POLITECNICO DI MILANO

Exam practice

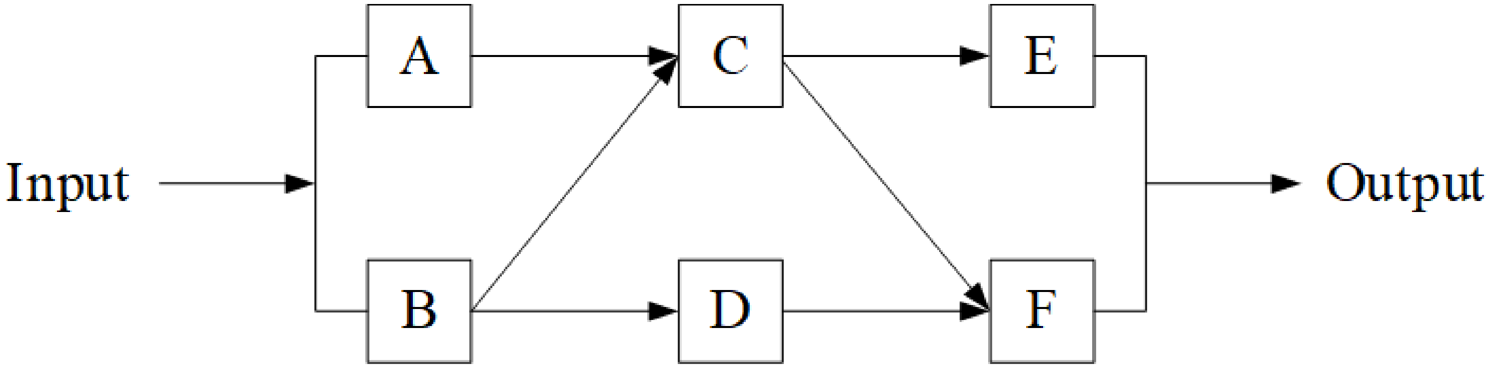
Stefano Marchetti

stefano.marchetti@polimi.it

Recap Exercises

Exercise 1

Consider the following structure of a simple system where input signal is processing via any path and is output.



Each component has a constant failure rate $\boldsymbol{\lambda} = \mathbf{2 \cdot 10}^{-5}$ h^{-1}.

Please answer the following questions:

1. Build the Fault Tree corresponding to the top event: 'no connection between Input and Output'.
2. Find the minimal cut sets
3. Estimate the reliability of system at t=1000h

Exercise 2

The mean time to failure of a component of a safety system is 1000 days. Testing the component requires $\tau = 6$ hours, whereas the time to repair can be considered negligible. The time T between the end of the previous test and the beginning of the next one is assumed to be 50 days. You are required to:

A) Plot the evolution of the instantaneous unavailability of the system.
B) Compute the average unavailability of the component.
C) Consider the operation time between tests (T) as a quantity to be optimized by the maintenance engineers. Which is the value of T that minimizes the average unavailability of the component?

Exercise 3

Consider a 2-out-of-4 redundant system of four identical components sharing a common load. In normal operation, four components equally share the load and each one of them may fail with failure rate λ_4 (exponential process). When one component fails, the remaining three working components have to carry the whole load and the failure rate immediately increases to λ_3 (exponential process); similarly, if another component fails, the remaining two working components have to carry the entire load with a failure rate increasing to λ_2 (exponential process). Since a single component alone is not capable of carrying the entire load, the system is immediately shut down when it experiences the third failure. The objective is to avoid severe damage to the surviving component. In case of shut down the repair process is such that the repairmen team restores the functionality of two components at the same time and simultaneously activate the component in shut down. The repairs start if at least two components are failed, and they last on average $^{1}/_{\mu_s}$.

1) Draw the Markov diagram of the system, upon proper definition of the system states;
2) Write the transition matrix and the probabilistic state transition process equation in matrix form;
3) Find the system MTTF;
4) Modify the Markov diagram to account for an external event hitting the system with rate λ_c. In case of external event, the probability that a working component fails is p.

Exam Simulation

Exam simulation (EX 1)

Consider the network system below, all components have equal failure probability $p = 5*10^{-2}$. The system fails when there is no connection between the source and terminal nodes. You are asked to:

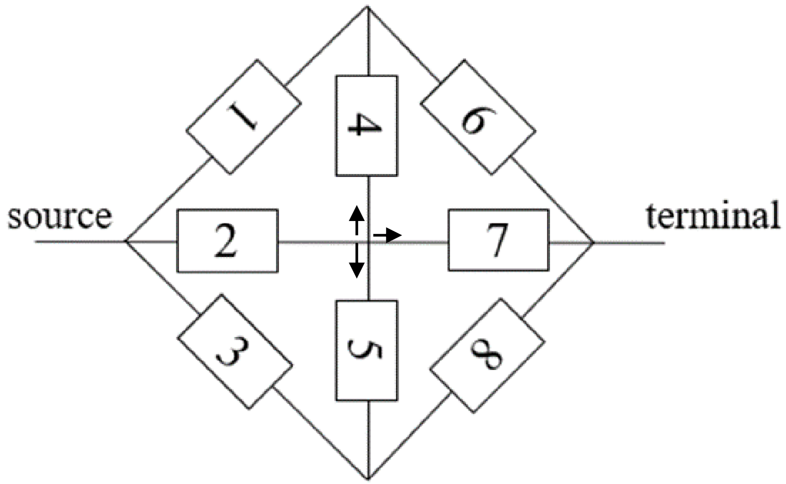
E1.1) Draw the fault tree of the system

E1.2) Write the structure function

E1.3) Identify the minimal cut sets

E1.4) Evaluate system failure probability

E1.5) (only reliability) Compute the Birnbaum's importance measure for components 2 and 7. Comment the result.



Exam simulation (EX 2) – ONLY RELIABILITY STUDENTS

A tank containing flammable substances in a chemical facility is equipped with a fire protection system made by two identical sprinklers, one safety valve and a fireproof coating. The probability of failure on demand of a sprinkler is p.

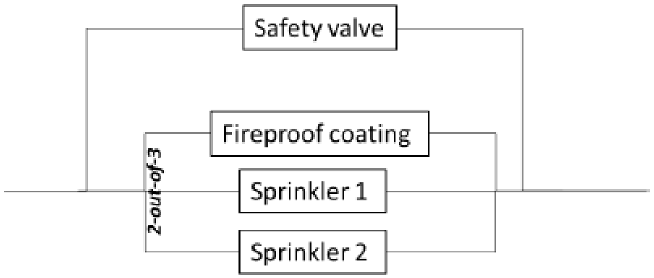
E2.1) Considering past experience on similar sprinklers, plant experts suggest using the following prior distribution for p: $P'(p{=}0.2){=}0.1$, $P'(p{=}0.3){=}0.6$,$P'(p{=}0.5){=}0.3$

A test is performed on 6 sprinklers and only one of them failed to activate on demand. You are required to find the posterior probability distribution of p;

E2.2) Assume now that the plant experts suggest using a Beta distribution with parameters $q{=}r{=}2$ as prior probability distribution of p, instead of the prior distribution suggested in E2.1). You are required to update the posterior probability distribution of p;

E2.3) Now, assume that in case of fire:

- if the safety valve of the fire protection system is functioning, the tank is not damaged;
- if the fireproof coating is not failed, one sprinkler is enough to mitigate the damage;
- if the fireproof coating is failed, both sprinklers are necessary to mitigate the damage.



Safety in Mobility students:

- Find the Fault Tree structure of the fire protection system
- Find the Minimal Cut Sets
- Compute the probability of damage to the tank in case of fire assuming:
  - Safety valve failure probability $P_{valve} = 0.1$
  - Fireproof coating failure probability $P_{coating} = 0.05$
  - Sprinklers failure probability $P_{sprinkler1} = P_{sprinkler2} = 0.2$

You are required to:

E2.3.1) Define a Bayesian Network to calculate the probability of damage of the tank in case of fire and the states and conditional probability tables of its nodes. With respect to the fire protection system, you can assume that the probability of failure of the sprinklers is equal to the expected value of the probability of failure on demand of the sprinkler of E2.1.

E2.3.2) Compute the probability of damage of the tank in case of fire using both the prior and posterior distribution (of E.2.1) for p and assuming that the safety valve and the fireproof coating are both failed.

Exam simulation (EX 2) – ONLY RELIABILITY STUDENTS

Basic random variable	Parameter	Prior and posterior distributions of parameter	Mean and Variance of Parameter	Posterior Statistics
Binomial $p_X(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$	θ	Beta $f_\Theta(\theta) = \frac{\Gamma(q+r)}{\Gamma(q)\Gamma(r)}\theta^{q-1}(1-\theta)^{r-1}$	$E(\Theta) = \frac{q}{q+r}$ $\mathrm{Var}(\Theta) = \frac{qr}{(q+r)^2(q+r+1)}$	$q'' = q' + x$ $r'' = r' + n - x$
Exponential $f_X(x) = \lambda e^{-\lambda x}$	λ	Gamma $f_\Lambda(\lambda) = \frac{\nu(\nu\lambda)^{k-1}e^{-\nu\lambda}}{\Gamma(k)}$	$E(\lambda) = \frac{k}{\nu}$ $\mathrm{Var}(\lambda) = \frac{k}{\nu^2}$	$\nu'' = \nu' + \sum_i x_i$ $k'' = k' + n$
Normal $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ (with known σ)	μ	Normal $f_M(\mu) = \frac{1}{\sqrt{2\pi}\sigma_\mu}\exp\left[-\frac{1}{2}\left(\frac{\mu-\mu_\mu}{\sigma_\mu}\right)^2\right]$	$E(\mu) = \mu_\mu$ $\mathrm{Var}(\mu) = \sigma_\mu^2$	$\mu_\mu'' = \frac{\mu_\mu'(\sigma^2/n) + \bar{x}\sigma_\mu'^2}{\sigma^2/n + (\sigma_\mu')^2}$ $\sigma_\mu'' = \sqrt{\frac{(\sigma_\mu')^2(\sigma^2/n)}{(\sigma_\mu')^2 + \sigma^2/n}}$
Normal $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$	μ, σ	Gamma-Normal $f(\mu, \sigma) = \left\{\frac{1}{\sqrt{2\pi}\sigma/\sqrt{n}}\exp\left[-\frac{1}{2}\left(\frac{\mu-\bar{x}}{\sigma/\sqrt{n}}\right)^2\right]\right\} \cdot \left\{\frac{[(n-1)/2]^{(n+1)/2}}{\Gamma[(n+1)/2]}\left(\frac{s^2}{\sigma^2}\right)^{(n-1)/2} \cdot \exp\left(-\frac{n-1}{2}\frac{s^2}{\sigma^2}\right)\right\}$	$E(\mu) = \bar{x}$ $\mathrm{Var}(\mu) = s^2\left[\frac{n-1}{n(n-3)}\right]$ $E(\sigma) = s\sqrt{\frac{n-1}{2}}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}$ $\mathrm{Var}(\sigma) = s^2\left(\frac{n-1}{n-3}\right) - E^2(\sigma)$	$n'' = n' + n$ $n''\bar{x}'' = n'\bar{x}' + n\bar{x}$ $(n''-1)s''^2 + n''\bar{x}''^2 = [(n'-1)s'^2 + n'\bar{x}'^2] + [(n-1)s^2 + n\bar{x}^2]$
Poisson $p_X(x) = \frac{(\mu t)^x}{x!}e^{-\mu t}$	μ	Gamma $f_M(\mu) = \frac{\nu(\nu\mu)^{k-1}e^{-\nu\mu}}{\Gamma(k)}$	$E(\mu) = \frac{k}{\nu}$ $\mathrm{Var}(\mu) = \frac{k}{\nu^2}$	$\nu'' = \nu' + t$ $k'' = k' + x$
Lognormal $f_X(x) = \frac{1}{\sqrt{2\pi}\zeta x} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right]$ (with known ζ)	λ	Normal $f_\Lambda(\lambda) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{\lambda-\mu}{\sigma}\right)^2\right]$	$E(\lambda) = \mu$ $\mathrm{Var}(\lambda) = \sigma^2$	$\mu'' = \frac{\mu'(\zeta^2/n) + \sigma^2\overline{\ln x}}{\zeta^2/n + \sigma^2}$ $\sigma'' = \sqrt{\frac{\sigma^2(\zeta^2/n)}{\sigma^2 + \zeta^2/n}}$

Exam simulation (MA)

1. (Only Reliability) In the method “event trees with boundary condition” for a system composed of 2 subsystems S1 and S2, with shared components C1 and C2, the event tree ET = {IE, S1, S2}:
   a. is modified as ET’ = {IE, C1, C2, S1, S2} and the minimal cut sets for each possible sequence of events are found
   b. is modified as ET’ = {IE, C1, S1, C2, S2} and the minimal cut sets for each possible sequence of events are found
   c. is modified as ET’ = {IE, C1, C2, S1, S2} and the minimal cut sets are found by merging the FT of S1 and S2 and developing the structure function
   d. is modified as ET’ = {IE, S1, C1, S2, C2} and the minimal cut sets for each possible sequence of events are found

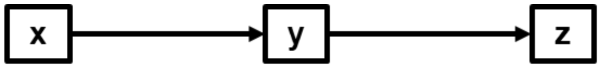
2. (Only Reliability) During a direct Monte Carlo simulation for a system composed by a parallel of two exponential components A and B, suppose that at time t component A is healthy whereas component B is failed, i.e. the state of the system is s=(1,0). If the transition rate to failed state of A is 0.01, the repair rate of B is 0.02, which among the following corresponds to the next system transition, considering the sampled random numbers [0.17, 0.84]:
   a. t’=t+6.21, s’=(0,0)
   b. t’=t+18.63, s’=(0,0)
   c. t’=t+6.21, s’=(1,1)
   d. t’=t+91.63, s’=(1,1)

3. Considering a system with the transition probability matrix $\underline{\underline{A}} = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ 0 & p_{11} & p_{12} \\ p_{20} & p_{21} & 0 \end{bmatrix}$, which among the following is true:
   a. $\Pi_0 = \frac{-p_{21}p_{12}}{p_{00}p_{11} - p_{21}p_{12} - p_{210}p_{02}}$
   b. $\Pi_0 = \frac{p_{21}p_{12}}{p_{00}p_{11} + p_{21}p_{12} - p_{210}p_{02}}$
   c. $\Pi_1 = \frac{p_{00}p_{11}}{p_{00}p_{11} + p_{21}p_{12} - p_{210}p_{02}}$
   d. $\Pi_1 = \frac{p_{20}p_{02}}{p_{00}p_{11} - p_{21}p_{12} + p_{210}p_{02}}$

4. Which minimal cut sets that can be found from the following structural function: T = 1-(1-L)[1-(1-(1-A)(1-C))(1-(1-B)(1-D))]?
   a. {AB, AC, AD, BD}
   b. {L, AB, AD, BC}
   c. {CD, CB, AB, AD}
   d. {CB, CD, AB, AD, L}

Exam simulation (MA)

5. (Only Reliability) Consider the BN in the Figure, composed by the variables x, y and z with their conditional probability tables.



	P(x)
x=0	0.4
x=1	0.6

P(y|x)	x=0	x=1
y=0	0.3	0.5
y=1	0.7	0.5

P(z|y)	y=0	y=1
z=0	0.1	0.8
z=1	0.9	0.2

What is the $P(z = 0|x = 0, y = 0)$?

a. 0.5
b. 0.1
c. 0.2
d. 0.7

6. In the technique of Principal Component Analysis (PCA), which of the following is true?

a. The number of principal components is always greater than the number of signals
b. The principal components having lower variances are ignored to reduce the number of dimensions
c. The number of principal components is always lower than the number of signals
d. The principal components having higher variances are ignored to reduce the number of dimensions

Consider a safety system made by two components (A and B) in parallel with constant failure rates equal to λ_A and λ_B. The testing and repair of each component lasts for τ_r and a staggered maintenance scheme is applied to the system:

- τ is the time interval between the end of the previous maintenance of component B and the beginning of the next maintenance of component B.
- maintenance of component A starts after a time interval of $\frac{\tau}{3}$ from the end of the maintenance of component B.

You are required to:

a. Evaluate the average system unavailability over the maintenance cycle (you can assume that $\tau \gg \tau_r$, $\lambda_A \tau \ll 1$ and $\lambda_B \tau \ll 1$)

b. Plot the time evolution of the system instantaneous unavailability (you can assume $\lambda_A = \lambda_B$)