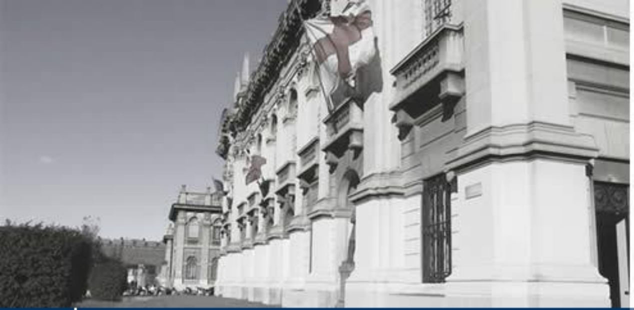

POLITECNICO DI MILANO

    

Importance Measures

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Introduction

- **Objective**: Importance Measures (IMs) aim at quantifying the contribution of components or basic events to the measure of system performance of interest
- **Examples**:
  - **Nuclear**: risk measure (Core Damage Frequency, Large Early Release Frequency)
  - **Aerospace**: unreliability
  - **Power generation**: unavailability
  - **Oil and gas platforms**: unavailability
- **Why**: great practical aid to system designers and managers: trace system bottlenecks and provides guidelines for effective system **improvement**

Introduction (continued)

- **How**: *ranking* and *categorization* with respect to:
  - *risk-significance*: if the component failure or unavailability contributes significantly to system **risk measure**
  - *safety-significance*: if the component plays an important role in the **prevention** of system undesired states (in system success)

- **Hypotheses**:
  - n binary components (X_j = 0 or 1, j = 1, …, n)
  - **X_j = 0 for component failure**
  - risk measures adopted: **system reliability** $R(t)$ or failure probability $F(t) = 1 - R(t)$
  - $\underline{r}(t) = (r_1(t), r_2(t), \ldots, r_n(t))$ vector of the reliabilities at time t of the individual components
  - $R(\underline{r}(t))$ = system reliability

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Birnbaum’s IM

Birnbaum's IM

- Definition

$$I_j^B(t) = \frac{\partial R(\underline{r}(t))}{\partial r_j(t)}$$

- It measures how much a change in the system reliability is due to a change in component *j's* reliability [**differential sensitivity measure**]
- Variation of reliability in some components results in the largest variations in system reliability

- Hypotheses:
  - $0 \le I_j^B(t) \le 1$
  - All the *n* components must be independent
  - When $R(\underline{r}(t))$ is a linear function of $\underline{r}(t)$ and if all components are independent, then $I_j^B(t)$ does not depend on $r_j(t)$, *j* = 1,2,...,*n*

Birnbaum's Importance Measure: Exercise 1

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	R	I_1^B	I_2^B	I_3^B
I	Series (1-2)				
II	Parallel (1-2)				
III	Parallel (1-2-3)				
IV	2-out-of-3 (1-2-3)				

Birnbaum's Importance Measure: Exercise 1

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	R	I_1^B	I_2^B	I_3^B
I	Series (1-2)				
II	Parallel (1-2)				
III	Parallel (1-2-3)				
IV	2-out-of-3 (1-2-3)				

- In a **series** $I_j^B(t)$ is larger for **less** reliable components
- In a **parallel** $I_j^B(t)$ is larger for **more** reliable components

- In a **series** $I_j^B(t)$ prioritizes components according to *increasing* reliability
- In a **parallel** $I_j^B(t)$ prioritizes components according to *decreasing* reliability

Birnbaum's Importance Measure: Exercise 1 - Solution

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$

Configuration #	*System configuration* *(system components)*	R	I_1^B	I_2^B	I_3^B
I	Series (1-2)				
II	Parallel (1-2)				
III	Parallel (1-2-3)				
IV	2-out-of-3 (1-2-3)				

Recall:

Series: $$R(t) = \prod_{i=1}^{N} r_i(t)$$

Parallel: $$R(t) = 1 - \prod_{i=1}^{N} [1 - r_i(t)]$$

Birnbaum's Importance Measure: Exercise 1 - Solution

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$

Series $r_1 - r_2$:

$$R(t) = \prod_{i=1}^{N} R_i(t) = r_1 r_2$$

$$I_1^B = \frac{\partial R}{\partial r_1} = r_2 \qquad I_2^B = \frac{\partial R}{\partial r_2} = r_1$$

Parallel $r_1 - r_2$:

$$R(t) = 1 - \prod_{i=1}^{N} [1 - r_i(t)] = r_1 + r_2 - r_1 r_2$$

$$I_1^B = \frac{\partial R}{\partial r_1} = 1 - r_2 \qquad I_2^B = \frac{\partial R}{\partial r_2} = 1 - r_1$$

Birnbaum's Importance Measure: Exercise 1 - Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Parallel: $r_1 - r_2 - r_3$ $$R(t) = 1 - \prod_{i=1}^{N} [1 - r_i(t)] = 1 - (1 - r_1)(1 - r_2)(1 - r_3)$$

$$R = r_1 + r_2 + r_3 - r_1 r_2 - r_1 r_3 - r_2 r_3 + r_1 r_2 r_3$$

$$I_1^B = \frac{\partial R}{\partial r_1} = 1 - r_2 - r_3 + r_2 r_3$$

$$I_2^B = \frac{\partial R}{\partial r_2} = 1 - r_1 - r_3 + r_1 r_3$$

$$I_3^B = \frac{\partial R}{\partial r_3} = 1 - r_1 - r_2 + r_1 r_2$$

Birnbaum's Importance Measure: Exercise 1 - Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

2-out-of-3: $r_1 - r_2 - r_3$

$$R = r_1 r_2 + r_1 r_3 + r_2 r_3 - 2 r_1 r_2 r_3$$

$$I_1^B = \frac{\partial R}{\partial r_1} = r_2 + r_3 - 2 r_2 r_3$$

$$I_2^B = \frac{\partial R}{\partial r_2} = r_1 + r_3 + 2 r_1 r_3$$

$$I_3^B = \frac{\partial R}{\partial r_3} = r_1 + r_2 + 2 r_1 r_2$$

Birnbaum's Importance Measure: Exercise 1 - Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	R	I_1^B	I_2^B	I_3^B
I	Series (1-2)	0.9408	r_2=0.96	$\mathbf{r_1=0.98}$	/
II	Parallel (1-2)	0.9992	$\mathbf{1-r_2=0.04}$	$1-r_1$=0.02	/
III	Parallel (1-2-3)	0.999952	**0.0024**	0.0012	0.0008
IV	2-out-of-3 (1-2-3)	0.9957	$r_2 + r_3 - 2r_2r_3$=**0.0952**	0.0776	**0.0584**

- In a **series** $I_j^B(t)$ is larger for **less** reliable components
- In a **parallel** $I_j^B(t)$ is larger for **more** reliable components

- In a **series** $I_j^B(t)$ prioritizes components according to *increasing* reliability
- In a **parallel** $I_j^B(t)$ prioritizes components according to *decreasing* reliability

Birnbaum's and Structure Function

$$X_T = \Phi(X_1, X_2, ..., X_n) = \Phi(\underline{X})$$

Pivotal decomposition

$$\Phi[\underline{X}(t)] = X_j(t)\Phi[\underline{X}(t), X_j = 1] + (1 - X_j(t))\Phi[\underline{X}(t), X_j = 0]$$
$$= X_j(t)\{\Phi[\underline{X}(t), X_j = 1] - \Phi[\underline{X}(t), X_j = 0]\} + \Phi[\underline{X}(t), X_j = 0]$$

Hp: component independence

$$R(\underline{r}(t)) = r_j(t) \cdot \{E[\Phi[\underline{X}(t), X_j = 1]] - E[\Phi[\underline{X}(t), X_j = 0]]\} + E[\Phi[\underline{X}(t), X_j = 0]] =$$
$$= r_j(t) \cdot \{R(r_j = 1, \underline{r}(t)) - R(r_j = 0, \underline{r}(t))\} + R(r_j = 0, \underline{r}(t)) =$$
$$= r_j(t) \cdot \{R_j^+(t) - R_j^-(t)\} + R_j^-(t)$$

$$R_j^+(t) = R(r_j = 1, \underline{r}(t)) = E[\Phi[\underline{X}(t), X_j = 1]]$$
$$R_j^-(t) = R(r_j = 0, \underline{r}(t)) = E[\Phi[\underline{X}(t), X_j = 0]]$$

Birnbaum's and Structure Function (2)

$$I_j^B(t) = \frac{\partial R(\underline{r}(t))}{\partial r_j(t)} = R\left[r_j = 1, \underline{r}(t)\right] - R\left[r_j = 0, \underline{r}(t)\right] = R_j^+(t) - R_j^-(t)$$

$R_j^+(t) = \textit{the increase reliability level with component j perfectly relaible}$
$R_j^-(t) = \textit{the decrease reliability level with component j assumed failed}$

$$I_j^B(t) = E\left\{\left[\Phi\left[\underline{X}(t), X_j = 1\right] - \Phi\left[\underline{X}(t), X_j = 0\right]\right]\right\}$$
$$= P\left\{\Phi\left[\underline{X}(t), X_j = 1\right] - \Phi\left[\underline{X}(t), X_j = 0\right] = 1\right\}$$

$$\boxed{\Phi\left[\underline{X}(t), X_j = 1\right] - \Phi\left[\underline{X}(t), X_j = 0\right] = \{0,1\}}$$

- $I_j^B(t)$ is the probability that $(\underline{X}(t), X_j = 1)$ is a **critical** path vector, i.e. **the other components of the system are in such a state that the system functions if *j* functions and the system is failed if *j* is failed**

$$I_j^B(t) = P\{j \text{ is critical}\}$$

Birnbaum's Importance Measure: Properties

- $0 \leq I_j^B(t) \leq 1$
- When $R(\underline{r}(t))$ is a linear function of $\underline{r}(t)$ and if all components are independent, then $I_j^B(t)$ does not depend on $r_j(t)$, $j = 1,2,...,n$

Birnbaum's and Structure Function - Example

We know:

$$I_j^B(t) = E\left\{\left[\Phi\left[\underline{X}(t), X_j = 1\right] - \Phi\left[\underline{X}(t), X_j = 0\right]\right]\right\}$$
$$= P\left\{\Phi\left[\underline{X}(t), X_j = 1\right] - \Phi\left[\underline{X}(t), X_j = 0\right] = 1\right\}$$

- Consider the series system of components 1 and 2

$$I_1^B = P\{\underline{X} = (1, X_2) \text{ is a critical path vector}\}$$

It is required that component 2 functions

$$I_1^B = P\{X_2 = 1\} = r_2$$

- Consider the parallel system of component 1 and 2

$$I_1^B = P\{\underline{X} = (1, X_2) \text{ is a a critical path vector}\}$$

It is required that component 2 is failed

$$I_1^B = P\{X_2 = 0\} = 1 - r_2$$

Birnbaum's and Structure Function – Dual formulation

- Let us denote the component's unreliability by:

$$q_j(t) = 1 - r_j(t)$$

$$I_j^B(t) = \frac{\partial R(\underline{r}(t))}{\partial r_j(t)} = R\left[r_j = 1, \underline{r}(t)\right] - R\left[r_j = 0, \underline{r}(t)\right] = \frac{\partial F(\underline{q}(t))}{\partial q_j(t)} =$$

$$= F\left[q_j = 1, \underline{q}(t)\right] - F\left[q_j = 0, \underline{q}(t)\right] = F_j^+(t) - F_j^-(t)$$

- $$F_j^+(t) = F\left[q_j = 1, \underline{q}(t)\right] = P\left\{\Phi\left[\underline{X}(t), X_j = 0\right] = 0\right\}$$

  $$F_j^-(t) = F\left[q_j = 0, \underline{q}(t)\right] = P\left\{\Phi\left[\underline{X}(t), X_j = 1\right] = 0\right\}$$

Birnbaum's IM: Possible Use

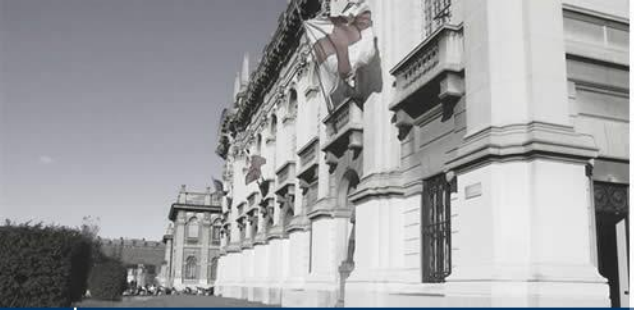
- Consider a component *i* characterized by a failure rate λ_i. $I_i^B(t)$ quantifies how much the system reliability will change by making a small variation of λ_i:

$$\frac{\partial R(\boldsymbol{r}(t))}{\partial \lambda_i} = \frac{\partial R(\boldsymbol{r}(t))}{\partial r_i} \cdot \frac{\partial r_i}{\partial \lambda_i} = I_i^B(t)\, \frac{\partial r_i(t)}{\partial \lambda_i}$$

- Practical reliability study of a complex system:
  - input parameter estimation (failure rates, repair rates, etc.) = time consuming task

- start with rough estimates
- calculate component Birnbaum's measure of importance

- No extra efforts for finding high-quality data for low importance components
- Improve parameter estimation for high importance components (look for more data,…)



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Criticality IM

Criticality Measure

- Let $C[X(t), X_j = 1]$ be the event such that *j* is **critical & works:**

$$P\{C[\underline{X}(t), X_j = 1]\} = I_j^B(t)$$

- Probability that *j* is **critical** and **failed** at time *t* is:

$$P\{C[\underline{X}(t), X_j = 1] \cap [X_j(t) = 0]\}$$

Criticality Measure

- Let $C[X(t), X_j = 1]$ be the event such that *j* is **critical & works:**

$$P\{C[\underline{X}(t), X_j = 1]\} = I_j^B(t)$$

- Probability that *j* is **critical** and **failed** at time *t* is:

$$P\{C[\underline{X}(t), X_j = 1] \cap [X_j(t) = 0]\} = I_j^B(t) \cdot (1 - r_j(t))$$

Component j is critical and it fails ≡ Component j has caused the system failure

Criticality Measure

- Let $C[X(t), X_j = 1]$ be the event such that *j* is **critical & works:**

$$P\{C[\underline{X}(t), X_j = 1]\} = I_j^B(t)$$

- Probability that *j* is **critical** and **failed** at time *t* is:

$$P\{C[\underline{X}(t), X_j = 1] \cap [X_j(t) = 0]\} = I_j^B(t) \cdot (1 - r_j(t))$$

Component j is critical and it fails ≡Component j has caused the system failure

- Let us assume that we know that the system is in a failed state at time *t*, we can compute the conditional probability:

$$I_j^{CR}(t) = P\{C[X(t), X_j = 1] \cap [X_j(t) = 0] | \Phi\left(\underline{X}(t)\right) = 0\}$$

$I_j^{CR}(t)$= *Probability that component j has caused the system failure, given that the system is failed at time t*

Criticality Measure

$$I_j^{CR}(t) = P\{C[X(t), X_j = 1] \cap [X_j(t) = 0] | \Phi\left(\underline{X}(t)\right) = 0\}$$

$$= \frac{P\{C[X(t), X_j = 1] \cap [X_j(t) = 0] \cap [\Phi\left(\underline{X}(t)\right) = 0]\}}{P[\Phi\left(\underline{X}(t)\right) = 0]}$$

- $C[\underline{X}(t), X_j = 1] \cap [X_j(t) = 0]$ → system failure

$$I_j^{CR}(t) = \frac{P\{C[X(t), X_j = 1] \cap [X_j(t) = 0]\}}{1 - R(\underline{r}(t))}$$

- $P\{C[\underline{X}(t), X_j = 1] \cap [X_j(t) = 0]\} = I_j^B(t) \cdot \left(1 - r_j(t)\right)$

$$I_j^{CR}(t) = \frac{I_j^B(t) \cdot \left(1 - r_j(t)\right)}{1 - R(\underline{r}(t))}$$

Criticality IM: Possible Use

$I_j^{CR}(t)$= *Probability that component j has caused the system failure, given that the system is failed at time t*

$$\equiv$$

When component j is repaired, the system will start functioning again

$$\equiv$$

Probability that if component j is repaired the system will start work, given that the system is failed at time t

- $I_j^{CR}(t)$ is used to prioritize maintenance actions (e.g. inspections and repairs) in complex systems

Exercise 2: Criticality

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	I_1^{cr}	I_2^{cr}	I_3^{cr}
I	Series (1-2)			
II	Parallel (1-2)			
III	2-out-of-3 (1-2-3)			

Exercise 2: Criticality

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	I_1^{cr}	I_2^{cr}	I_3^{cr}
I	Series (1-2)			
II	Parallel (1-2)			
III	2-out-of-3 (1-2-3)			

- In a **series** system, the most important component according to I^{cr} is the least reliable one (=I^B)
- In a simple **parallel**, the criticality of all component is the same (if the system is failed, it will start operates, independently from the component which is repaired)
- In a 2 out of 3 configuration, I^{cr} increases with decreasing component reliability, ($\neq I^B$)

Exercise 2: Criticality - Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94
- Recall the result of the I_j^B in Exercise 1:

Configuration #	*System configuration* *(system components)*	*R*	I_1^B	I_2^B	I_3^B
I	Series (1-2)	0.9408	r_2=0.96	**r_1=0.98**	/
II	Parallel (1-2)	0.9992	**1-r_2=0.04**	1-r_1=0.02	/
III	Parallel (1-2-3)	0.999952	**0.0024**	0.0012	0.0008
IV	2-out-of-3 (1-2-3)	0.9957	$r_2 + r_3 - 2r_2r_3$=**0.0952**	0.0776	**0.0584**

$$I_j^{CR}(t) = \frac{I_j^B(t)\cdot\left(1-r_j(t)\right)}{1-R(\underline{r}(t))}$$

Series $r_1 - r_2$:

$$I_1^{cr} = \frac{I_1^B(1-r_1)}{1-R} = \frac{0.96(1-0.98)}{1-0.9408} = 0.3243; \quad I_2^{cr} = \frac{I_2^B(1-r_2)}{1-R} = \frac{0.98(1-0.96)}{1-0.9408} = 0.6622$$

Parallel $r_1 - r_2$:

$$I_1^{cr} = \frac{I_2^B(1-r_1)}{1-R} = \frac{0.04(1-0.98)}{1-0.9992} = 1; \quad I_2^{cr} = \frac{I_2^B(1-r_2)}{1-R} = \frac{0.02(1-0.96)}{1-0.9992} = 1$$

Exercise 2: Criticality - Solution

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$

Configuration #	*System configuration* *(system components)*	I_1^{cr}	I_2^{cr}	I_3^{cr}
I	Series (1-2)	$\frac{I_1^B(1-r_1)}{1-r_1r_2} = 0.3243$	$\frac{I_2^B(1-r_2)}{1-r_1r_2} = 0.662$	/
II	Parallel (1-2)	$\frac{I_1^B(1-r_1)}{1-r_1-r_2+r_1r_2} = 1$	1	/
II	Parallel (1-2-3)	1	1	1
IV	2-out-of-3 (1-2-3)	**0.4428**	0.7219	**0.8149**

- In a **series** system, the most important component according to I^{cr} is the least reliable one ($=I^B$)
- In a simple **parallel**, the criticality of all component is the same (if the system is failed, it will start operates, independently from the component which is repaired)
- In a 2 out of 3 configuration, I^{cr} increases with decreasing component reliability, ($\neq I^B$)

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Fussel-Vesely’s IM

Fussel-Vesely importance measure

- The component contributes to system failure when a minimal cut set (mcs) containing the component occurs
- $I_j^{FV}(t) =$ probability that at least one mcs containing *j* is verified at time *t*, given that the system is failed at *t*
- Let:
  - m_j = number of mcs containing component *j*, *j* = 1, 2, …, *n*
  - M_h^j = {*h*-th mcs among those containing component *j* is verified at time *t*}
  - $D_j(t)$ = event that {at least one mcs that contains component *j* is verified at time *t* }:

$$D_j(t) = M_1^j(t) \cup M_2^j(t) \cup \cdots \cup M_{m_j}^j(t)$$

Fussel-Vesely importance measure (2)

- $$I_j^{FV}(t) = P\left\{D_j(t) \mid \Phi\left[\underline{X}(t)\right]=0\right\} = \frac{P\left\{D_j(t) \cap \Phi\left[\underline{X}(t)\right]=0\right\}}{P\left\{\Phi\left[\underline{X}(t)\right]=0\right\}} = \frac{P\left\{D_j(t)\right\}}{P\left\{\Phi\left[\underline{X}(t)\right]=0\right\}}$$

- **Hypotheses for a rough approximation**:
  - **Independent** components $\longrightarrow$ $P(M_h^j(t)) = \prod_{l \in M_h^j} (1 - r_l(t))$
  - **Independence** of cut-sets containing *j* $\rightarrow$ $P(D^j(t)) \cong 1 - \prod_{h=1}^{m_j} \left(1 - P\left(M_h^j(t)\right)\right)$

$$I_j^{FV}(t) \cong \frac{1 - \prod_{h=1}^{m_j} \left(1 - P\left(M_h^j(t)\right)\right)}{1 - R(\underline{r}(t))}$$

  - **Rare-event** approximation

$$I_j^{FV}(t) \cong \frac{\sum_{h=1}^{m_j} P\left(M_h^j(t)\right)}{1 - R\left(\underline{r}(t)\right)} = \frac{F(t) - F_J^-(t)}{F(t)}$$

$F(t) = the\ present\ (\text{nominal})\ failure\ (Unreliability)\ level$

$F_j^-(t) = the\ decreased\ failure\ (Unreliability)\ level\ with\ component\ j\ perfectly\ relaible$

Exercise 3: Fussel-Vesely

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	I_1^{FV}	I_2^{FV}	I_3^{FV}
I	Series (1-2)			
II	Parallel (1-2)			
III	Parallel (1-2-3)			
IV	2-out-of-3 (1-2-3)			

Exercise 3: Fussel-Vesely

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$

Configuration #	*System configuration* *(system components)*	I_1^{FV}	I_2^{FV}	I_3^{FV}
I	Series (1-2)			
II	Parallel (1-2)			
III	Parallel (1-2-3)			
IV	2-out-of-3 (1-2-3)			

- Value similar to I^{cr}: both aim at quantifying the contribution of the failure of component *j* to the system failure probability
- In the **parallel** system configuration, the system itself constitutes a minimal cut set → $I_1^{FV} = I_2^{FV} = 1.$

Exercise 3: Fussel-Vesely – Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94
- Recall:

Series $r_1 - r_2$: $R(t) = r_1 r_2$

$$F(t) = 1 - r_1 r_2 \quad F(t) = q_1 + q_2 - q_1 q_2$$

Parallel $r_1 - r_2$: $R(t) = r_1 + r_2 - r_1 r_2$

$$F(t) = 1 - (r_1 + r_2 - r_1 r_2) \quad F(t) = q_1 q_2$$

Parallel $r_1 - r_2 - r_3$: $R(t) = r_1 + r_2 + r_3 - r_1 r_2 - r_1 r_3 - r_2 r_3 + r_1 r_2 r_3$

$$F(t) = 1 - (r_1 + r_2 + r_3 - r_1 r_2 - r_1 r_3 - r_2 r_3 + r_1 r_2 r_3) \quad F(t) = q_1 q_2 q_3$$

Exercise 3: Fussel-Vesely – Solution

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$

Series $r_1 - r_2$: $F(t) = q_1 + q_2 - q_1 q_2$ $\quad F_1^-(t) = q_2$ $\quad F_2^-(t) = q_1$

$$I_1^{FV}(t) = \frac{F(t) - F_1^-(t)}{F(t)} = \frac{q_1 + q_2 - q_1 q_2 - q_2}{F(t)} = \frac{q_1 - q_1 q_2}{F(t)} \approx \frac{q_1}{1 - R(t)} = \frac{1 - r_1}{1 - r_1 r_2} = 0.3378$$

$$I_2^{FV}(t) = \frac{F(t) - F_2^-(t)}{F(t)} = \frac{q_1 + q_2 - q_1 q_2 - q_1}{F(t)} = \frac{q_2 - q_1 q_2}{F(t)} \approx \frac{q_2}{1 - R(t)} = \frac{1 - r_2}{1 - r_1 r_2} = 0.6757$$

Parallel $r_1 - r_2$: $F(t) = q_1 q_2$ $\quad F_1^-(t) = 0$ $\quad F_2^-(t) = 0$

$$I_1^{FV}(t) = \frac{F(t) - F_1^-(t)}{F(t)} = \frac{q_1 q_2 - 0}{F(t)} = \frac{q_1 q_2}{F(t)} = \frac{q_1 q_2}{q_1 q_2} = 1$$

Exercise 3: Fussel-Vesely – Solution

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$

Configuration #	*System configuration* *(system components)*	I_1^{FV}	I_2^{FV}	I_3^{FV}
I	Series (1-2)	$\frac{1-r_1}{1-r_1 r_2} = 0.3378$	$\frac{1-r_2}{1-r_1 r_2} = 0.6757$	/
II	Parallel (1-2)	1	1	/
III	Parallel (1-2-3)	1	1	1
III	2-out-of-3 (1-2-3)	0.4651	0.7442	0.8372

- Value similar to I^{cr}: both aim at quantifying the contribution of the failure of component *j* to the system failure probability
- In the **parallel** system configuration, the system itself constitutes a minimal cut set → $I_1^{FV} = I_2^{FV} = 1.$

Exercise 3: Fussel-Vesely

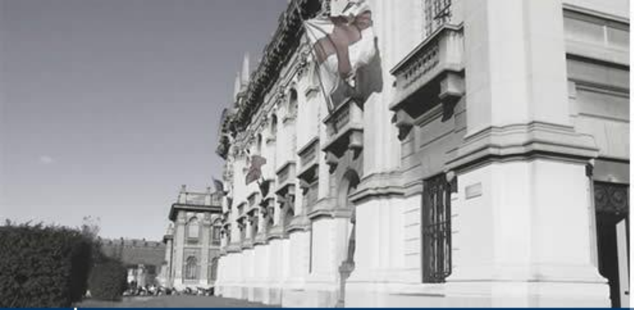
- Consider the difference between I_1^{CR} and I_1^{FV} for the series:
  - $I_1^{CR} = \frac{r_2(1-r_1)}{1-r_1 r_2}$
  - $I_1^{FV} = \frac{(1-r_1)}{1-r_1 r_2}$

Same denominators = System Unreliability

Different Numerators!

- FV: component 1 contributes to the system failure when it fails (its single component cut set is verified), independently from the state of component 2
- CR: component 1 contributes to the risk only when it is component 1 that causes the system failure, i.e. it is failed but component 2 is working

FV Importance measure takes into account that a component may contribute to the system risk measure without being critical



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Risk Achievement Worth (RAW) IM & Risk Reduction Worth (RRW) IM

Risk Achievement Worth (RAW)

- Problem: which is the worth of a component in achieving the present risk?
- Method: remove the component and then determine how much the risk is increased
- Definition:

$$RAW_j(t) = \frac{F\left[q_j = 1, \underline{q}(t)\right]}{F(t)} = \frac{F_j^+(t)}{F(t)}$$

$$F(t) = the\ present\ (\text{nominal}) failure\ (Unreliability)\ level$$
$$F_j^+(t) = the\ increased failure\ (Unreliability) level\ with\ component\ j\ always\ failed$$

Ratio of the risk when component *j* is considered **always failed** in (0,*t*) (q_j = 1, X_j = 0) and the current value of risk

- The RAW is also known by the term **risk increase factor**.
- It highlights the importance of **maintaining** the current level of reliability of component *j*
- It is used for maintenance and surveillance decisions.
- Appropriate for **temporary** changes, may be misleading for **permanent** changes (often not complete unavailability).

Risk Reduction Worth (RRW)

- Problem: which is the worth of a component in reducing the present risk?
- Method: "optimize" the component and then determine how much the risk is decreased
- Definition:

$$RRW_j(t) = \frac{F(t)}{F\left[q_j(t)=0, \underline{q}(t)\right]} = \frac{F(t)}{F_j^-(t)}$$

$F_j^-(t) = the\ decreased failure\ (Unreliability) level\ with\ component\ j\ perfectly\ relaible$

Ratio of the current value of risk and the risk when component j is always available in (0,t) $(q_j = 0, X_j = 1)$

- The RRW is known also by the term **risk decrease factor**.
- It measures the potential of component *j* in reducing the risk
- Useful for identifying or optimizing improvements towards risk reduction (e.g. useful in plant upgrading programs and backfitting activities)

Exercise 4: RAW & RRW

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration* *(system components)*	RAW_1	RAW_2	RAW_3
I	Series (1-2)			
II	Parallel (1-2)			
III	2-out-of-3 (1-2-3)			

Configuration #	*System configuration* *(system components)*	RRW_1	RRW_2	RRW_3
I	Series (1-2)			
II	Parallel (1-2)			
III	2-out-of-3 (1-2-3)			

Exercise 4: RAW & RRW – Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

- Recall:

Series $r_1 - r_2$: $F(t) = q_1 + q_2 - q_1 q_2$

Parallel $r_1 - r_2$: $F(t) = q_1 q_2$

Parallel $r_1 - r_2 - r_3$: $F(t) = q_1 q_2 q_3$

$$RAW_j(t) = \frac{F_j^+(t)}{F(t)} \qquad RRW_j(t) = \frac{F(t)}{F_j^-(t)}$$

Exercise 4: RAW & RRW – Solution

- Consider various configurations of 3 binary components with $r_1 = 0.98$, $r_2 = 0.96$, $r_3 = 0.94$
- FOR:

Series $r_1 - r_2$: $F(t) = q_1 + q_2 - q_1 q_2$

$$F_1^+(t) = 1 + q_2 - 1.q_2 = 1 \qquad F_1^-(t) = q_2$$

$$F_2^+(t) = q_1 + 1 - q_1.1 = 1 \qquad F_2^-(t) = q_1$$

$$RAW_1(t) = \frac{F_1^+(t)}{F(t)} = \frac{1}{q_1 + q_2 - q_1 q_2} = \frac{1}{(1-0.98)+(1-0.96)-(1-0.98)(1-0.96)} = 16.89$$

$$RAW_2(t) = \frac{F_2^+(t)}{F(t)} = \frac{1}{q_1 + q_2 - q_1 q_2} = 16.89$$

$$RRW_1(t) = \frac{F(t)}{F_1^-(t)} = \frac{q_1 + q_2 - q_1 q_2}{q_2} = 1.48 \qquad RRW_2(t) = \frac{F(t)}{F_2^-(t)} = \frac{q_1 + q_2 - q_1 q_2}{q_1} = 2.96$$

Exercise 4: RAW & RRW – Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94
- FOR:

Parallel $r_1 - r_2$: $F(t) = q_1 q_2$

$$F_1^+(t) = 1.q_2 = q_2 \qquad F_1^-(t) = 0$$

$$F_2^+(t) = q_1.1 = q_1 \qquad F_2^-(t) = 0$$

$$RAW_1(t) = \frac{F_1^+(t)}{F(t)} = \frac{q_2}{q_1 q_2} = \frac{1}{q_1} = \frac{1}{(1-0.98)} = 50$$

$$RAW_2(t) = \frac{F_2^+(t)}{F(t)} = \frac{q_1}{q_1 q_2} = \frac{1}{q_2} = \frac{1}{(1-0.96)} = 25$$

$$RRW_1(t) = \frac{F(t)}{F_1^-(t)} = \frac{q_1 q_2}{0} = \infty \qquad RRW_2(t) = \frac{F(t)}{F_2^-(t)} = \frac{q_1 q_2}{0} = \infty$$

Exercise 4: RAW & RRW – Solution

- Consider various configurations of 3 binary components with r_1 = 0.98, r_2 = 0.96, r_3 = 0.94

Configuration #	*System configuration (system components)*	RAW_1	RAW_2	RAW_3
I	Series (1-2)	$\frac{1}{q_1+q_2-q_1q_2}=16.89$	$\frac{1}{q_1+q_2-q_1q_2}=16.89$	/
II	Parallel (1-2)	$\frac{q_2}{q_1q_2}=\frac{1}{q_1}=50$	$\frac{1}{q_2}=25$	/
III	2-out-of-3 (1-2-3)	22.67	18.31	13.75

largest if the most reliable component is taken out of service

Configuration #	*System configuration (system components)*	RRW_1	RRW_2	RRW_3
I	Series (1-2)	$\frac{q_1+q_2-q_1q_2}{q_2}=1.48$	$\frac{q_1+q_2-q_1q_2}{q_1}=2.96$	/
II	Parallel (1-2)	$\frac{q_1q_2}{0}=\infty$	$\frac{q_1q_2}{0}=\infty$	/
III	2-out-of-3 (1-2-3)	1.79	3.58	5.38

Largest for the components which contribute most to the system failure, i.e, the least reliable

Exercise 4: RAW & RRW – Solution

- In a **series**, components have the same RAW and are ranked by RRW in increasing order of failure probability

- In a **parallel**, components have the same RRW and are ranked by RAW in decreasing order of failure probability

- In a **parallel** the achievement in risk (RAW) is highest if the most reliable component is **taken out of service**

- In a **series** the reduction in risk (RRW) achievable by improving the component to **perfection** is highest for the components which contribute most to the system failure, i.e, the least reliable

Comments

Which IMs are more appropriate to rank or categorize components by risk-significance or by safety-significance?

- Let us write the risk metric *F:*

$$F(t) = q_j(t) \cdot \left(F_j^+(t) - F_j^-(t)\right) + F_j^-(t)$$

pivotal decomposition

$$F = \alpha_j \cdot q_j + \beta_j$$

- q_j = unavailability of j

$\alpha_j = F_j^+ - F_j^-$ coefficient of q_j in the risk equation

$\beta_j = F_j^-$ collection of all the other terms of F with $q_j = 0$

Comments (2)

- **Fussel-Vesely**

$$I_j^{FV} \approx \frac{F - F_j^-}{F} = \frac{\alpha_j q_j + \beta_j - \beta_j}{\alpha_j q_j + \beta_j} = \frac{\alpha_j q_j}{\alpha_j q_j + \beta_j}$$

  - High risk installation (very redundant) → failure of a single component is typically not critical → $\alpha_j q_j \ll \beta_j$

$$I_j^{FV} \approx \frac{\alpha_j}{\beta_j} q_j$$

- proportional to the unreliability of component *j*
- *It represents the contribution of component j to risk*

How much the component failure/unavailability is contributing to the systerm risk?

Fussel-Vesely = risk significance IM

Comments (2)

Risk Achievement Worth:

$$RAW_j = \frac{F_j^+}{F} = \frac{\alpha_j + \beta_j}{\alpha_j q_j + \beta_j}$$

- High risk installation (very redundant) → failure of a single component is typically not critical → $\alpha_j q_j \ll \beta_j$

$$RAW_j \approx \frac{\alpha_j}{\beta_j} + 1$$

- RAW_j is independent on q_j → degree of defence against failure provided by the rest of the installation
- A large RAW_j means component *j* is highly safety-significant since the risk increase due to the unavailability of the component is high → prevention!

How much the component is important in the prevention of the system failure?

Risk Achievement Worth = safety significance IM

Comments (3)

- **Birnbaum** → appropriate for establishing test and maintenance programs (jointly to RAW to assess the impact in terms of loss of safety of taking the component out of service)
- **Fussel-Vesely** and **Birnbaum** → appropriate for the system design phase:
  - I^{FV} is used for selecting the components candidate for improvement because most contributing to the risk
  - I^{B} allows identifying for which components the improvements are more effective
- **Criticality** → appropriate for identifying components most probably causing system failure (help set up a repair priority checklist)

Drawbacks

1. IMs deal with changes in risk only **at the extremes (0,1)** of the defined ranges of probability

2. IMs rank only individual components or basic events, whereas they are not directly applicable to combinations or groups (e.g. change in technical specifications or component’s failures made up of more basic events)

3. IMs have been mainly applied to systems made up of binary components (i.e. functioning or faulty)

4. IMs have been mainly applied to systems made up of binary components (i.e. functioning or faulty)

Lead to the development of

Generalized Importance Measure

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Generalized Importance Measure

1. Generalized risk importance measure

- Consider the following **relative change in risk** due to a change in the probability of the basic failure event *j* from the value q_j to the value q_j^n

Pivotal Decomposition:

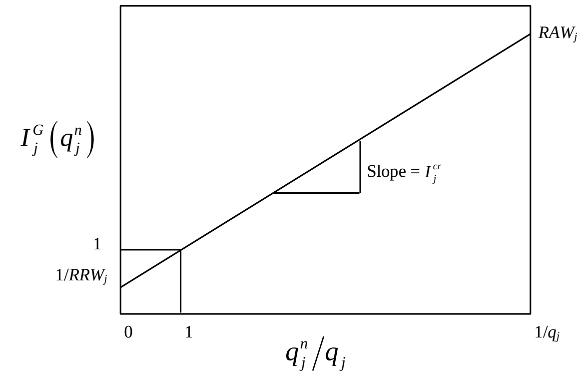
$$F(t) = F_j^-(t) + q_j(t)\cdot\left(F_j^+(t) - F_j^-(t)\right)$$
$$F_j^n(t) = F_j^-(t) + q_j^n(t)\cdot\left(F_j^+(t) - F_j^-(t)\right)$$

$$\frac{\Delta F_j}{F} = \frac{F_j^n - F}{F} = \frac{\left(F_j^+(t) - F_j^-(t)\right)\cdot\left(q_j^n - q_j\right)}{F}$$

$$I_j^G\left(q_j^n\right) = \frac{F_j^n}{F} = \left\{F\left[q_j = 1, \underline{q}(t)\right] - F\left[q_j = 0, \underline{q}(t)\right]\right\}\left(\frac{q_j^n - q_j}{F}\right) + 1$$

$$I_j^G\left(q_j^n\right) = \frac{F_j^n}{F} = I_j^{cr}\left(\frac{q_j^n}{q_j}\right) + \frac{1}{RRW_j}$$

Risk Impact Curve



2. Importance measures for multiple basic events

- No simple relation between the importance measures of single components and a group → **no general approach!**
- Let us consider the indicator variable T of the top event of a given fault tree can be written as:

$$T = AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH$$

(C_k , *k =1,…, 4* are different failure modes of component C or different components in the same group)

$$RAW(C_1) = \frac{F_j^+}{F} = \frac{AB(1 + C_3) + DE(C_2 + C_4) + F(1 + C_3)(C_2 + C_4) + GH}{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH} \qquad \boxed{C_1 = 1}$$

- **RAW of the components of group C:**
  - Substitute $C_i = 1$? **NO**: a value of 2 appears in structure function
  - Adding RAWs for basic events? **NO**
  - Substitute $C_i = C$ and Boolean reduction and $C = 1$? **YES**

$$T = ABC + DEC + FC + GH$$

$$RAW(C) = \frac{AB + DE + F + GH}{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH}$$

2. Importance measures for multiple basic events (2)

- **Birnbaum**:

$$I_j^B = F\left[q_j = 1, \underline{q}(t)\right] - F\left[q_j = 0, \underline{q}(t)\right]$$

  - Substitute C_i = 1 with **no** Boolean reduction?

$$I_C^B = 2AB + 2DE + 4F$$

  - Substitute C_i = C and Boolean reduction and C = 1?

$$I_C^B = AB + DE + F$$

  - Both are **unappropriate!**

- **Fussel-Vesely**:

$$I_C^{FV}(C) = \frac{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4)}{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH}$$

  - Any cutset with a contribution from any C_k of the group is included!
  - Already appropriate measure of group importance!
  - No additivity ((i.e., not equal to the summation of all individual measures I_{C_k})

2. Importance measures for multiple basic events (3)

- **Differential Importance measure (DIM)** :
  - Sensitivity measure that ranks the parameters of the risk model according to the fraction of the total change in the risk that is due to a small change in the parameters' values, taken one at a time
  - Additivity!

$$F = F(p_1, p_2, \ldots, p_{Np})$$

$$dF = \frac{\partial F}{\partial p_1} \cdot dp_1 + \frac{\partial F}{\partial p_2} \cdot dp_2 + \ldots + \frac{\partial F}{\partial p_{N_P}} \cdot dp_{N_P}$$

$$\mathrm{DIM}\left(p_i\right) = \frac{dF_{p_i}}{dF} = \frac{\frac{\partial F}{\partial p_i} \cdot dp_i}{\frac{\partial F}{\partial p_1} \cdot dp_1 + \frac{\partial F}{\partial p_2} \cdot dp_2 + \ldots + \frac{\partial F}{\partial p_{N_P}} \cdot dp_{N_P}}$$

3. IMs uncertainty

- F and F_j- are to be considered as random variables characterized by given probability distributions

- IMs is a random variable for which specific statistics can be calculated!

4. Binary components

- Extensions to **multi-state** systems!