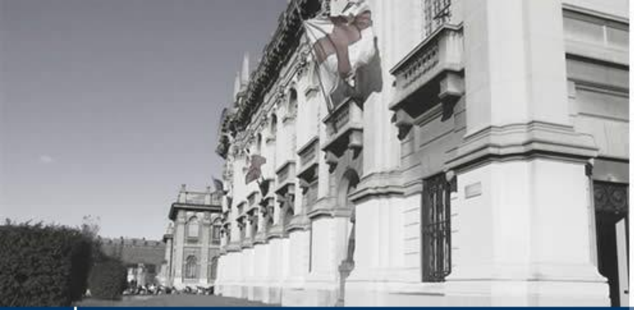

POLITECNICO DI MILANO

Dependent Failures: Parameter Estimation

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Data classification and screening

- For the estimation of the parameters of the failure models used in probabilistic risk assessment, the data sources available are typically of two kinds:

  - generic raw data
  - plant specific data.

For the parameters of a common cause failure model, in principle:

1) A complete set of events should be available for each of the common cause component groups.

2) A complete set of events should be found for each of the common cause component groups.

Data classification and screening

- ***Binary Impact Vector*** for each event that has occurred in a group of size *m*

$$I(j) = [\, I_1(j), \ldots, I_m(j)]$$

e.g. , 2 components have failed in a group of size 3:

$$I(j) = [0,1,0]$$

- Event descriptions are not clear → classification of the event requires establishing hypotheses representing different interpretations of the event. Probability are associated to the hypotheses.

Data classification and screening

Component Group Size	Hypothesis	Hypothesis Probability	I_0	I_1	I_2	I_3	Shock type	Fault Mode
3	H_1	0.9	0	0	1	0	Non lethal	Failure during operation
	H_2	0.1	0	0	0	1		
	Average Impact Vector ($\bar{I}$)		P_0	P_1	P_2	P_3		
			0	0	0.9	0.1		

- ***Average impact vector*** (not binary): for a given event *j*:

$$P(j) = [P_1(j), \ldots, P_m(j)]$$

- Several events → computation of n_k = the total number of events involving the failure of *k* similar components in the group

$$n_k = \sum_j P_k(j)$$

Parameter estimation

- **Task:** use the data available on dependent failures to estimate:

1. the basic event probabilities directly (within the basic parameter model)
2. the parameters of the common cause failure models (beta factor, BFR, etc.).

The information provided by the set of impact vectors derived from recorded data amounts to the number of events in which 1,2,3,…*m* components failed.

Beta-factor estimation for a two-train redundant standby safety system tested for failures

- Available recorded evidence:
  - n_1 failures of single components
  - n_2 failures of both components
  - Q_t : total single component failure probability

$$\beta = \frac{Q_2}{Q_t} = \frac{Q_2}{Q_1 + Q_2} = \frac{\frac{n_2}{N_2}}{\frac{n_1}{N} + \frac{n_2}{N_2}}$$

N_2: number of tests for common-cause failures

N: number of single-component demands to start

Unknown!

Beta-factor estimation for a two-train redundant standby safety system tested for failures (TEST STRATEGY I)

- Estimation of N (number of single-component demands to start)

Total number of failures

$$Q_t = \frac{n_1 + 2n_2}{N} \Rightarrow N = \frac{n_1 + 2n_2}{Q_t}$$

It is assumed to be known

n_1 : failures of single components
n_2 : failures of both components

- Estimation of N_2 → surveillance testing strategy
  - Both components are tested at the same time

N_2: number of tests for common-cause failures

Tests = Demands to start

	Day 0	Day 15	Day 30	Day 45	Day 60
Comp. 1	S	S	F	F	S
Comp. 2	S	F	S	F	S
N	2	4	6	8	10
N_2	1	2	3	4	5
n_1	0	1	2	2	2
n_2	0	0	0	1	1

Beta-factor estimation for a two-train redundant standby safety system tested for failures (TEST STRATEGY I)

- Estimation of N (number of single-component demands to start)

Total number of failure

$$Q_t = \frac{n_1 + 2n_2}{N} \Rightarrow N = \frac{n_1 + 2n_2}{Q_t}$$

- Estimation of N_2 → depends on the testing strategy
  - Both components are tested at the same time

$$N = 2N_2$$

number of single-component demands to start

number of tests for common-cause failures

$$N_2 = \frac{N}{2} \longrightarrow Q_2 = \frac{n_2}{N/2} \longrightarrow \beta = \frac{Q_2}{Q_t} = \frac{\dfrac{n_2}{N/2}}{\dfrac{n_1}{N} + \dfrac{n_2}{N_2/2}} = \frac{2n_2}{n_1 + 2n_2}$$

Beta-factor estimation for a two-train redundant standby safety system tested for failures (TEST STRATEGY II)

- The components are tested at staggered intervals, <u>if there is a failure, the second component is tested</u> immediately. N_2 is known. N is linked to N_2 from:

Number of single-component demands to start

Number of tests for common-cause failures

Number of failures of a single component

Number of failures involving both components

$$N = N_2 + n_1 + n_2$$

Tests = Demands to start

If both components are failed is for CCF!

	Day 0	Day 15	Day 30	Day 45	Day 60
Comp. 1	S	S	F	F	S
Comp. 2	NO TEST	NO TEST	S	F	NO TEST
N	1	2	4	6	7
N_2	1	2	3	4	5
n_1	0	0	1	1	1
n_2	0	0	0	1	1

Beta-factor estimation for a two-train redundant standby safety system tested for failures (TEST STRATEGY II)

- The components are tested at staggered intervals, if there is a failure, the second component is tested immediately. N_2 is known. N is linked to N_2 from:

Number of single-component demands to start

$$N = N_2 + n_1 + n_2$$

$$Q_2 = \frac{n_2}{N_2} \approx \frac{n_2}{N} \quad (n_1 \text{ and } n_2 << N)$$

$$\beta = \frac{Q_2}{Q_t} \cong \frac{\frac{n_2}{N}}{\frac{n_1 + n_2}{N}} = \frac{n_2}{n_1 + n_2}$$

Estimates of β are based on the assumptions on the testing strategies

Binomial failure rate (BFR) model

- System composed of *m* identical components operating for time T.
  - Each component can fail at random times, independently of each other, with failure rate λ.
  - A common cause shock can hit the system with occurrence rate μ.
  - Whenever a shock occurs, each of the *m* individual components may fail with probability *p*, independent of the states of the other components (*p*=1→β-model)
  - Shocks and individual failures occur independently of each other;
  - All failures are immediately discovered and repaired, with negligible repair time

- N_i random number of occurrences of i simultaneous failures

$$N_i \sim Poisson(\lambda_i T), i = 1, \dots, m$$

Binomial failure rate (BFR) model

- N_i random number of occurrences of i simultaneous failures

$$N_i \sim Poisson(\lambda_i T), i = 1, .., m$$

with

$$\lambda_1 = m\lambda + \mu r_1$$

Total contibution due to independent failure

Rate of single –unit failures from common cause shocks

and

$$\lambda_i = \mu r_i \quad i = 2, \dots, M$$

Rate of i –unit failures from common cause shocks

$$r_i = \binom{m}{i} p^i (1-p)^{m-i}, i = 1, .., m$$

Probability that i on m component ,where each component can fail with probability p

Binomial failure rate model parameter estimation

- Parameter to be estimated: p, μ, λ_1
- μ is not directly available because:
  - Shocks that do not cause any failure are not observable
  - Single failures from common-cause shocks may not be distinguishable from single independent failures
- Let introduce the random variable N_+ which counts the number of dependent failures of any multiplicity order

$$N_+ = \sum_{i=2}^{m} N_i$$

$$N_+ \sim Poisson(\lambda_+ T)$$

$$\lambda_+ = \sum_{i=2}^{m} \lambda_i = \mu[1 - r_0 - r_1]$$

Rate of dependennt failures of any multiplicity order

Binomial failure rate model parameter estimation

- Observation available

$$(n_1, .. n_m), n_+ = \sum_{i=2}^{m} n_i$$

- Method for the estimation: **maximizing the likelihood**

$$\mathbb{P}(N_1 = n_1, \dots, N_m = n_m) = \mathbb{P}(N_1 = n_1)\mathbb{P}(N_2 = n_2, .., N_m = n_m) =$$
$$= \mathbb{P}(N_1 = n_1)\ \mathbb{P}(N_2 = n_2, .., N_m = n_m | N_+ = n_+)\ \mathbb{P}(N_+ = n_+)$$

$L_1(\lambda_1)$ $\qquad$ $L_2(p)$ $\qquad$ $L_3(\lambda_+)$

- $N_1 \sim Poisson(\lambda_1 T)$
- $N_+ \sim Poisson(\lambda_+ T)$
- $N_2 = n_2, .., N_m = n_m | N_+ \sim Multinomial(n_+, z_2, \dots, z_m)\ z_i = r_i/(1 - r_0 - r_1)$

Binomial failure rate model parameter estimation

$$Log(L) = \log(L_1(\lambda_1)) + \log(L_2(p)) + \log(L_3(\lambda_+))$$

3 independent maximization!

- One can find that

$$\hat{\lambda}_1 = \frac{n_1}{T}, \hat{\lambda}_+ = \frac{n_+}{T}$$

- To find, $\hat{p}$ one have to solve numerically the following equation in p

$$\frac{dlog(L_2)}{dp} = S - \frac{mn_+[1-(1-p)^{m-1}]}{1-(1-p)^3 - mp(1-p)^{m-1}} = 0 \; m > 2, S = \sum_{i=2}^{m} in_i$$

- From $\lambda_1, \lambda_+ \; and \; p$ it is possible to estimate μ from

$$\lambda_+ = \sum_{i=2}^{m} \lambda_i = \mu[1 - r_0 - r_1]$$