Monte Carlo Simulations: Exercise Session

04.04.25 | Luca Pinciroli

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EXERCISE 1

Exercise 1

Consider the Weibull distribution:

$$F_T(t) = 1 - e^{-\beta t^{\alpha}}, \qquad f_T(t) = \alpha\beta t^{\alpha-1} e^{-\beta t^{\alpha}}$$

with $\alpha = 1.5, \beta = 1$

1. Sample N=400 values from $f_T(t)$
2. Verify whether the obtained distribution provides a good approximation of the Weibull distribution. To this aim, you are required to:
   A. find the empirical probability density function (pdf) of the sampled value in 1
   B. compare the empirical pdf found in 2A. with the analytical Weibull distribution.
3. Provide an estimate G_N of $\int_0^{+\infty} t f_T(t) dt$
4. Estimate the variance of G_N

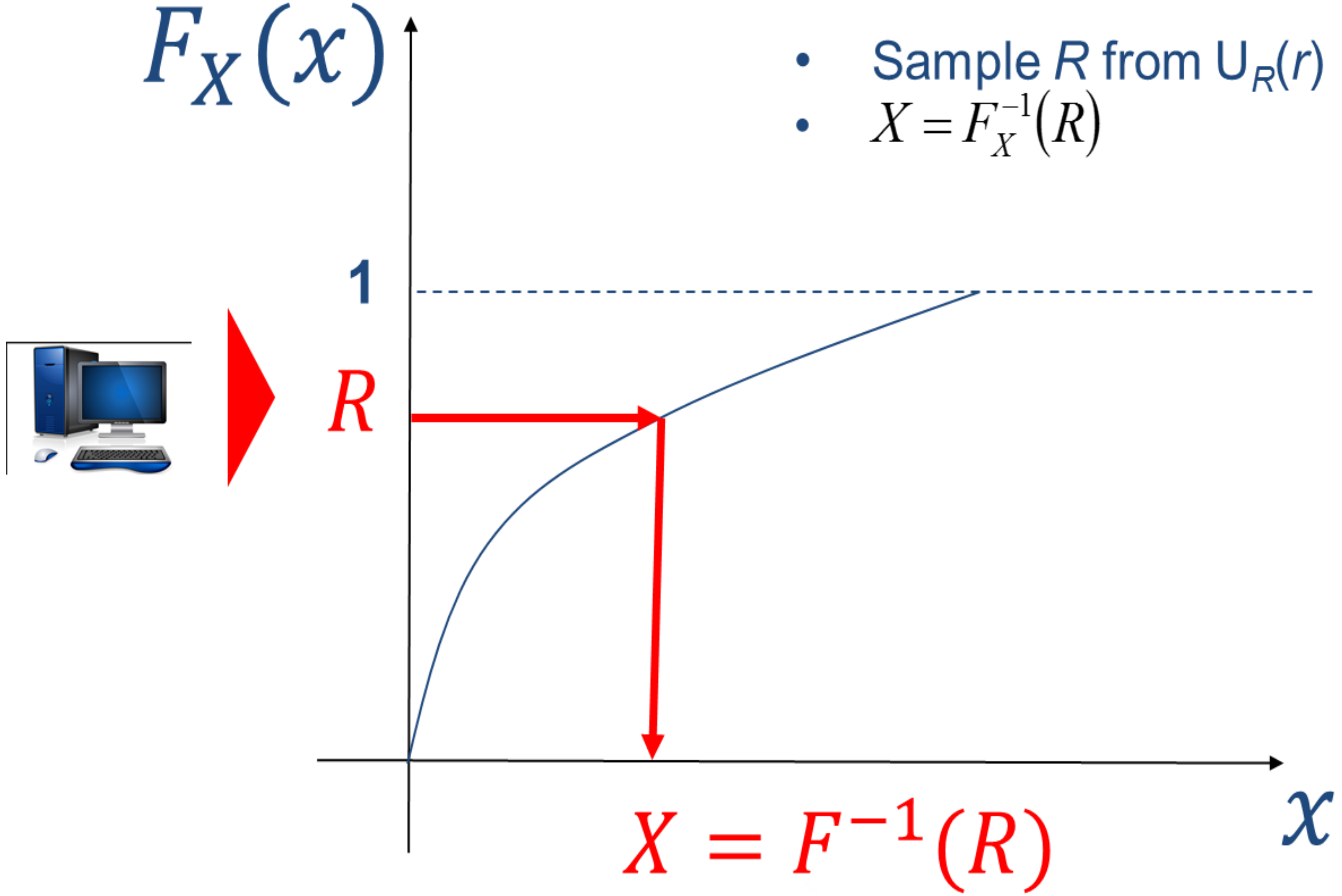
Useful commands

- np.random.rand(N): provides N random numbers sampled from a uniform distribution in the range [0,1)

- num_samples = matplotlib.pyplot.hist(Y, bins) bins the elements of Y into the bins defined by bins and returns the number of elements in each counter.

Sampling random number from $F_X(x)$

- Sample R from $U_R(r)$
- $X = F_X^{-1}(R)$



Example: Weibull Distribution

- Time-dependent hazard rate $\lambda(t) = \beta\alpha t^{\alpha-1}$

cdf: $$F_T(t) = P\{T \le t\} = 1 - e^{-\beta t^{\alpha}}$$

pdf: $$f_T(t) \cdot dt = P\{t \le T < t + dt\} = \alpha\beta t^{\alpha-1} e^{-\beta t^{\alpha}} \cdot dt$$

- Sampling a failure time *T* (by the inverse transform)

$$R \equiv F_R(r) = F_T(t) = 1 - e^{-\beta t^{\alpha}}$$

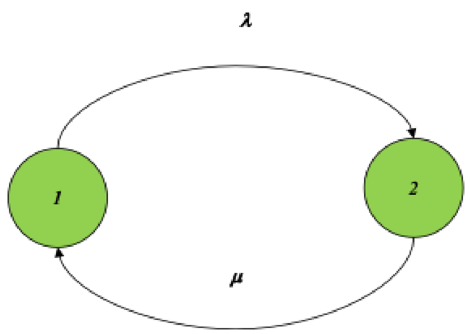
$$T = F_T^{-1}(R) = \left(-\frac{1}{\beta} \ln(1-R) \right)^{\frac{1}{\alpha}}$$

EXERCISE 2

Exercise 2

Write the MC code for the estimation of the **time dependent reliability** and **instantaneous availability** of a continuously monitored component with constant failure (λ) and repair (μ) rates

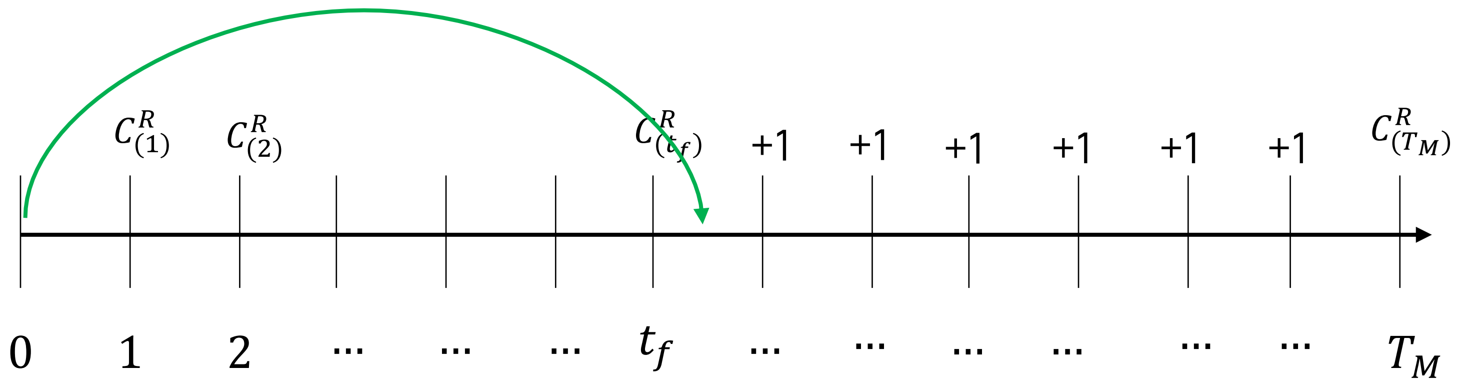
	values
λ	$3 \cdot 10^{-3}\ h^{-1}$
μ	$25 \cdot 10^{-3}\ h^{-1}$



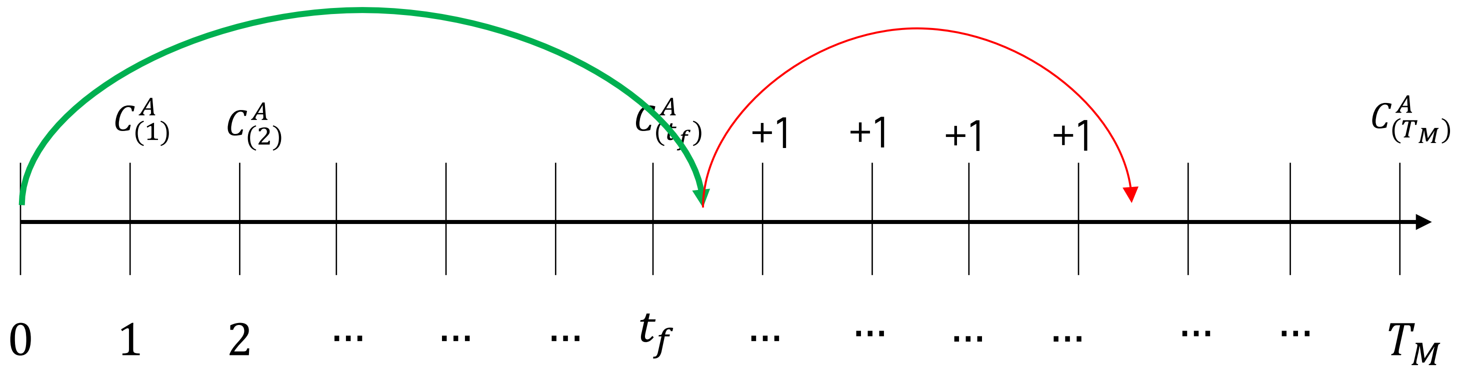
- You can assume a mission time of 10^3 time units
- You can compute the time dependent reliability and the instantaneous availability at all times: 0,1,2,3,…10^3

Exercise 2

Estimation of the System Reliability



Estimation of the System Availability

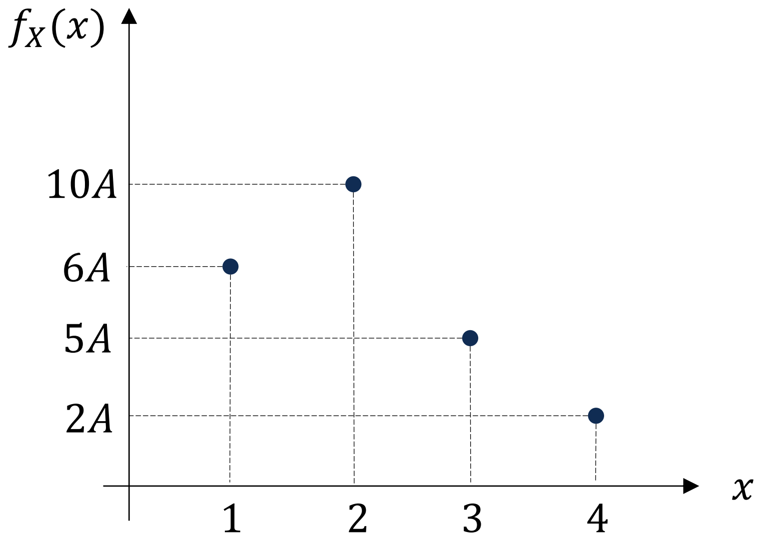


EXERCISE 3

Exercise 3

Consider the discrete probability distribution $f_X(x)$ in the graph:

1) Identify the value of the parameter A;
2) Compute the corresponding cumulative distribution;
3) Write a Matlab/Python code to sample N=10000 values from $f_X(x)$;
4) Verify that the samples are distributed according to $f_X(x)$.

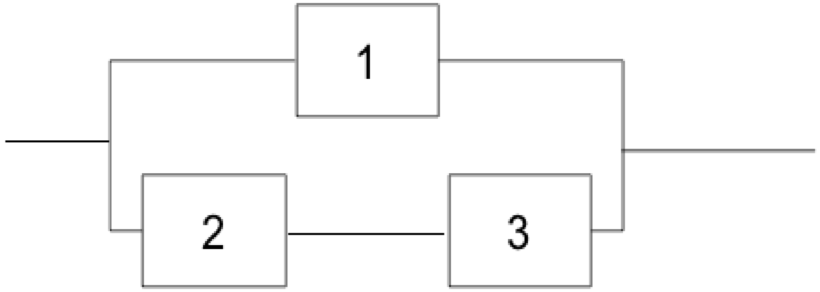


EXERCISE 4

Exercise 4

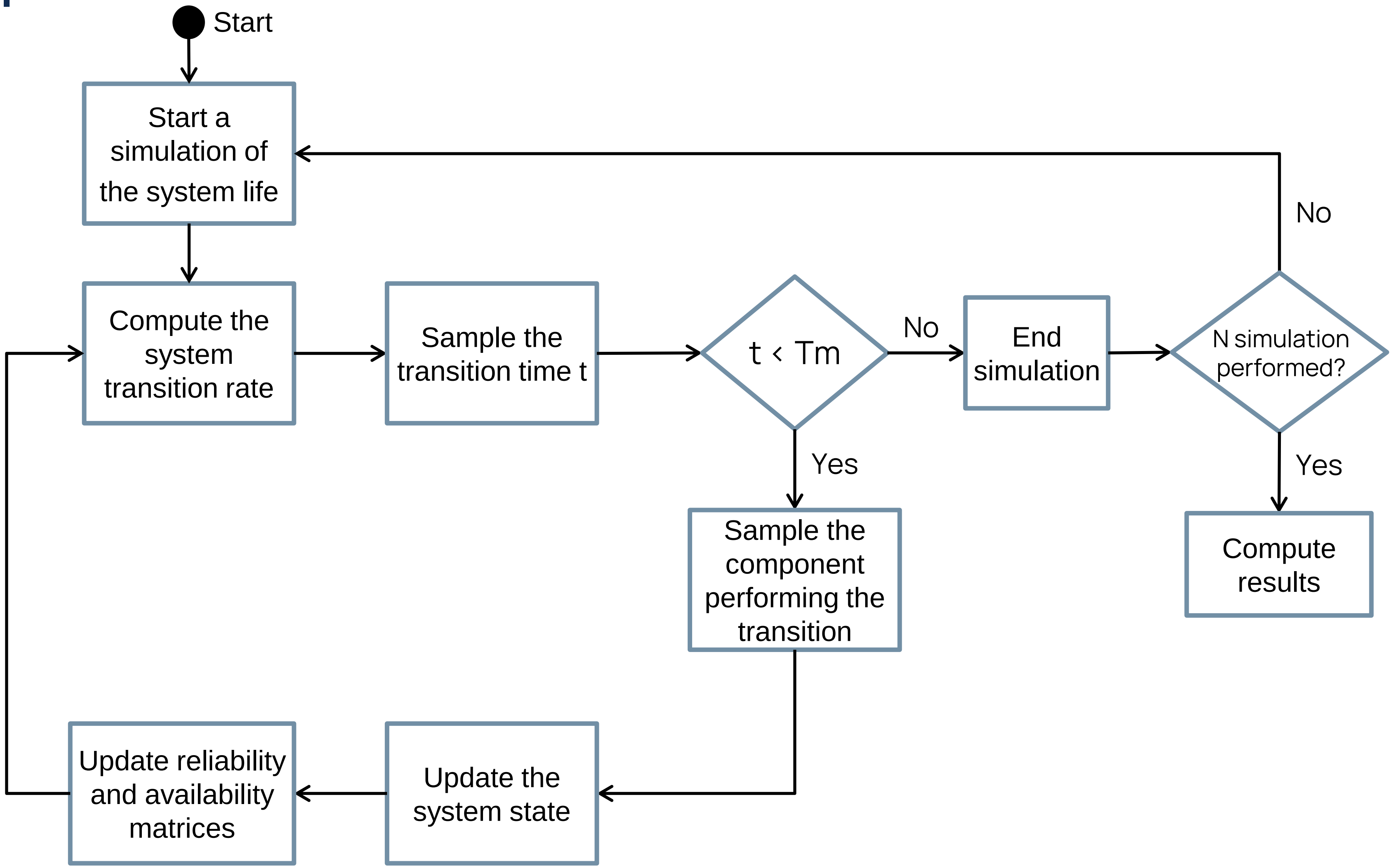
Consider the system in figure composed of three components (A, B, C). Each component can be in two different health states (1-nominal, 2-failed) with exponentially distributed transition times between them. Assuming a mission time $T = 500\ hours$, write the MC code for the estimation of:

- The time dependent reliability
- The instantaneous availability.
- The estimators uncertainty



	1	2	3
λ	$1\cdot 10^{-3}\ h^{-1}$	$2\cdot 10^{-2}\ h^{-1}$	$5\cdot 10^{-2}\ h^{-1}$
μ	$3\cdot 10^{-2}\ h^{-1}$	$5\cdot 10^{-2}\ h^{-1}$	$5\cdot 10^{-3}\ h^{-1}$

Flow diagram



Sampling the time of transition

- The rate of transition of the system out of its current configuration
- (1, 1, 1) is:

$$\lambda^{(1,1,1)} = \lambda_{1\to2}^{A} + \lambda_{1\to3}^{A} + \lambda_{1\to2}^{B} + \lambda_{1\to3}^{B} + \lambda_{1\to2}^{C} + \lambda_{1\to3}^{C}$$

- We are now in the position of sampling the first system transition time t_1, by applying the **inverse transform method**:

$$t_1 = t_0 - \frac{1}{\lambda^{(1,1,1)}} \ln(1 - R_t)$$

where $R_t \sim U[0,1)$

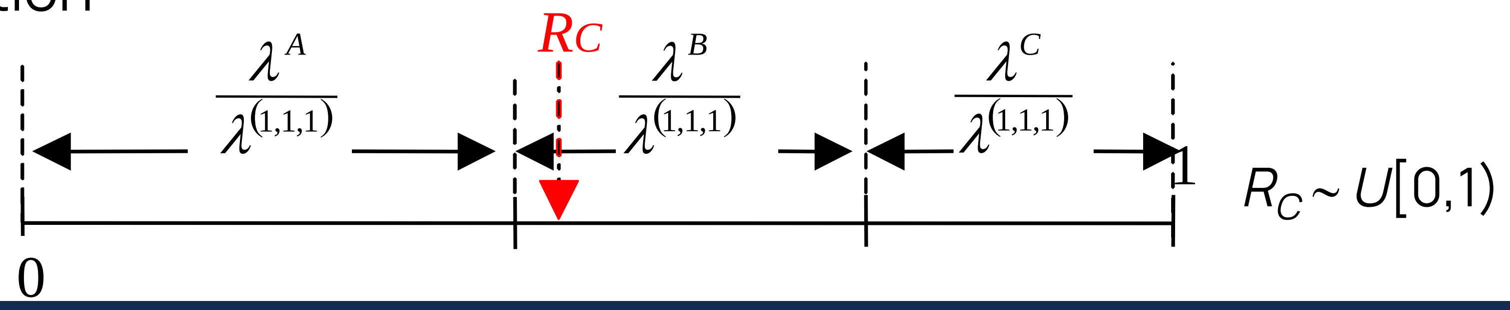
Sampling the component performing the Transition

- Assuming that $t_1 < T_M$ (otherwise we would proceed to the successive trial), we now need to determine which component has undergone the transition
- The probabilities of components A, B, C undergoing a transition out of their initial nominal states 1, given that a transition occurs at time t_1, are:

$$\frac{\lambda^A}{\lambda^{(1,1,1)}}, \quad \frac{\lambda^B}{\lambda^{(1,1,1)}}, \quad \frac{\lambda^C}{\lambda^{(1,1,1)}}$$

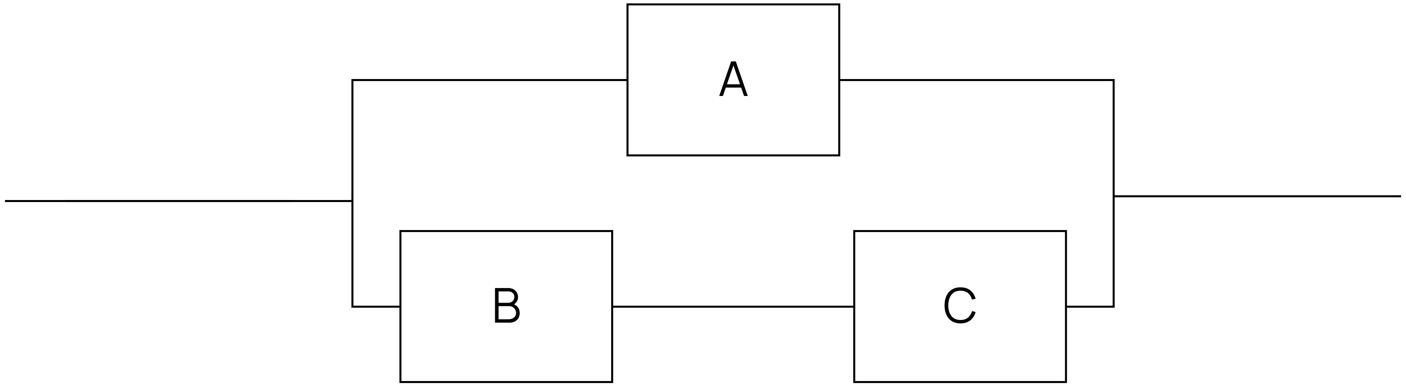
$$\lambda^A = \lambda^A_{1\to 2} + \lambda^A_{1\to 3} \qquad \lambda^B = \lambda^B_{1\to 2} + \lambda^B_{1\to 3} \qquad \lambda^C = \lambda^C_{1\to 2} + \lambda^C_{1\to 3}$$

- Thus, we can apply the inverse transform method to the discrete distribution



EXERCISE 5

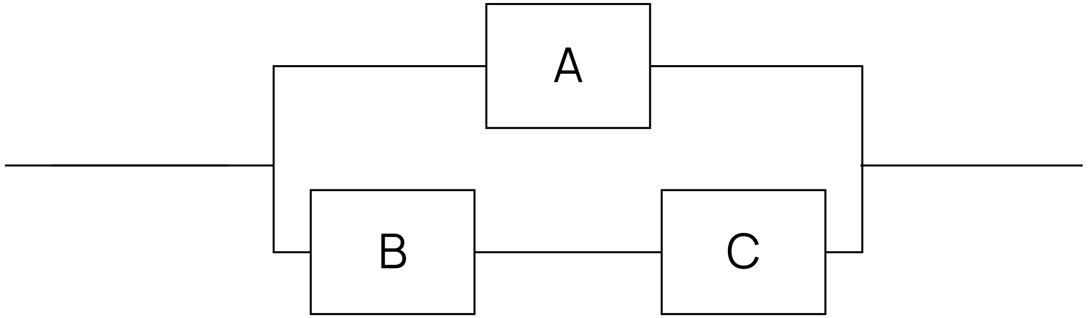
Exercise 5



- Components can be in three states and the time of transition from one state to another is exponentially distributed:

Initial \ Arrival	1	2	3
1(nominal)	0	$\lambda_{1\to2}^{A(B,C)}$	$\lambda_{1\to3}^{A(B,C)}$
2 (degraded)	0	0	$\lambda_{2\to3}^{A(B,C)}$
3 (failed)	$\lambda_{3\to1}^{A(B,C)}$	$\lambda_{3\to2}^{A(B,C)}$	0

Exercise 5



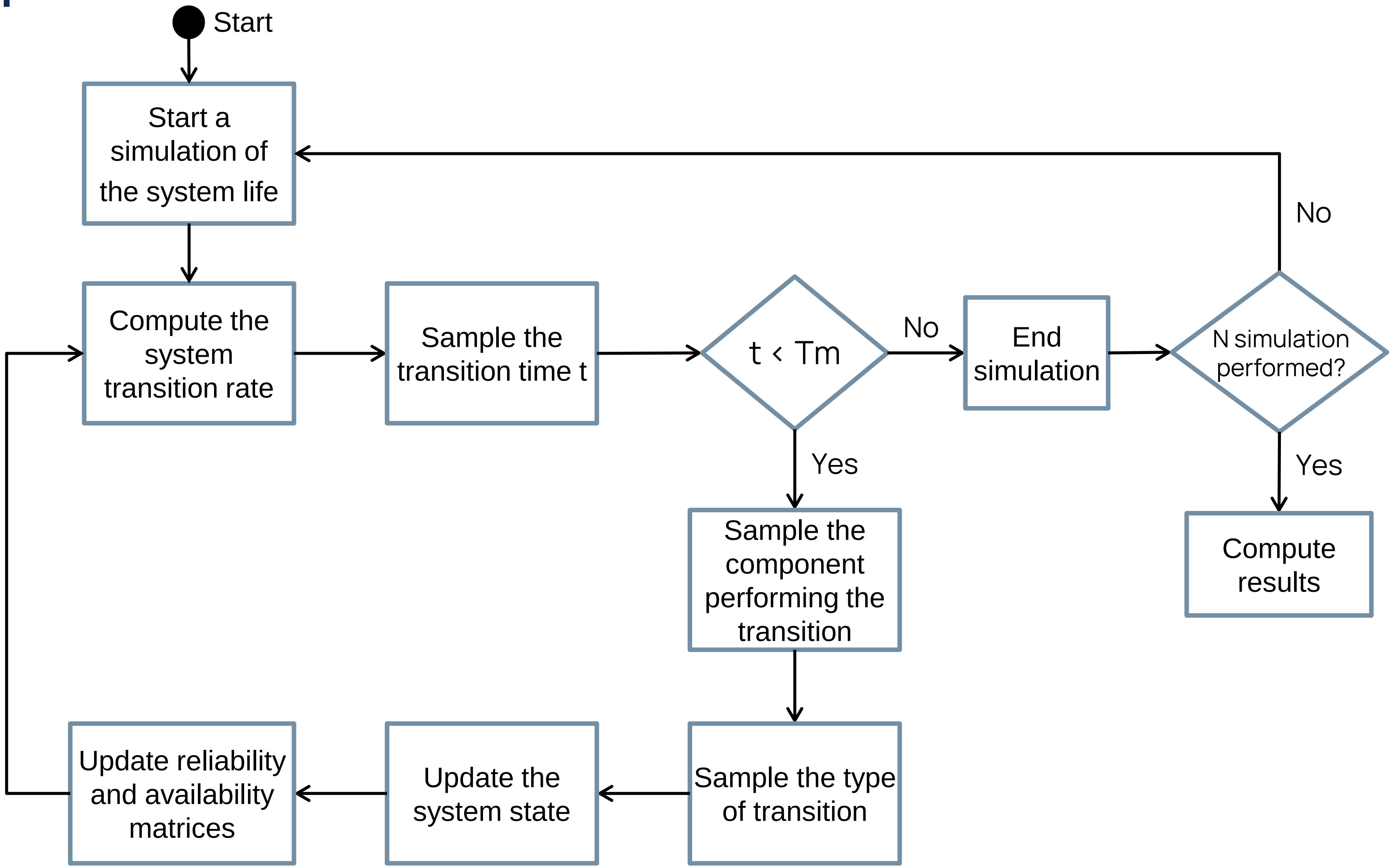
A	1	2	3
1	-	$3\ 10^{-3}$	10^{-3}
2	-	-	$6\ 10^{-3}$
3	$8\ 10^{-3}$	$5\ 10^{-3}$	-

B	1	2	3
1	-	$1\ 10^{-3}$	$5\ 10^{-3}$
2	-	-	$4\ 10^{-3}$
3	$7.5\ 10^{-3}$	$3.5\ 10^{-3}$	-

C	1	2	3
1	-	$8\ 10^{-3}$	$2.5\ 10^{-3}$
2	-	-	$2\ 10^{-3}$
3	$4\ 10^{-3}$	$1.5\ 10^{-3}$	-

- Estimate the **reliability** of the system at $T_{miss} = 4000$
- Estimate the **time dependent reliability** R(t)
- Estimate the **instataneous availability** A(t)

Flow diagram



Sampling the time of transition

- The rate of transition of the system out of its current configuration
- (1, 1, 1) is:

$$\lambda^{(1,1,1)} = \lambda^{A}_{1\to2} + \lambda^{A}_{1\to3} + \lambda^{B}_{1\to2} + \lambda^{B}_{1\to3} + \lambda^{C}_{1\to2} + \lambda^{C}_{1\to3}$$

- We are now in the position of sampling the first system transition time t_1, by applying the **inverse transform method**:

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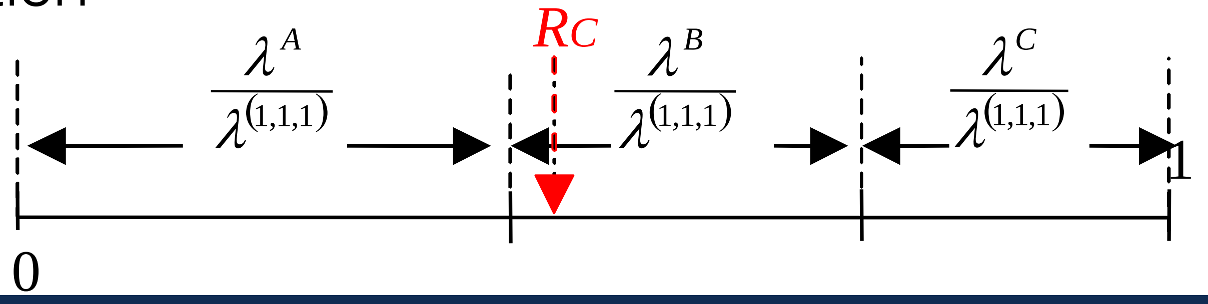
- Thus, we can apply the inverse transform method to the discrete distribution



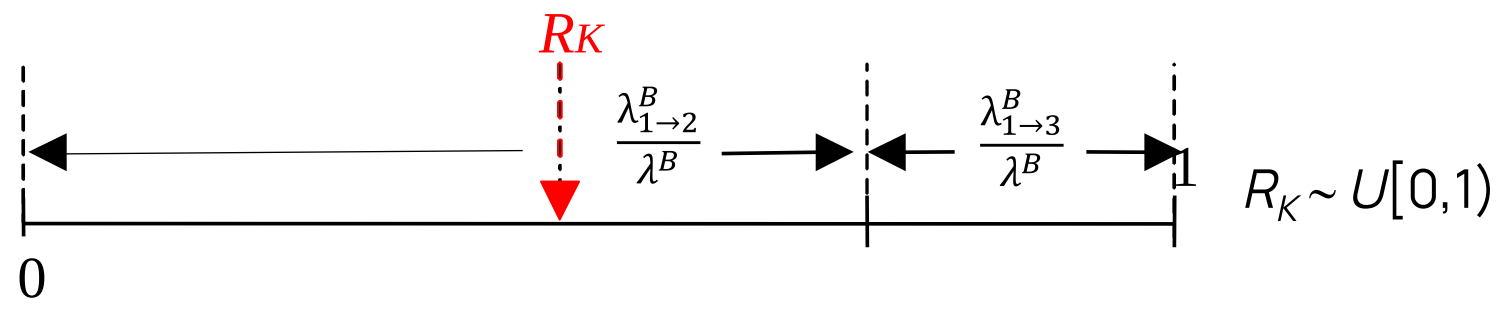
$R_C \sim U[0,1)$

Sampling the kind of Transition

- Since component B is the one undergoing the transition we need to sample the new state of component B.
- The probabilities of components B undergoing a transition out of their initial nominal states 1 given that a transition occurs at time t_1, are:

$$\frac{\lambda^B_{1\to2}}{\lambda^B} \qquad \frac{\lambda^B_{1\to3}}{\lambda^B}$$

- Thus, we can apply the inverse transform method to the discrete distribution



Thank you for your kind attention

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