Exercise: Decision analysis and risk-informed optimization

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May 8, 2025

Exercise 1

Calculate the VaR_α and CVaR_α risk measures for an exponential system loss distribution, $X \sim \mathrm{Exp}(\lambda)$. Recall that the exponential parameter has range $\lambda > 0$ with $\mathbb{E}[X] = \frac{1}{\lambda}$, and that the exponential CDF is given by

$$F(X) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Exercise 2

Consider the random loss version of the dike construction problem (DCP-randomloss) as illustrated in the following Figure (the same problem as in the compendium), here we repeat the problem setting:

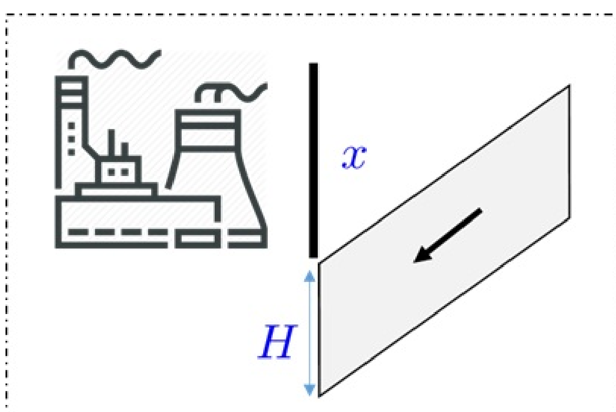

Figure 1: Illustration of the dike construction problem

- The investment cost of the dike is proportional to its height with a constant $c = 0.9$ [M€/m/yr].

- The height of the dike x can be any value in the range of $[0, 3]$ meter, where $x = 0$ means not constructing the dike
- The economic loss of the power plant from a flooding event is proportional to the overflow height with a random coefficient a [M€/m/yr] which roughly follows a log-normal distribution $a \sim \text{Lognormal}(0.7, 0.16^2)$.
- The historical annual maximum overflow height H (when there wasn't a dike) follows a Gamma distribution $H \sim \text{Gamma}(\alpha = 8, \beta = 4.5)$

Questions:

1) Sampling the distribution of the historical annual economic loss from flooding events, calculate its empirical VaR and CVaR at confidence level $\alpha = 0.9$
2) Assume the objective is to select an optimal dike height so that the summation of the investment cost and the *expected* economic loss is minimized. Formulate the sample average approximation problem, and solve it with different sample size N and validate your choice using the out-of-sample validation
3) Suppose we want to control the tail risk by minimizing the CVaR of the total cost (investment cost + economic loss), formulate the optimization problem and solve it with the sample set your chosen in problem (2). Compare the results with problem (2)