LASAR[3]

POLITECNICO MILANO 1863

Exercise Session 27/02/2026
Probabilistic Models

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Exercise 1

Ten compressors, each one with a failure probability of 0.1, are tested independently.

1. What is the expected number of compressors that are found failed?
2. What is the variance of the number of compressors that are found failed?
3. What is the probability that none will fail?
4. What is the probability that two or more will fail?

Exercise 2

A machine has been observed to survive a period of 100 hours without failure with probability 0.5. Assume that the machine has a constant failure rate λ.

1. Determine the failure rate λ.
2. Find the probability that the machine will survive 500 hours without failure.
3. Determine the probability that the machine fails within 1000 hour, assuming that the machine has been observed to be functioning at 500 hours.

Exercise 3

Consider a system of two independent components with exponentially distributed failure times. The failure rates are λ_1 and λ_2 , respectively.

Determine the probability that component 1 fails before component 2.

Exercise 4

The reliability engineer of a nuclear power plant is unsure that the installed alarm system, composed by a single alarm, is reliable enough. If the reactor enters an unsafe condition, the probability that the alarm triggers is 0.99. Assume also that if the reactor is safe, the probability that the alarm will not trigger is still 0.99.

Suppose that the reactor is in unsafe conditions only one day out of 100.

1. What is the probability that the reactor is in unsafe conditions if the alarm goes off?
2. If we add a second alarm (identical to the first one), what is the probability that the reactor is in unsafe conditions if also the second alarm goes off? Comment the results.

Exercise 5

Consider the occurrence of misprints in a book and suppose that they occur at the rate of 2 per page.

1. What is the probability that the first misprint will not occur in the first page?
2. What is the expected number of pages until the first misprint appears?
3. Comment on the applicability of the Poisson assumption (independence, homogeneity, fixed period) in this case.

Exercise 6

An aircraft flight panel is fitted with two types of artificial horizon indicators. The time to failure t of each indicator from the start of a flight follow and exponential distribution with a mean value of 15 hours for the first type and 30 hours for the second type. A flight lasts for a period of 3 hours.

1. What is the probability that the pilot will be without an artificial horizon indication by the end of a flight?
2. What is the mean time to this event, if the flight is of a long duration?

Exercise 7

In considering the safety of a building, the total force acting on the columns of the building must be examined. This would include the effect of the dead load D (due to the weight of the structure), the live load L (due to the human occupancy, movable furniture…) and the wind load W. Assume that the load effects on the individual columns are statistically independent and follow a Gaussian distribution with:

$$\mu_D = 4.2\ kips \qquad \sigma_D = 0.3\ kips$$
$$\mu_L = 6.5\ kips \qquad \sigma_L = 0.8\ kips$$
$$\mu_W = 3.4\ kips \qquad \sigma_W = 0.7\ kips$$

1. Determine the mean and standard deviation of the total load acting on a column.
2. If the strength R of the column is also Gaussian with a mean equal to 1.5 times the total mean force, what is the probability of failure of the column? Assume that the coefficient of variation of the strength δ_R is 15% and the strength and load effects are statistically independent.

Exercise 8

The following relationship arises in the study of the earthquake-resistant design:

$$Y = ce^X$$

where Y is the ground motion intensity at the building site, X is the magnitude of an earthquake and c is related to the distance between the site and center of the earthquake.

If X is exponentially distributed,

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x \geq 0$$

find the cumulative distribution function of Y, $F_Y(y)$.

Exercise 9 (work at home)

Suppose that, from a previous traffic count, an average of **60 cars per hour** was observed to make left turns at an intersection.

What is the probability that exactly 10 cars will be making left turns in a 10 minute interval?

Discretize the time interval of interest to approach the problem with the binomial distribution. Show that the solution of the problem tends to the exact solution obtained with the Poisson distribution as the time discretization gets finer.

Exercise 10 (work at home)

A capacitor is placed across a power source. Assume that surge voltages occur on the line at a rate of one per month and they are normally distributed with a mean value of 100 volts and a standard deviation of 15 volts. The breakdown voltage of the capacitor is 135 volts.

1. Find the Mean Time To Failure (MTTF) for this capacitor;
2. Find its reliability for a time period of one month.

Where to find more exercises…