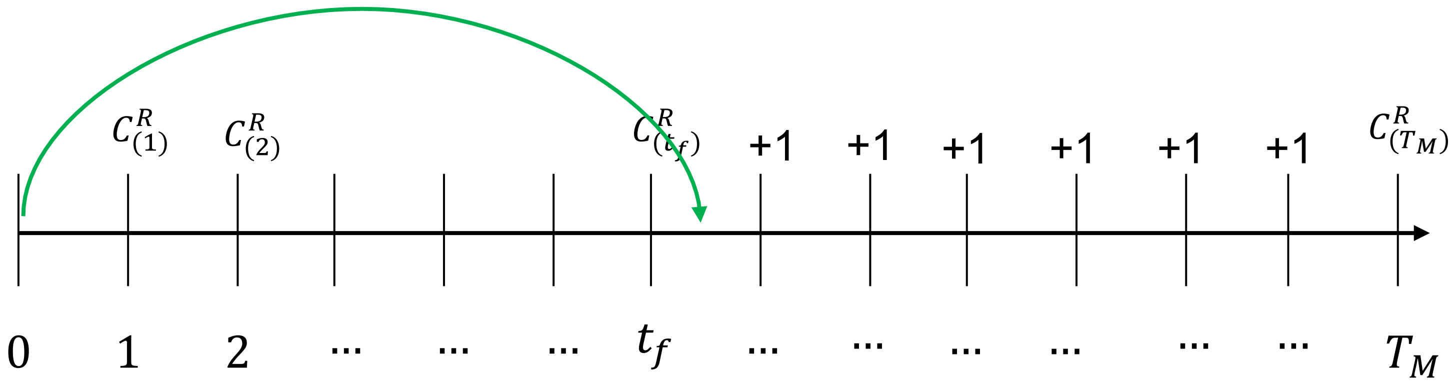
1. The instantaneous availability
2. The time dependent reliability



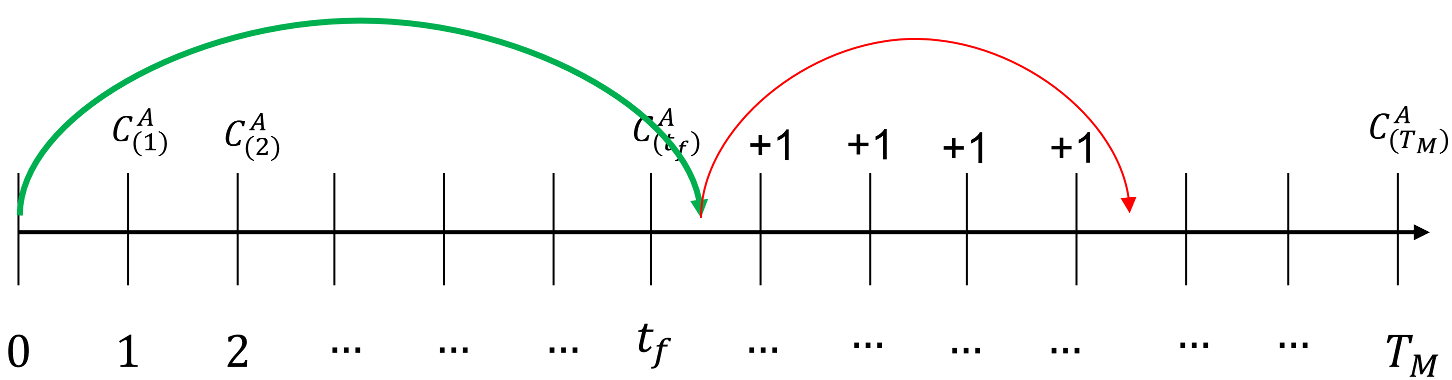
	values
λ	3· 10^{-3} h^{-1}
μ	25· 10^{-3} h^{-1}

Exercise 2

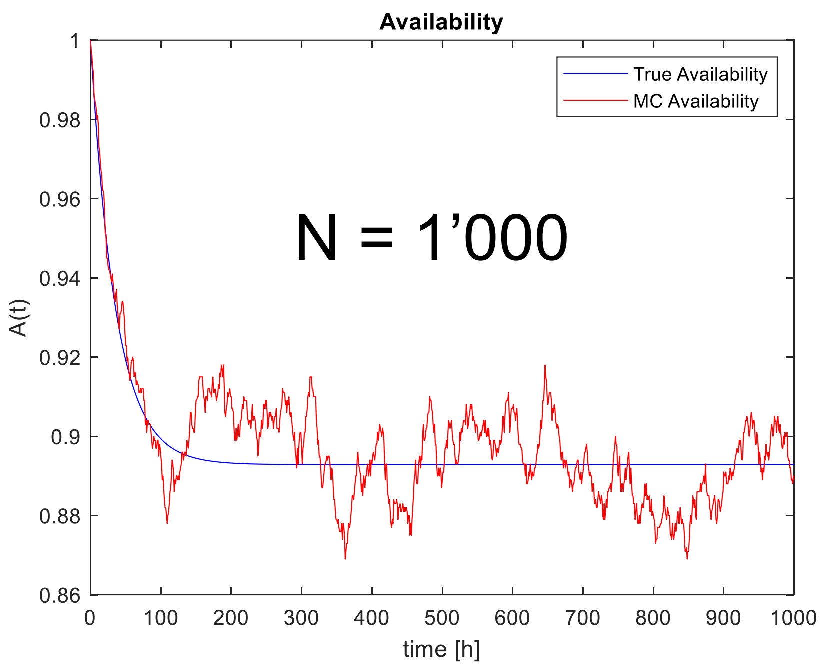
Estimation of the System Reliability

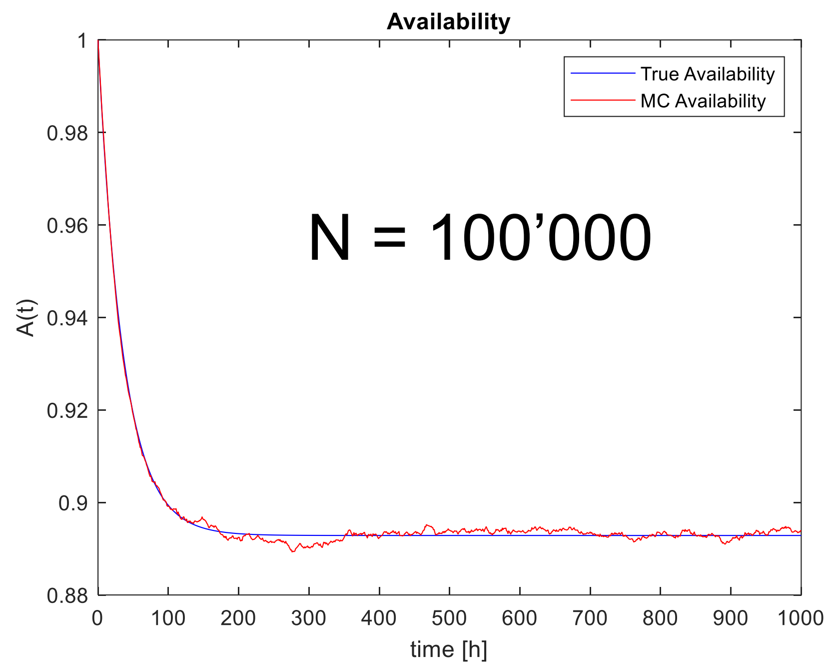


Estimation of the System Availability

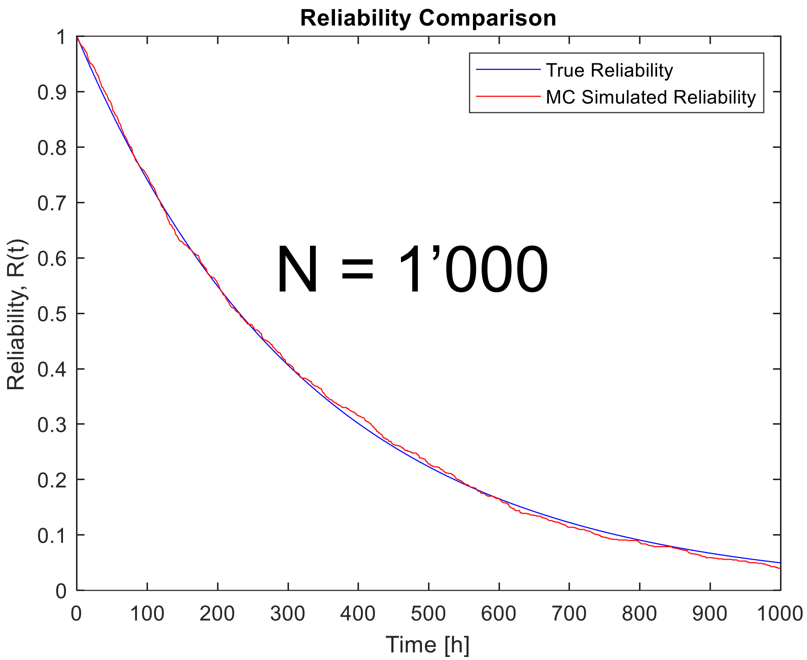


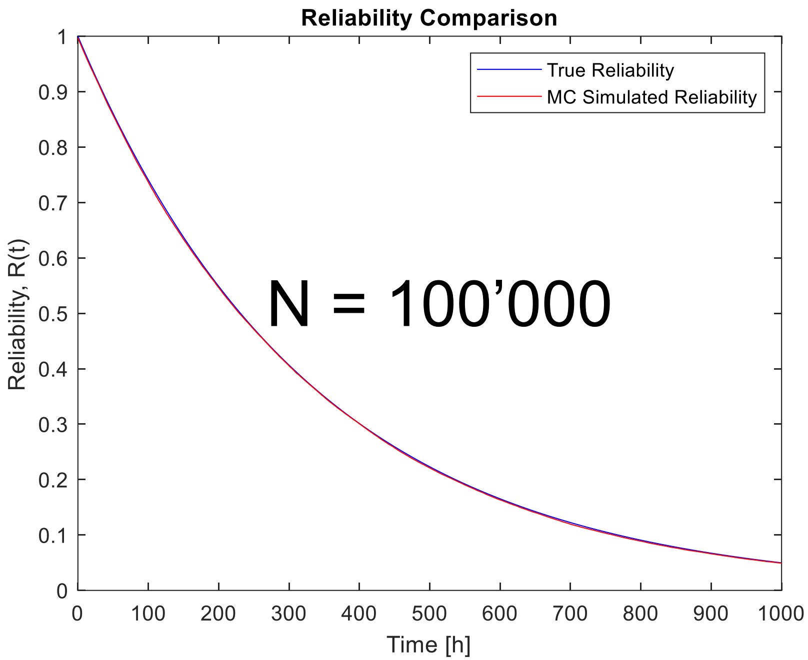
Exercise 2 – Availability plots





Exercise 2 – Reliability plots





EXERCISE 1
part 2

Exercise 1

Consider the Weibull distribution:

$$f_T(t) = \frac{\beta}{\tau}\left(\frac{t}{\tau}\right)^{\beta-1} e^{-\left(\frac{t}{\tau}\right)^{\beta}} \qquad F_T(t) = 1 - e^{-\left(\frac{t}{\tau}\right)^{\beta}}$$

with $\beta = 1{,}5$ and $\tau = 1{,}0$

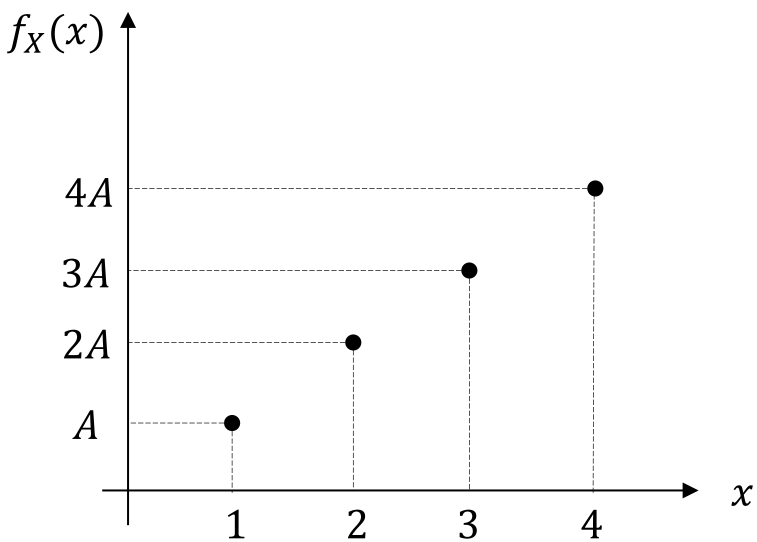
1. Sample *N*=400 values from $f_T(t)$
2. Verify whether the obtained distribution provides a good approximation of the Weibull distribution. To this aim, you are required to:
   A. find the empirical probability density function (pdf) of the sampled values in 1
   B. compare the empirical pdf found in 2A. with the analytical Weibull distribution.
3. Provide an estimate G_N of $\int_0^{+\infty} t f_T(t)dt$
4. Estimate the variance of G_N

EXERCISE 3

Exercise 3

Consider the discrete probability distribution $f_X(x)$ in the figure:

1) Identify the value of the parameter A;
2) Compute the corresponding cumulative distribution;
3) Write a Matlab/Python code to sample N=1000 values from $f_X(x)$;
4) Verify that the samples are distributed according to $f_X(x)$.

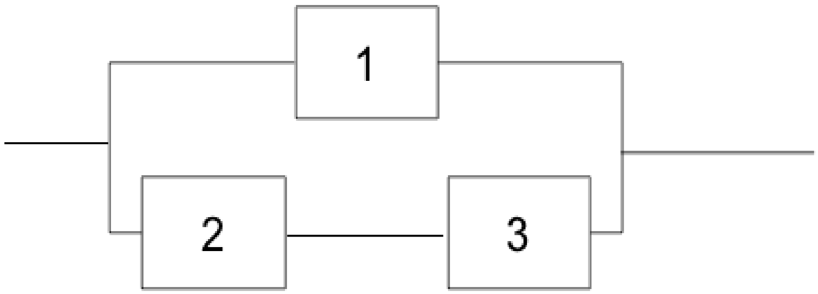


EXERCISE 4

Exercise 4

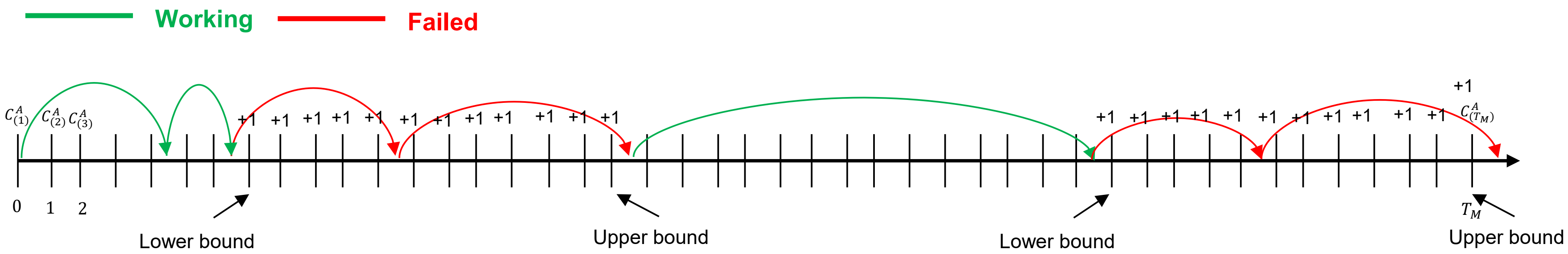
Consider the system in figure composed of three components(A, B, C). Each component can be in two different health states (1-nominal, 2-failed) with exponentially distributed transition times (table) between them. Assuming a mission time $T = 500\ hours$, write the MC code for the estimation of:

- The time dependent reliability
- The instantaneous availability.
- The estimators uncertainty

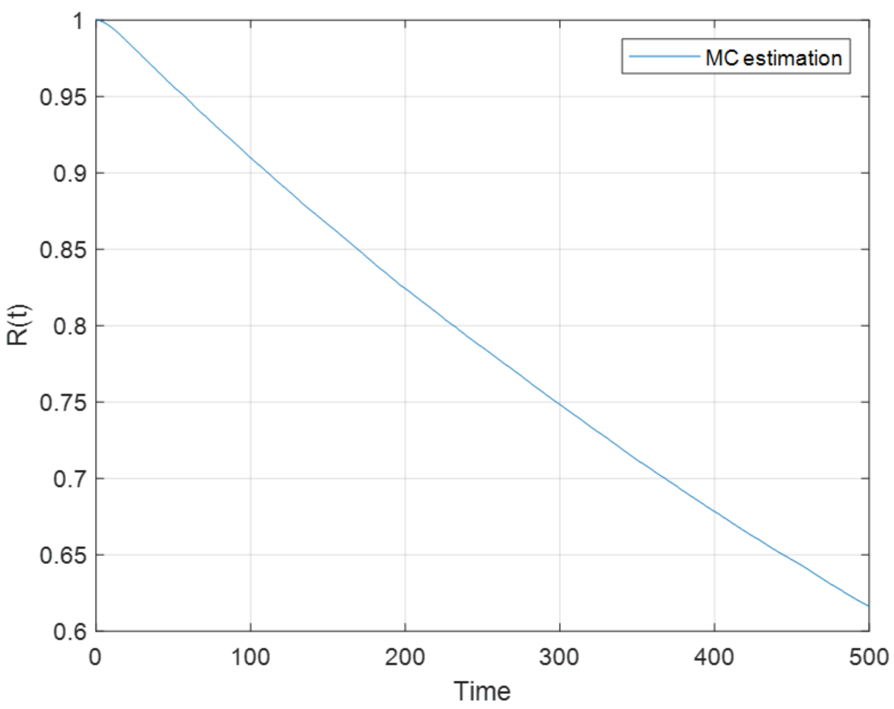


	1	2	3
λ	$1\cdot 10^{-3}\ h^{-1}$	$2\cdot 10^{-2}\ h^{-1}$	$5\cdot 10^{-2}\ h^{-1}$
μ	$3\cdot 10^{-2}\ h^{-1}$	$5\cdot 10^{-2}\ h^{-1}$	$5\cdot 10^{-3}\ h^{-1}$

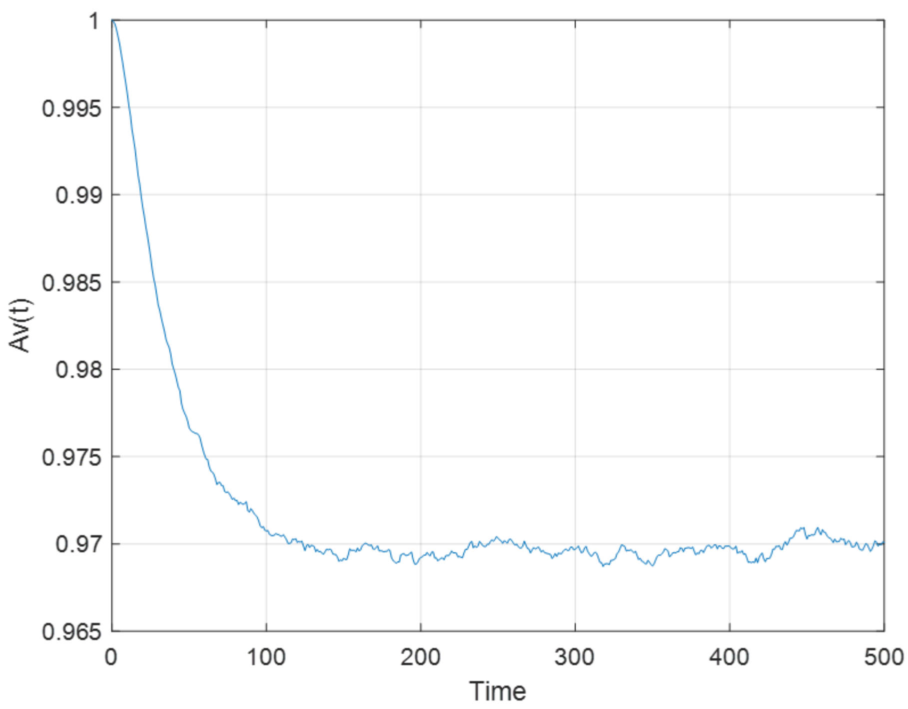
Exercise 4 – How to update the counters



Reliability

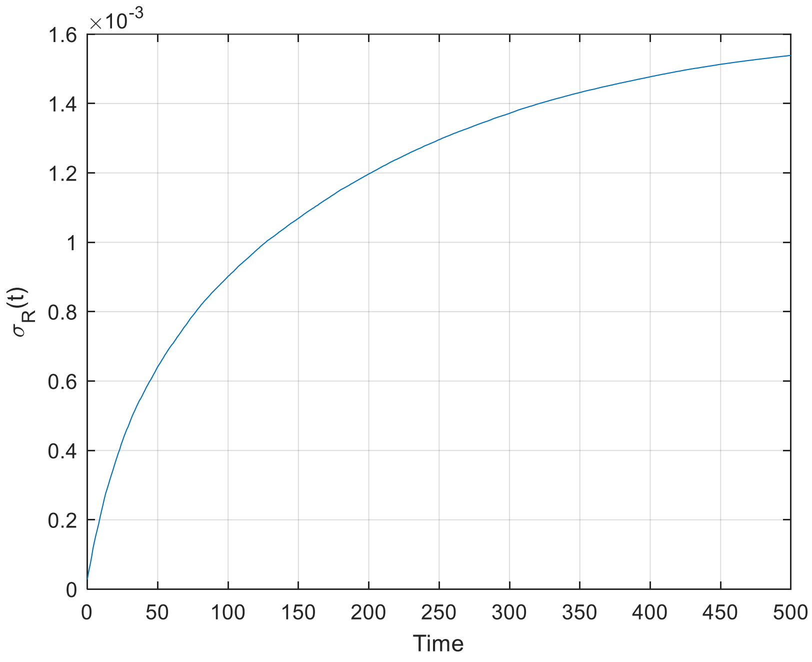


Availability



Exercise 4 –results

Reliability



Availability