

POLITECNICO
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FAULT DETECTION

Part 1 & 2

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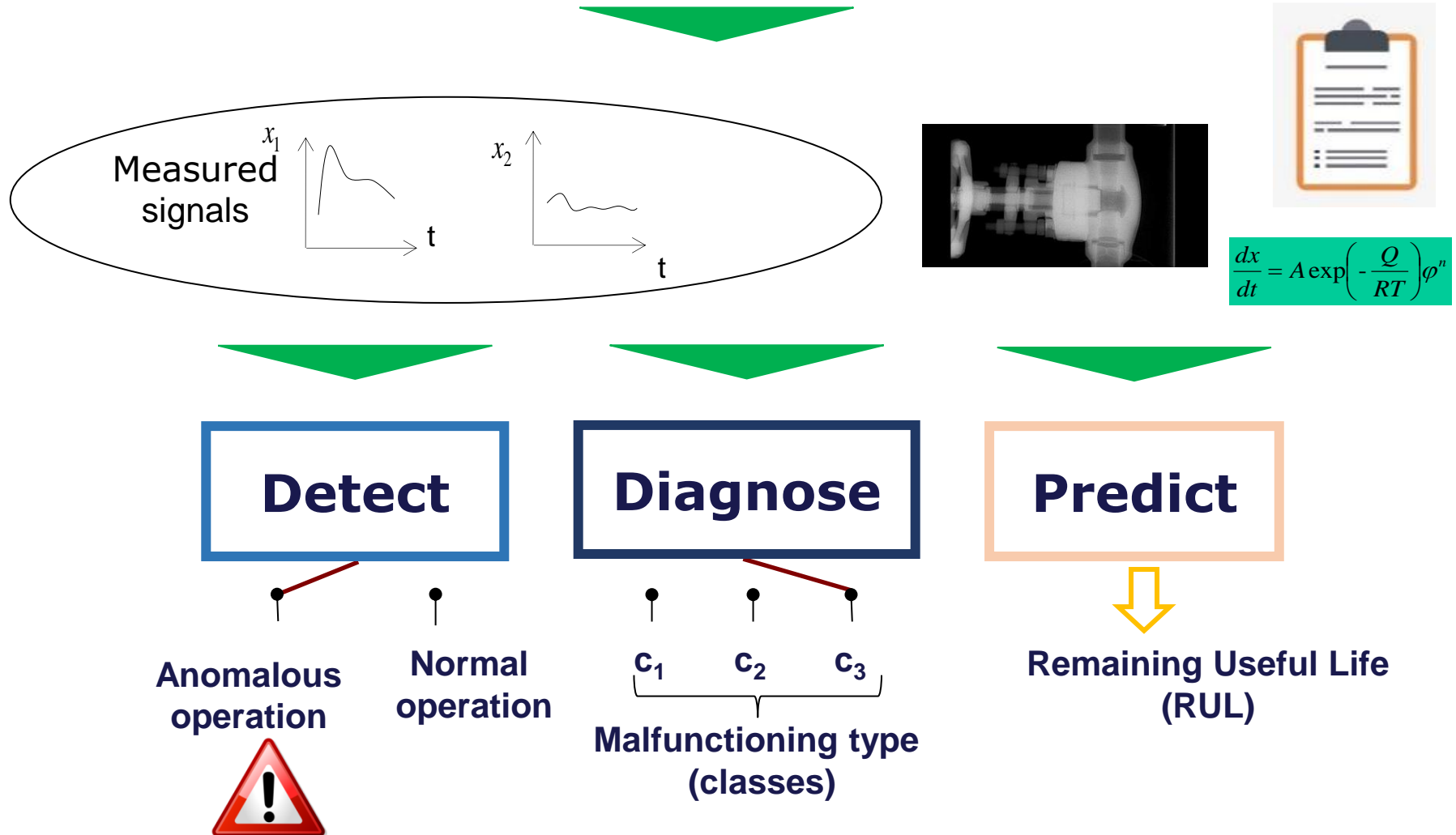
- Part 1: Model of the Equipment Behavior in Normal Condition
 - 1A) Auto Associative Kernel Regression (AAKR)
 - 1B) Principal Component Analysis (PCA)
- Part 2: Statistical Test
 - 2A) Thresholds-Based
 - 2B) Q-Statistics
 - 2C) Sequential Probability Ratio Test (SPRT)

Overview

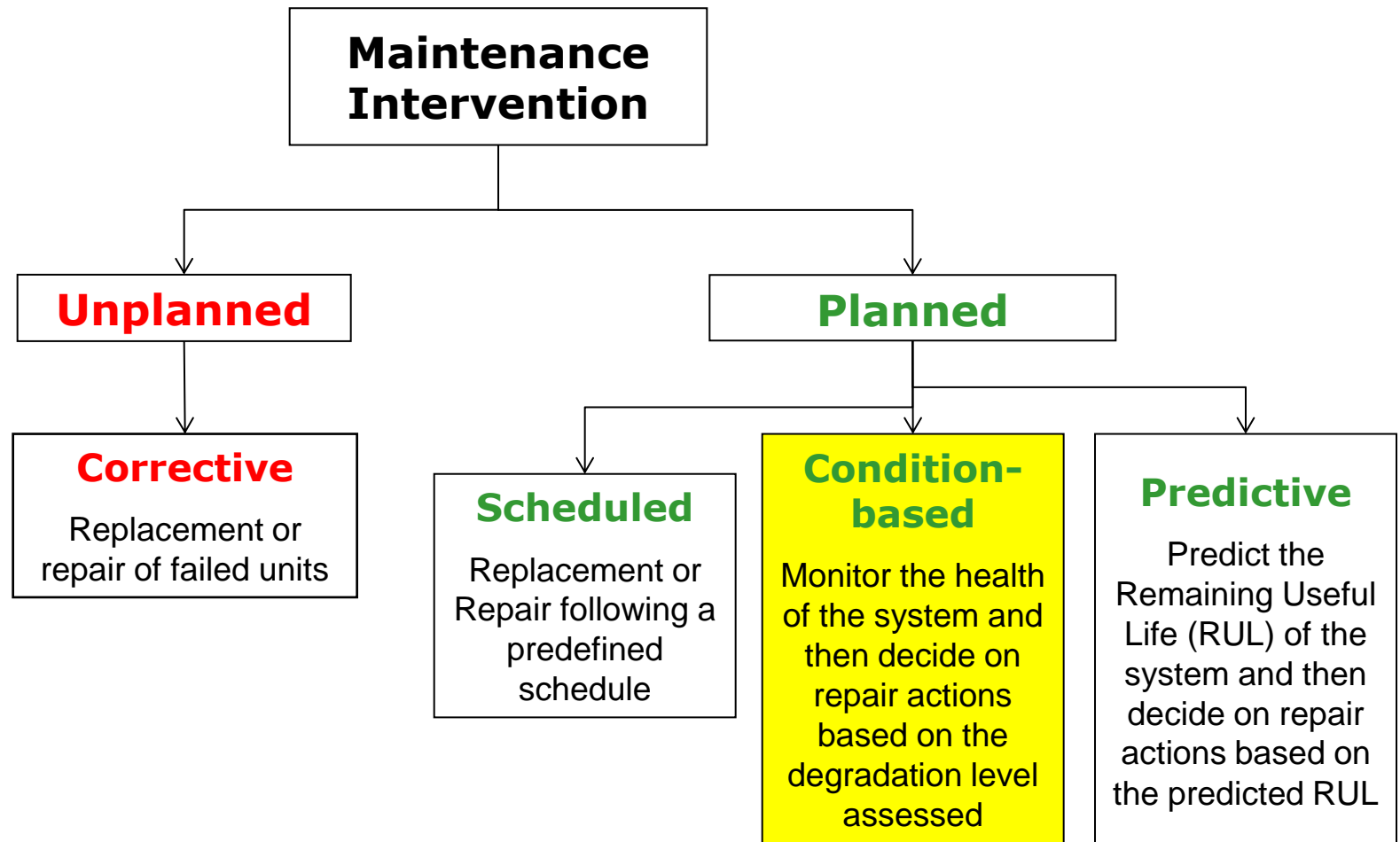


Context: Prognostics and Health Management (PHM)

Equipment (System, Structure or Component)

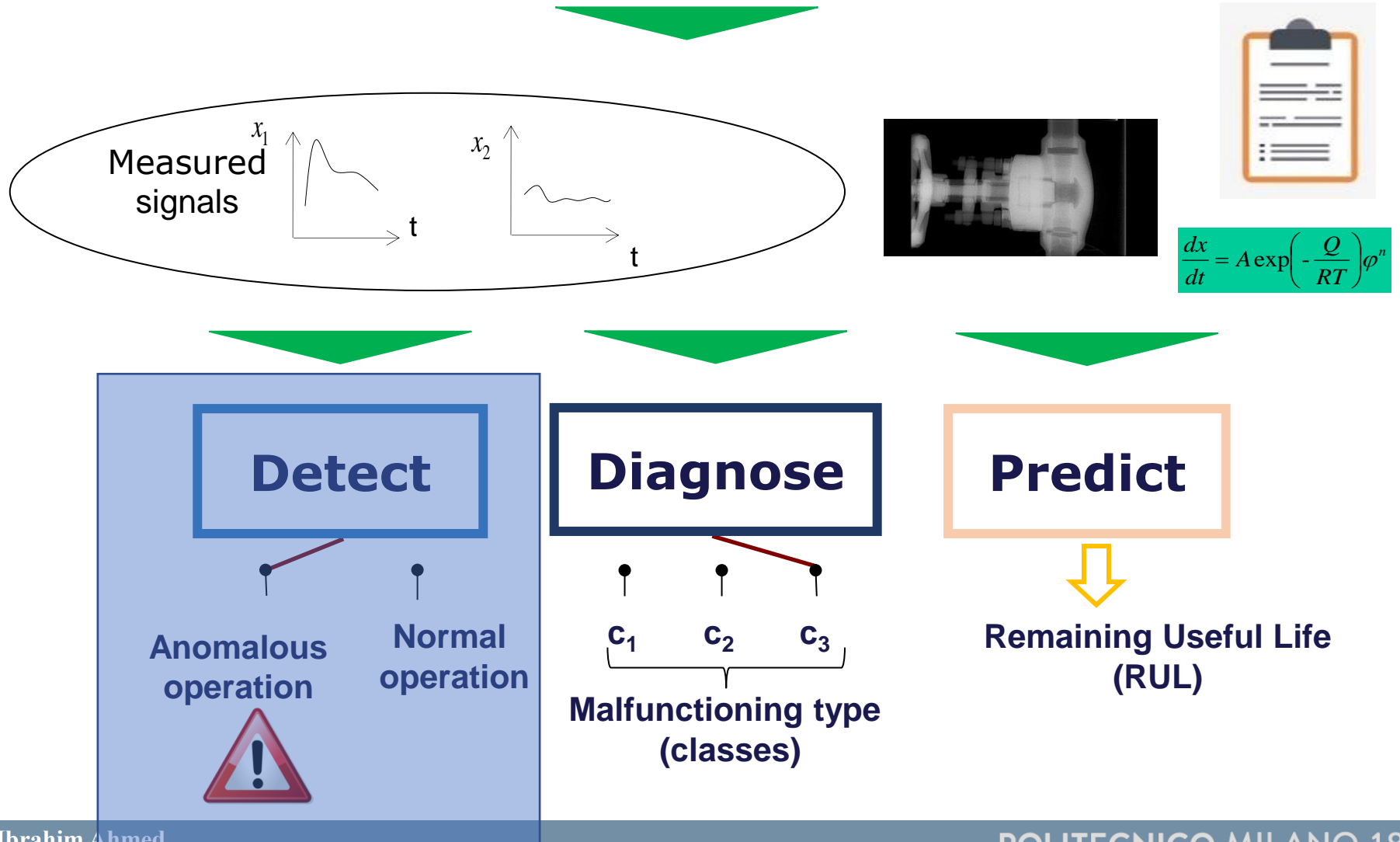


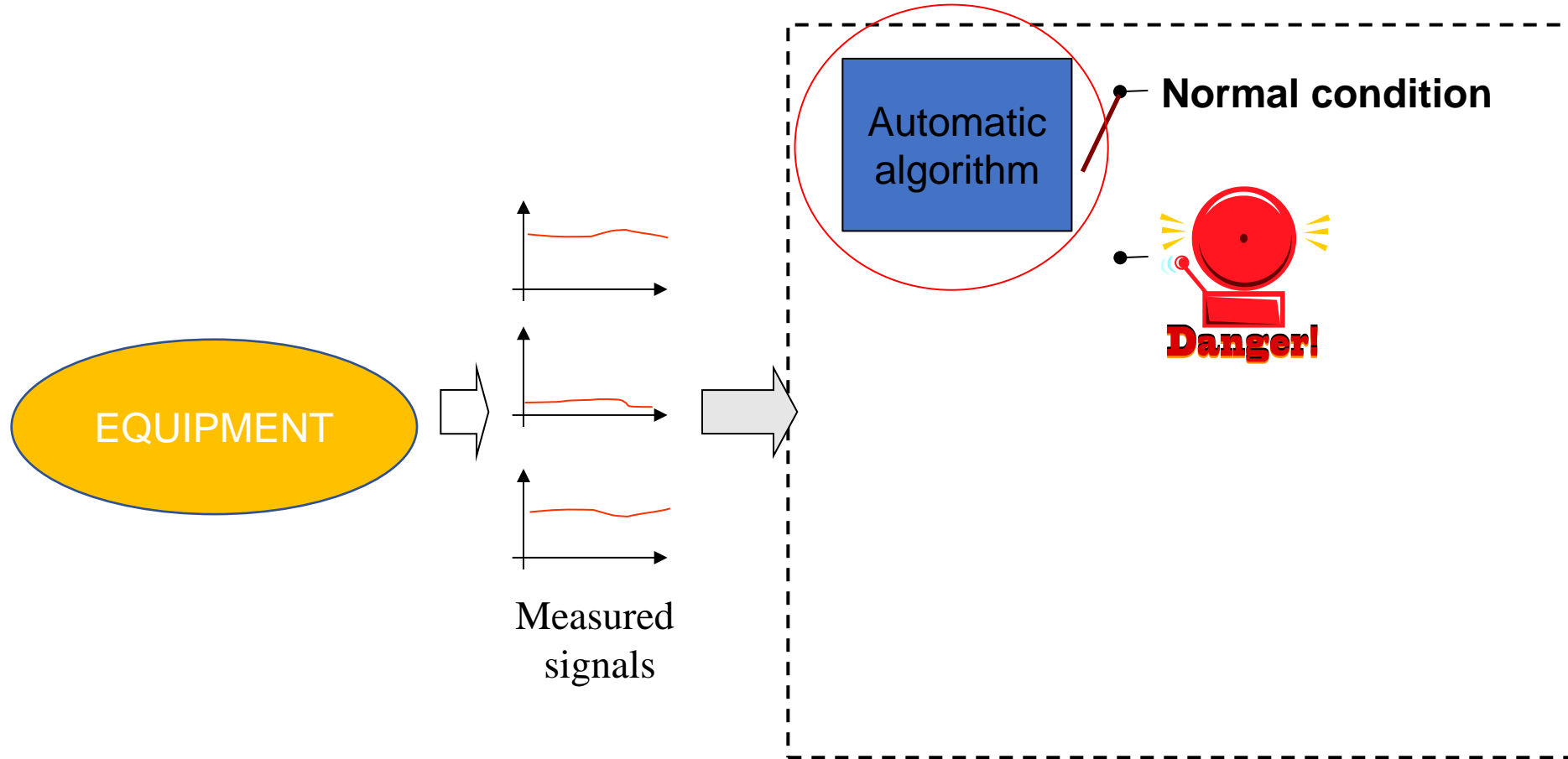
Context: Maintenance Interventions & PHM

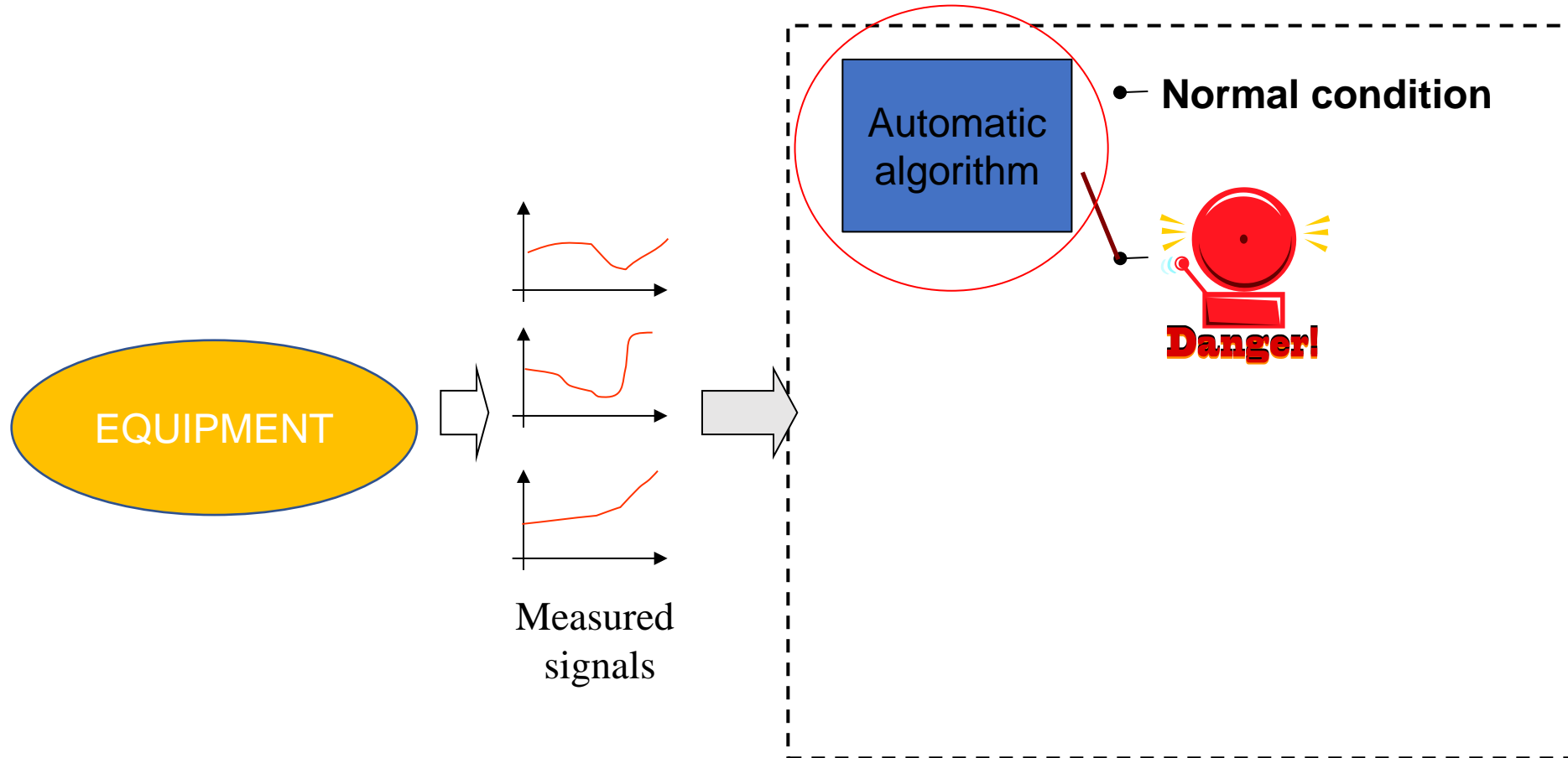


In This Lecture: Fault Detection

Equipment (System, Structure or Component)

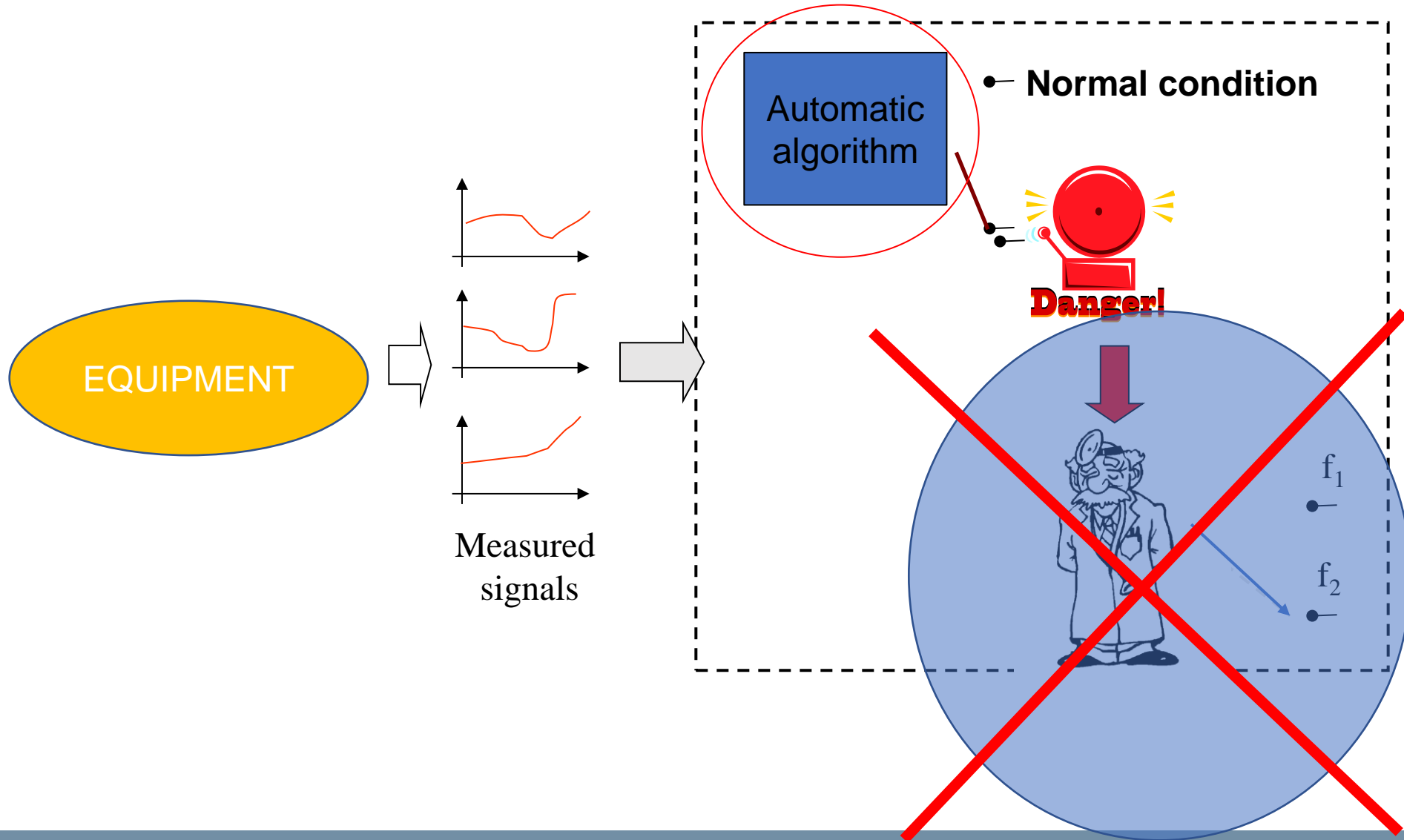






Fault Detection: What is not?

9



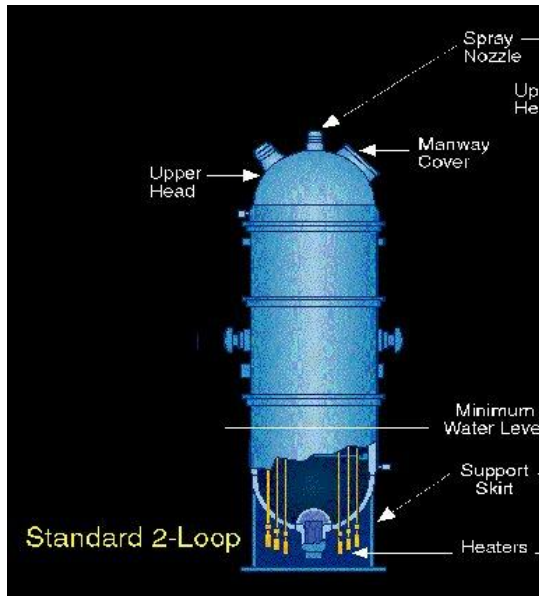
Fault Detection: Methods

- Limit-based
- Model-based
- Data-driven

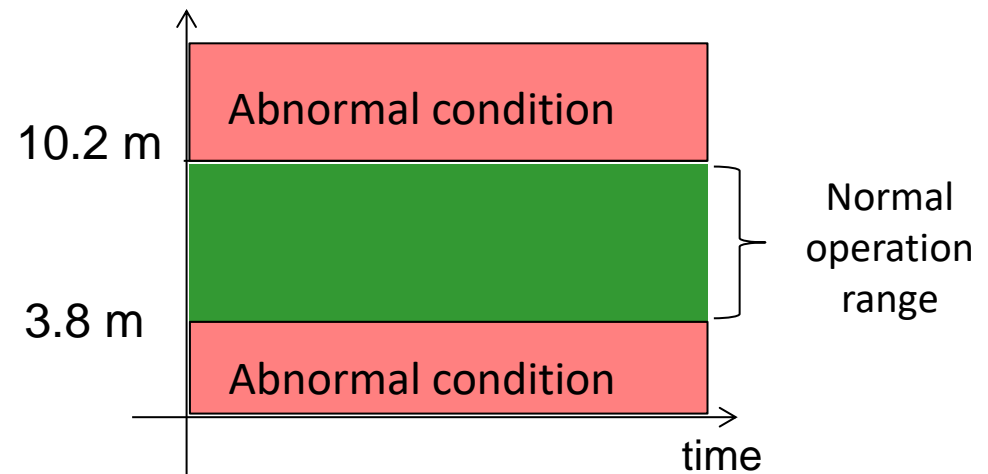
- Normal operation ranges of key signals

Example:

Pressurizer of a nuclear reactor



Water level



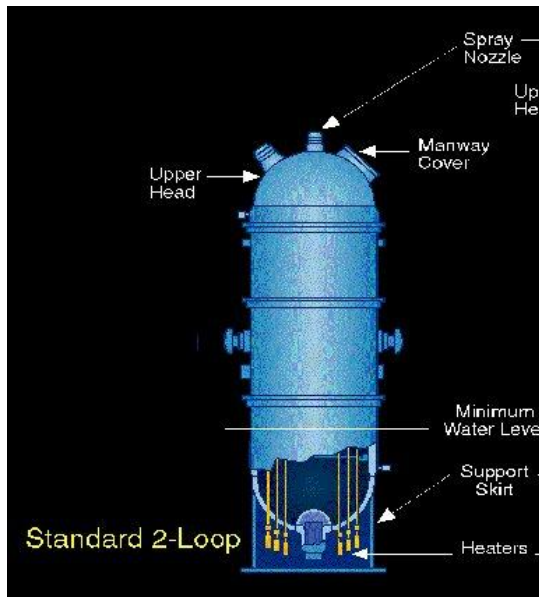
- Normal operation ranges of key signals



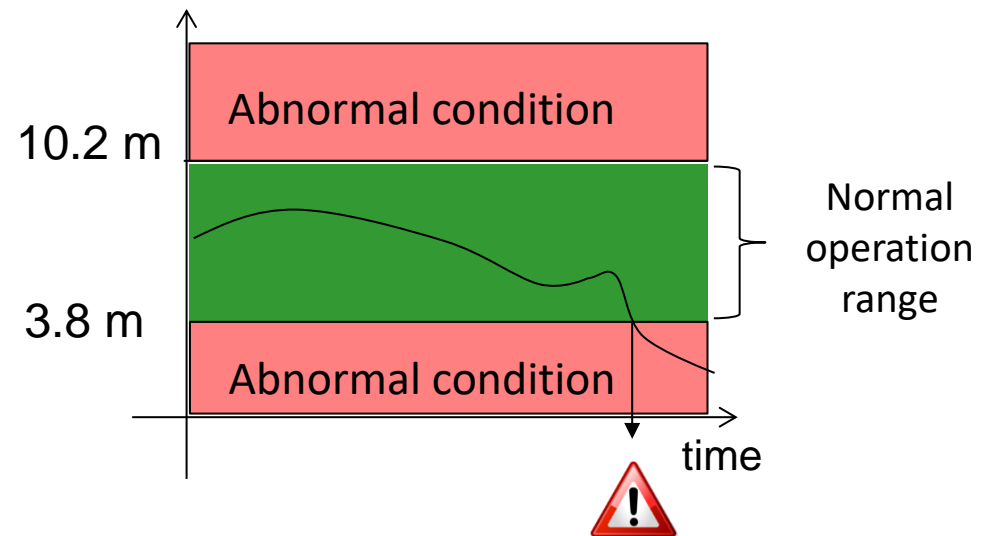
- Limit Value-Based Fault Detection

Example:

Pressurizer of a nuclear reactor



Water level



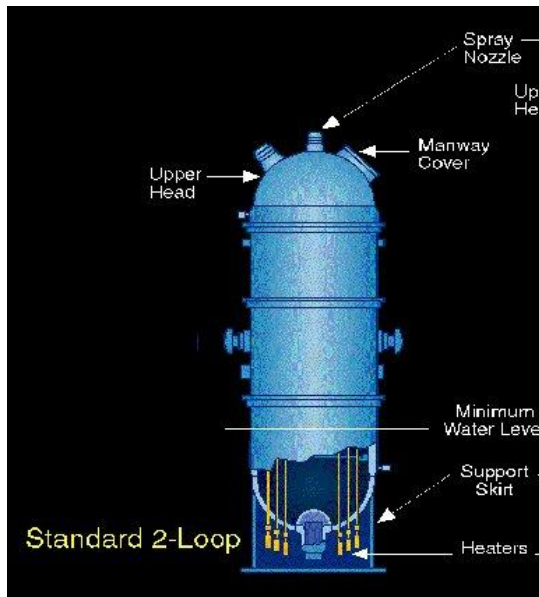
- Normal operation ranges of key signals



- Limit Value-Based Fault Detection

Example:

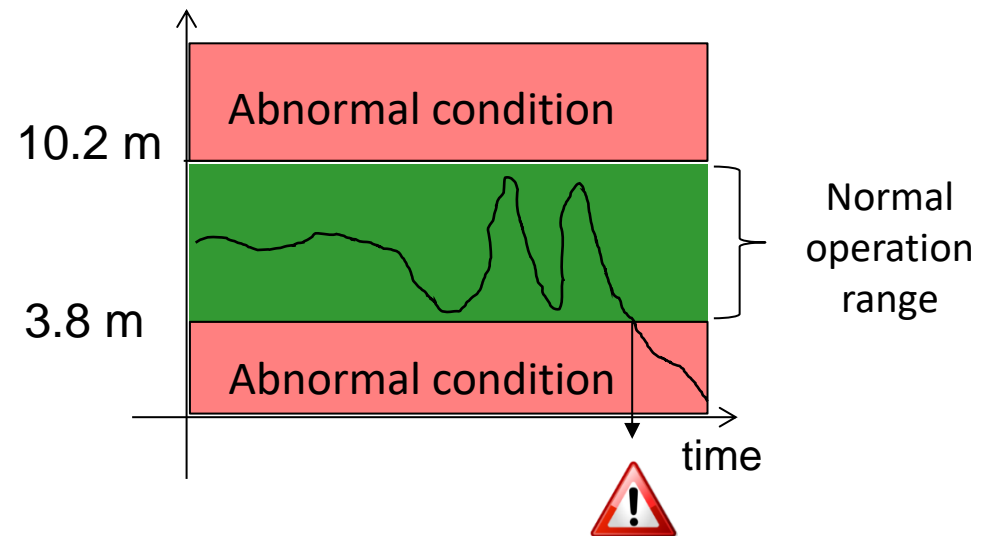
Pressurizer of a PWR nuclear reactor



Limitations:

- No early detection
- Not applicable to fault detection during operational transients
- Control systems operations may hide small anomalies (the signal remains in the normal range although there is a process anomaly)
- Considering signal individually can delay detection

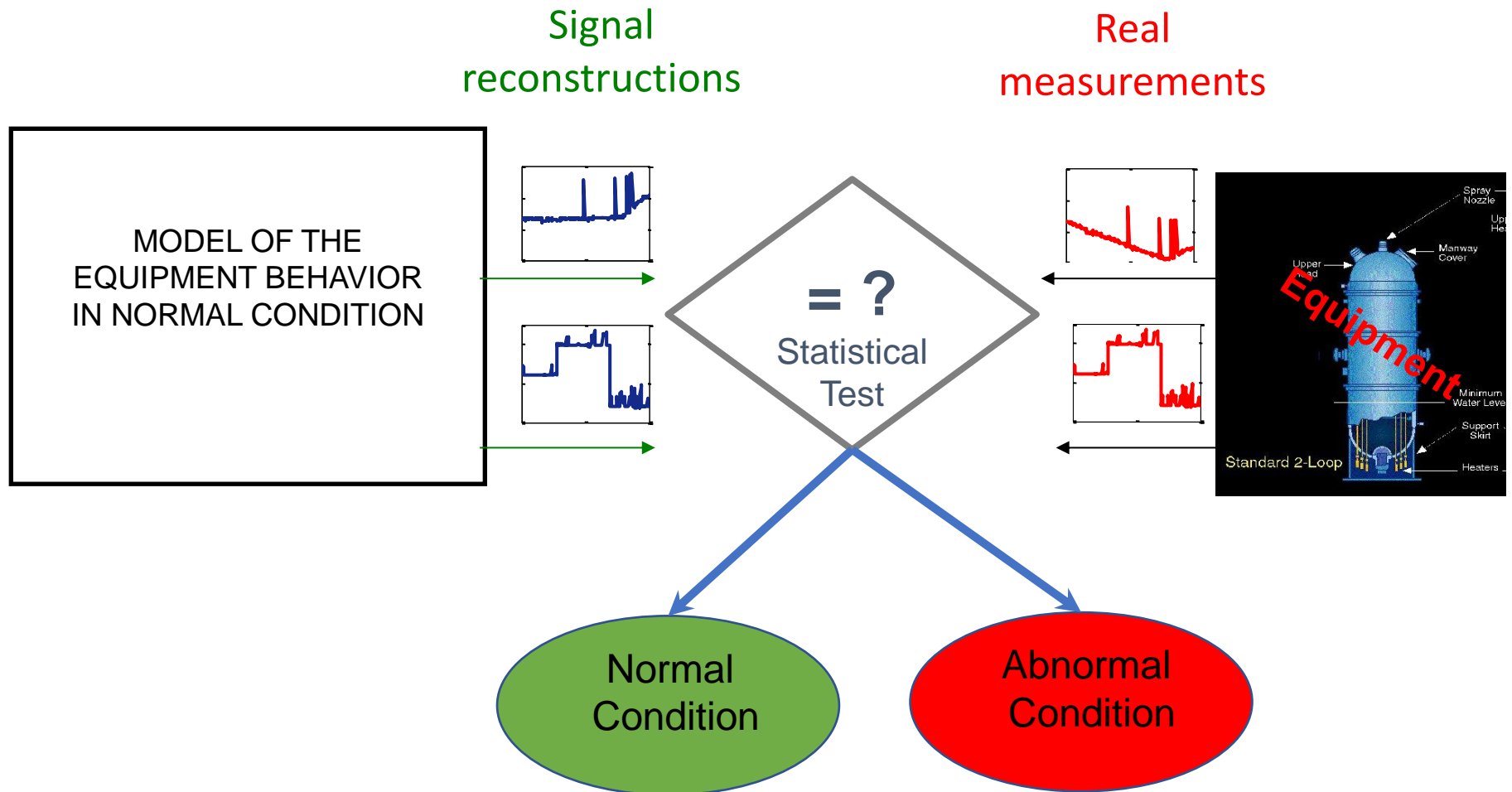
Water level



Fault Detection: Approaches

- Limit-based
- Model-based
- Data-driven

Model-based & Data-driven fault detection: basic idea



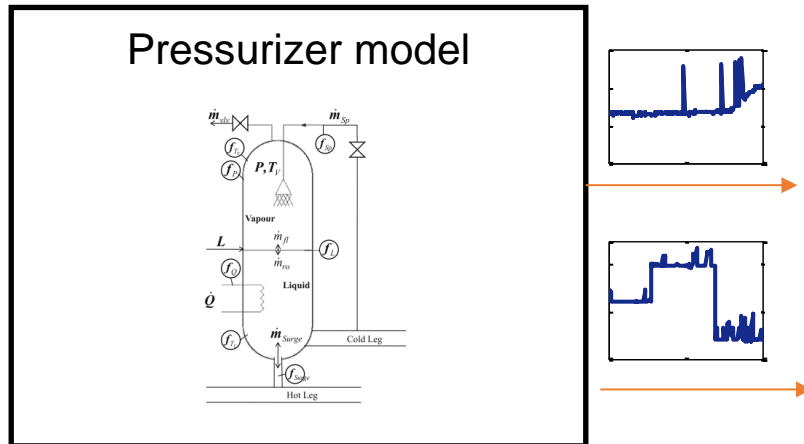
Fault Detection: Approaches

- Limit-based
- Model-based
- Data-driven

- Physics-based model of the process (used to reproduce the expected behavior of the signals in normal condition)

Example:

Signal
reconstructions

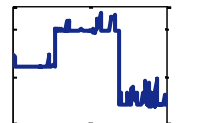
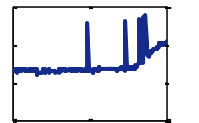
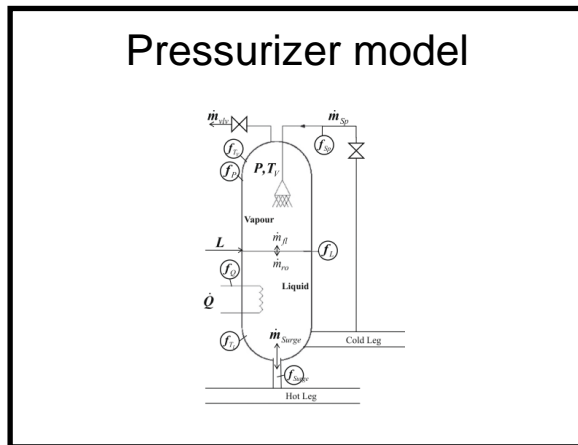


- Physics-based model of the process (used to reproduce the expected behavior of the signals in normal condition)

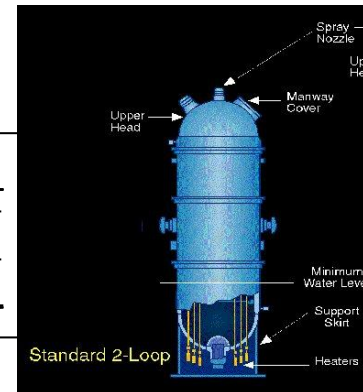
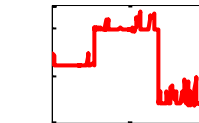
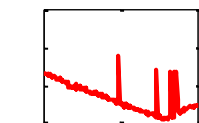
Example:

Signal
reconstructions

Real
measurements



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- Typically not available for complex systems
- Long computational time

Abnormal Condition

Fault Detection: Approaches

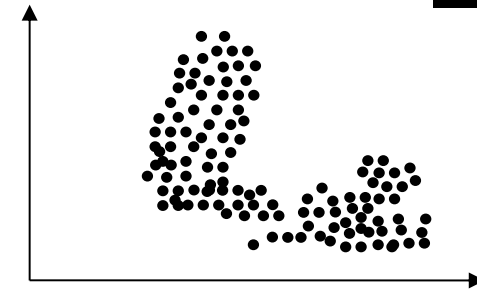
- Limit-Based
- Model Based
- Data-driven

- Historical signal measurements in normal operation

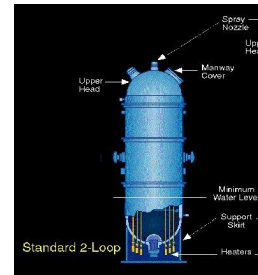
Example:

Pressure	Liquid temperature	Steam temperature	Spray flow	Surge line flow	Heaters power	Level
150.2	321	362	539	244	0	7.2
150.4	322	363	681	304	0	7.5
150.3	323	364	690	335	1244	7.7
...

Pressure

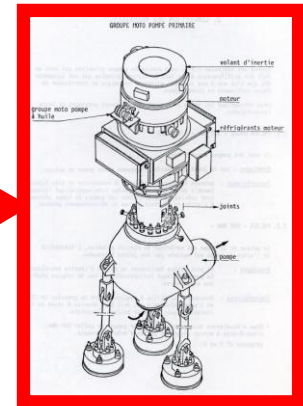
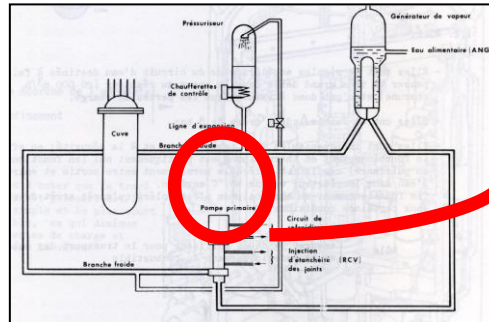


Water level



COMPONENT TO BE MONITORED

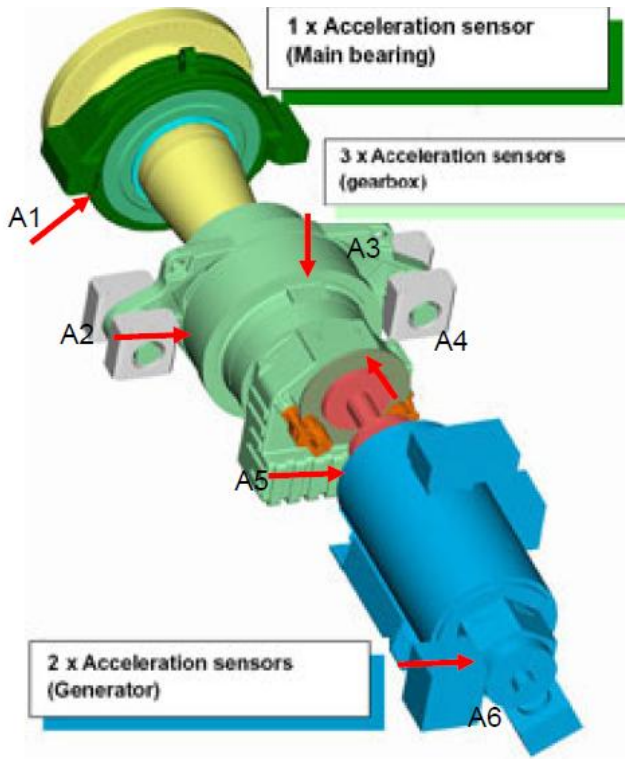
Reactor Coolant Pump of PWR Nuclear Power Plant



x4

Measured signals	48 (Temperatures, pressures, flows,...)
Available data	Historical signal measurements in normal plant condition [1 year, frequency=1/30 Hz]

* Work developed with EDF-R&D



Measured signals	6 vibration signals measured by accelerometers
Available data	Historical signal measurements in normal plant condition [3 years, frequency=5 kHz]

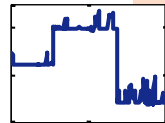
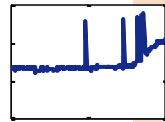
- Statistical Approaches:
 - AutoAssociative Kernel Regression (AAKR)
 - Principal Component Analysis (PCA)-based
 - ...
- Artificial Intelligence (AI)-based
 - Feedforward Neural Networks (FNNs)
 - AutoAssociative Neural Networks (AANNs)
 - AutoEncoders (AEs)
 - Self Organizing Maps
 - ...

- **Part 1: Model of the Equipment Behavior in Normal Condition**
 - 1A) Auto Associative Kernel Regression (AAKR)
 - 1B) Principal Component Analysis (PCA)
- **Part 2: Statistical Test**
 - 2A) Thresholds-Based
 - 2B) Q-Statistics
 - 2C) Sequential Probability Ratio Test (SPRT)

Part 1

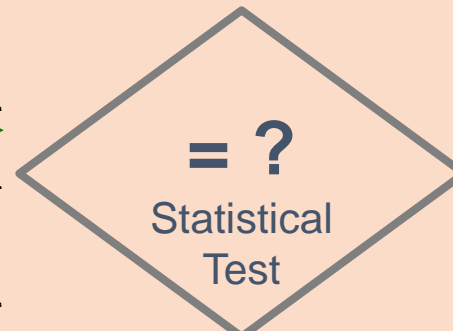
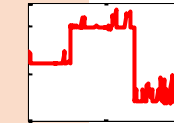
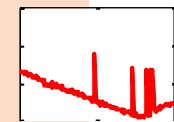
MODEL OF THE
EQUIPMENT BEHAVIOR
IN NORMAL CONDITION

Signal
reconstructions



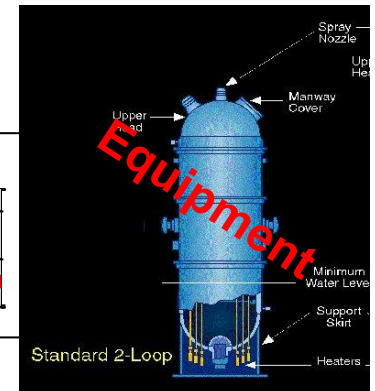
Part 2

Real
measurements



Normal
Condition

Abnormal
Condition



PART 1: Model of the Equipment Behaviour in Normal Condition

- Auto Associative Kernel Regression (AAKR)
- Principal Component Analysis (PCA)

Data in normal conditions

Signal

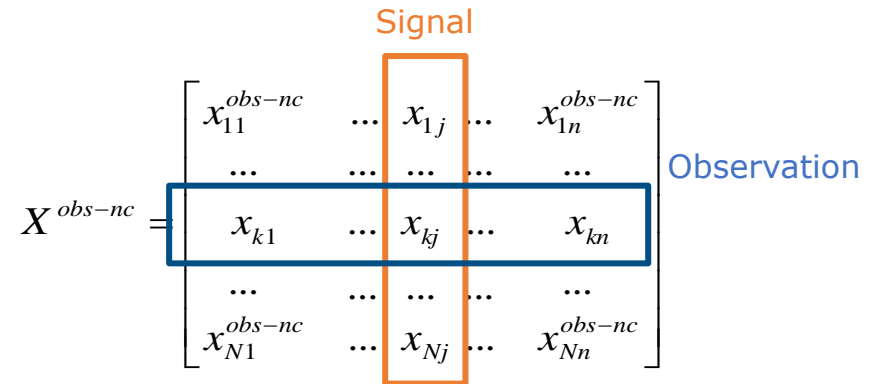
$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$$

Observation

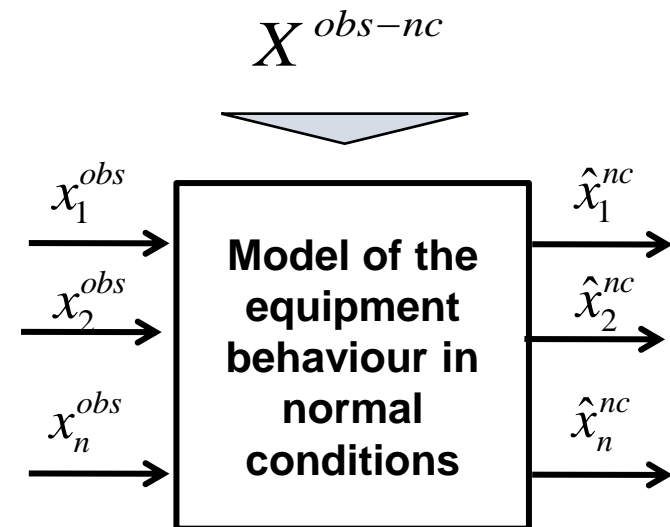
The diagram shows a matrix X^{obs-nc} with rows and columns. The j -th column is highlighted with an orange box and labeled 'Signal'. The k -th row is highlighted with a blue box and labeled 'Observation'. The intersection of these two boxes is the element x_{kj} . The matrix elements are labeled as x_{ij}^{obs-nc} for most entries, but the highlighted row and column use x_{ij} notation.

obs-nc = observation in normal condition

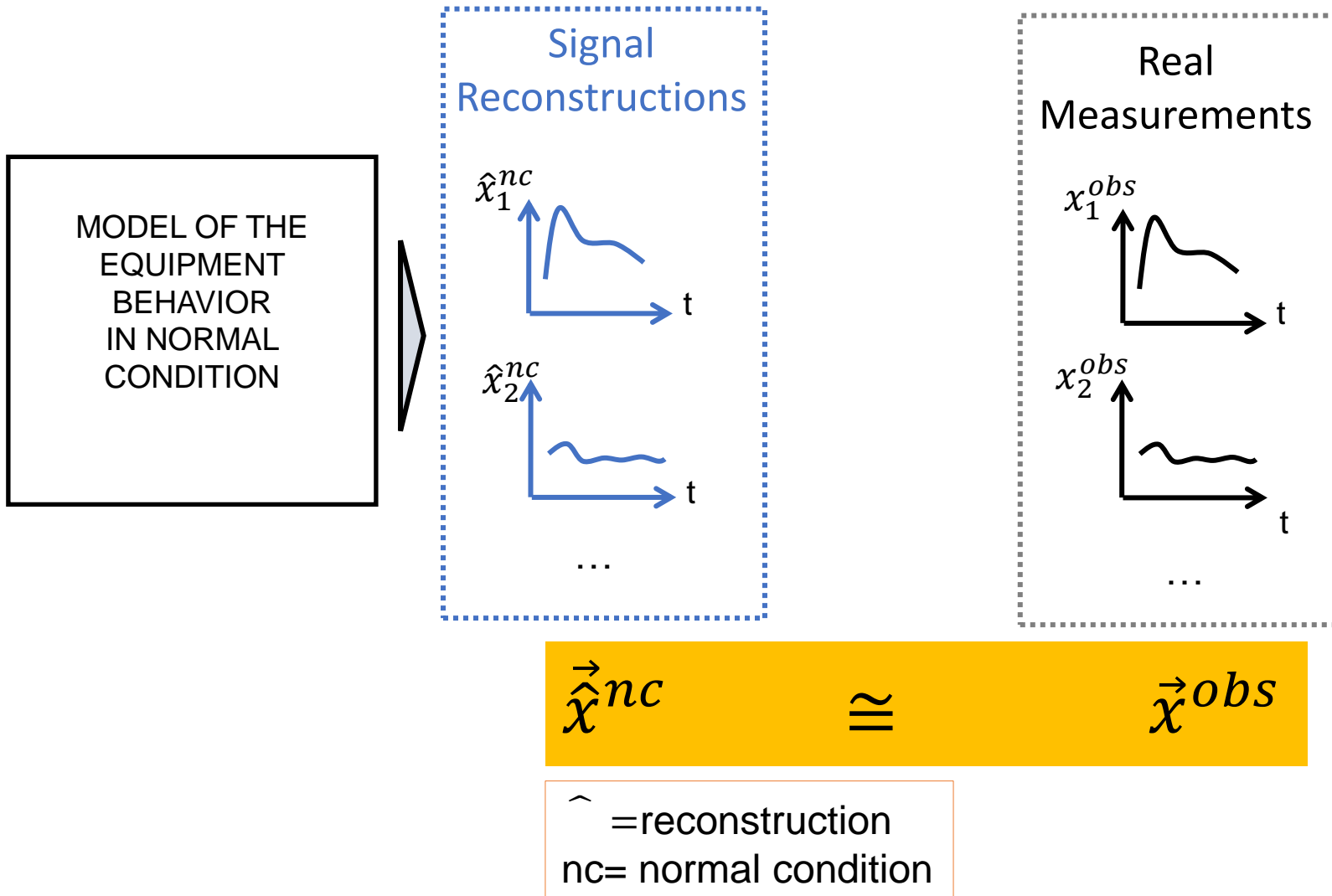
- Training patterns:
Historical signal measurements
in normal condition



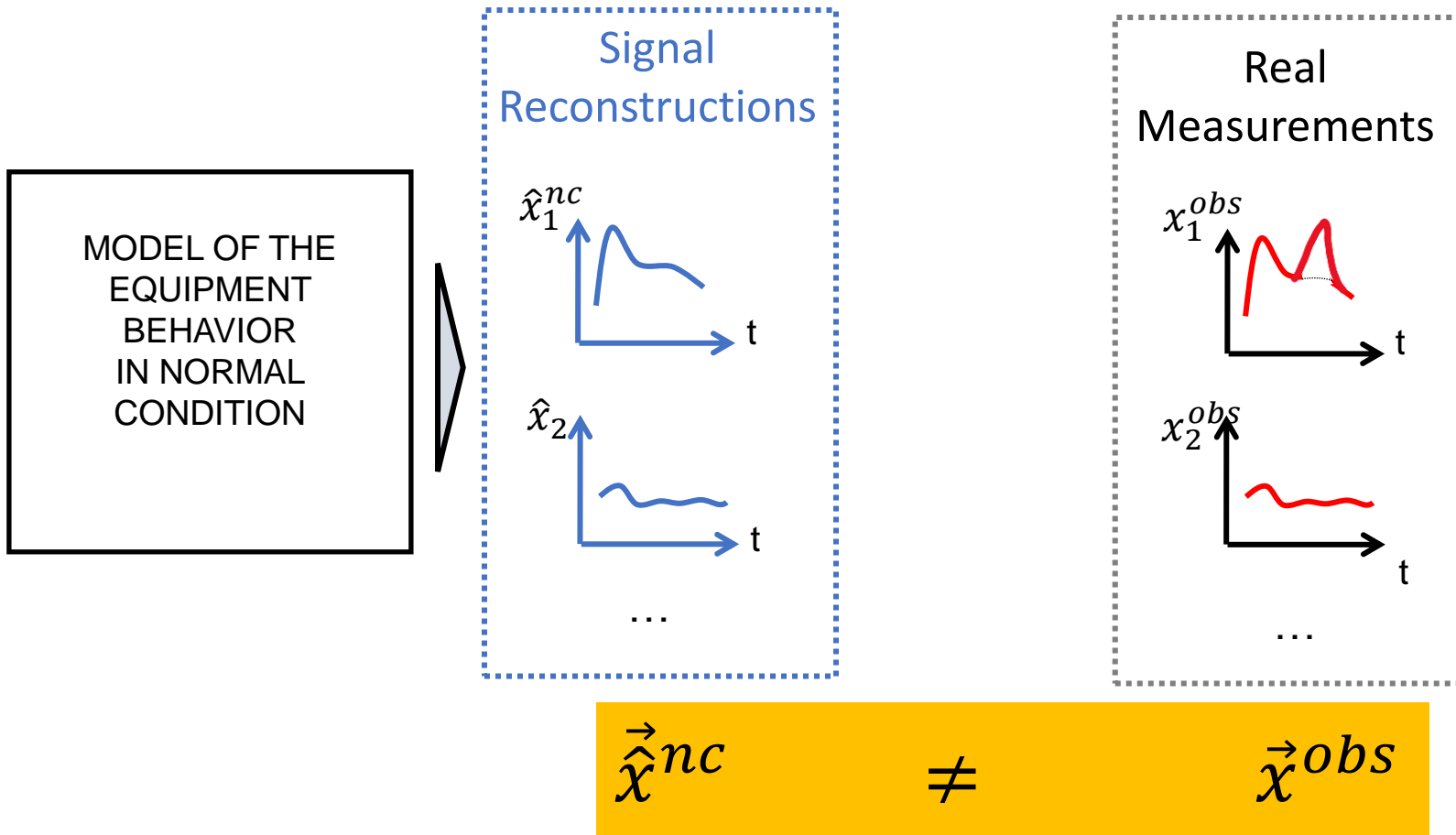
- Test input: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$
Signals measured at current time
- Test Output: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$
Signal reconstructions
(expected values of the signals
in normal condition)



- Equipment is in normal condition



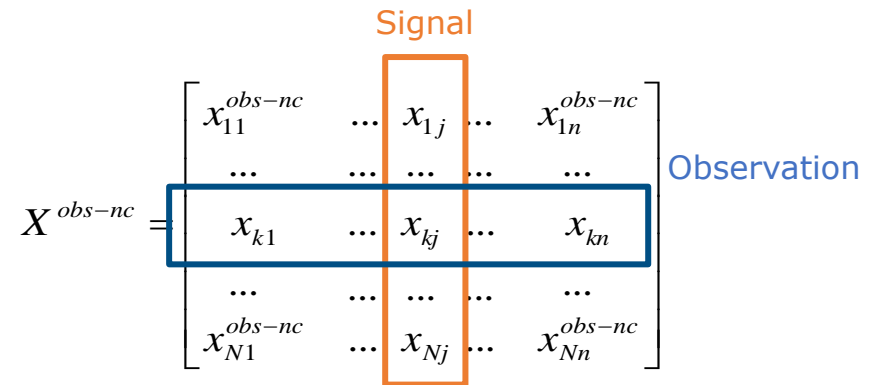
- Equipment is in **abnormal condition**



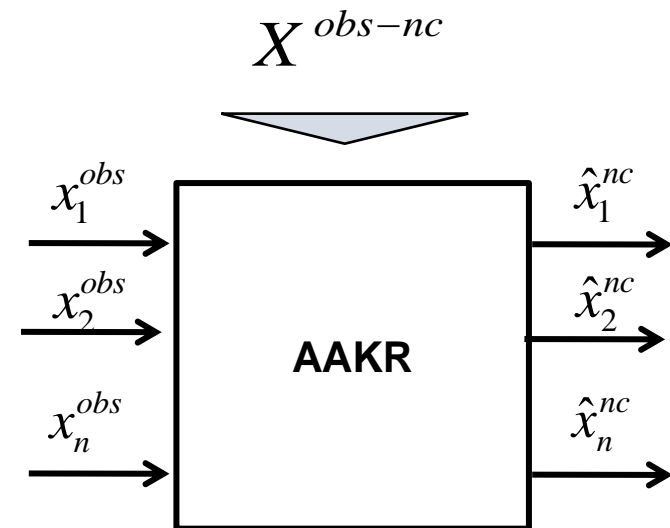
PART 1: Model of the Equipment Behaviour in Normal Condition

- **1A) Auto Associative Kernel Regression (AAKR)**
- 1B) Principal Component Analysis (PCA)

- Training patterns:
Historical signal measurements
in normal condition



- Test input: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$
Signals measured at current time
- Test Output: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$
Signal reconstructions
(expected values of the signals
in normal condition)

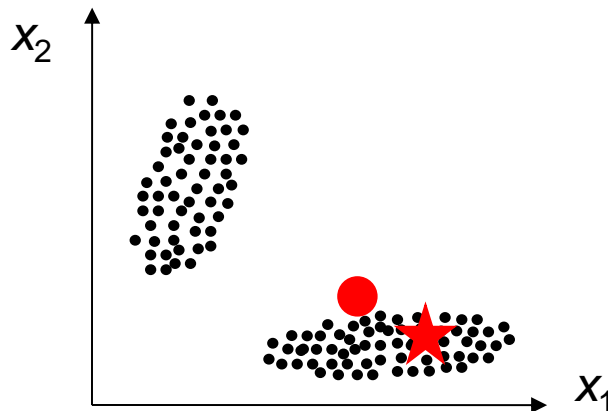


- Training patterns:

$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j}^{obs-nc} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1}^{obs-nc} & \dots & x_{kj}^{obs-nc} & \dots & x_{kn}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj}^{obs-nc} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$$

- Test input ●: measured signals at current time $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$

- Test output ★: weighted sum of the training patterns $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$



On all the training pattern

$$\hat{x}_j^{nc} = \frac{\sum_{k=1}^N w(k) \cdot x_{kj}^{obs-nc}}{\sum_{k=1}^N w(k)}$$

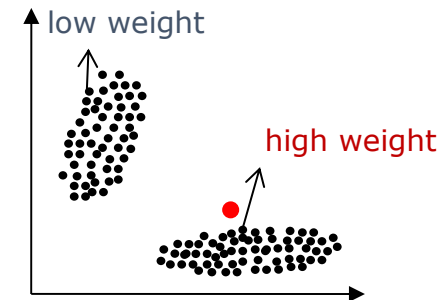
AAKR: the algorithm (2)

- Output $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$ = weighted sum of the training patterns:

On all the training pattern $\hat{x}_j^{nc} = \frac{\sum_{k=1}^N w(k) \cdot x_{kj}^{obs-nc}}{\sum_{k=1}^N w(k)}$

- weights $w(k)$ = similarity measures between \vec{x}^{obs} and \vec{x}_k^{obs-nc} (the test and the k -th training pattern):

$$w(k) = \frac{1}{\sqrt{2\pi}h} e^{-\frac{d^2(k)}{2h^2}}$$



h = bandwidth parameter (it controls the decay speed)

AAKR: parameter h setting

$$d=0 \rightarrow w=0.40/h$$

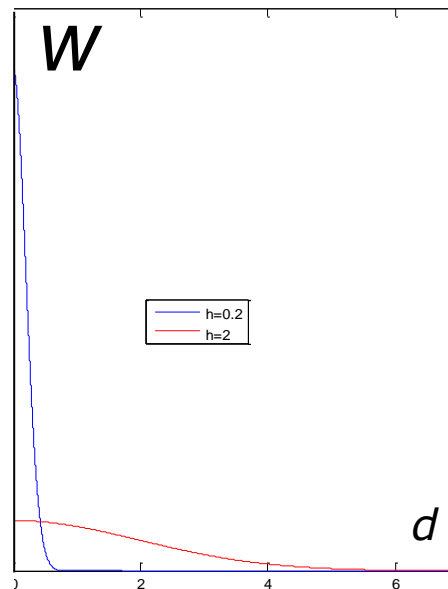
$$d=h \rightarrow w=0.24/h$$

$$d=2h \rightarrow w=0.05/h$$

$$d=3h \rightarrow w=0.004/h$$



$$\frac{w(d=0)}{w(d=3h)} = \frac{0.40}{0.004} = 100$$



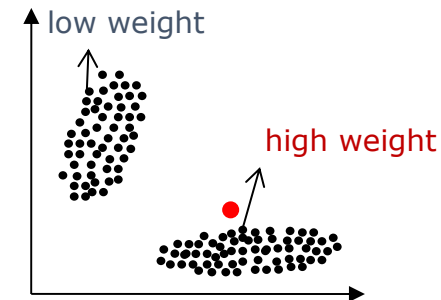
AAKR: the algorithm (2)

- Output $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$ = weighted sum of the training patterns:

$$\hat{x}_j^{nc} = \frac{\sum_{k=1}^N w(k) \cdot x_{kj}^{obs-nc}}{\sum_{k=1}^N w(k)}$$

- weights $w(k)$ = similarity measures between \vec{x}^{obs} and \vec{x}_k^{obs-nc} (the test and the k -th training pattern):

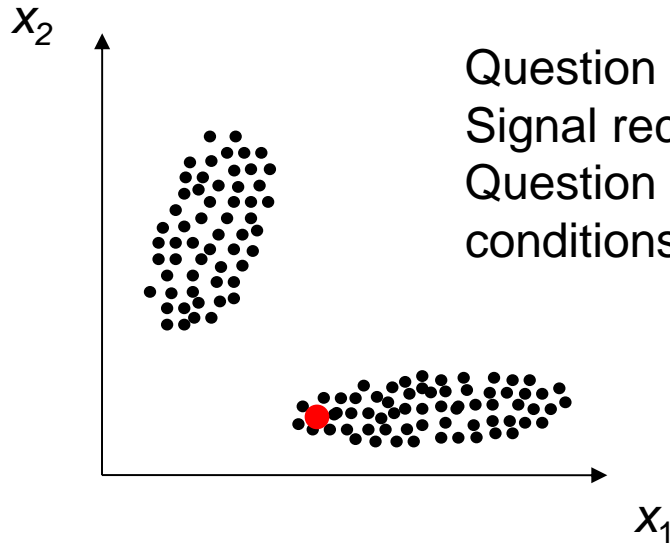
$$w(k) = \frac{1}{\sqrt{2\pi}h} e^{-\frac{d^2(k)}{2h^2}}$$



- with $d^2(k) = \sum_{j=1}^n (x_j^{obs} - x_{kj}^{obs-nc})^2$ Euclidean distance between \vec{x}^{obs} and \vec{x}_k^{obs-nc}

AAKR: Exercise 1

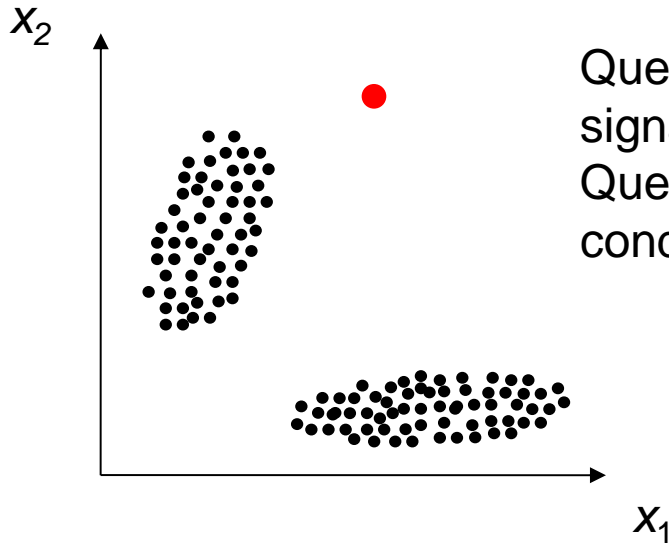
- Signal values at current time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$ ●
- Historical signal measurements in normal plant condition: •



Question 1) Where do you expect to be the Signal reconstruction ★ $\hat{x}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$?
Question 2) Is the plant in normal or abnormal conditions?

AAKR: Exercise 2

- Signal values at current time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$ ●
- Historical signal measurements in normal plant condition: •



Question 1) Where do you expect to be the signal reconstruction ★ $\hat{x}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$?

Question 2) Is the plant in normal or abnormal conditions?

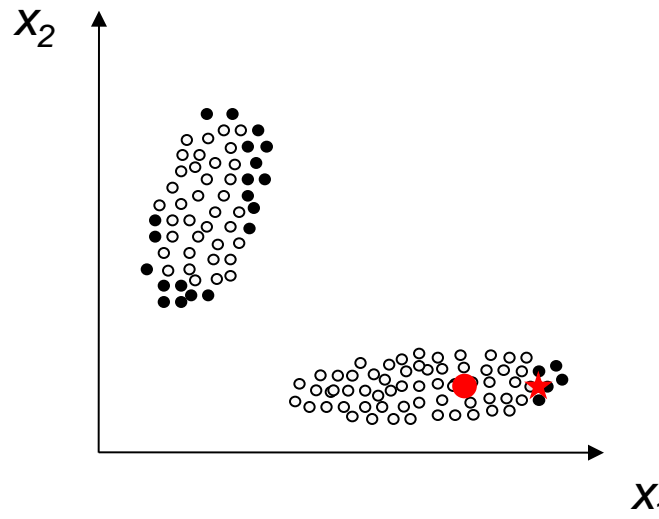
AAKR remarks: Computational Time

- Computational time:
 - No training of the model
 - Test:
computational time depends on
 - a) the number of training patterns N ;
 - b) the number of signals n .

$$d^2(k) = \sum_{j=1}^n (x_j^{obs} - x_{kj}^{obs-nc})^2$$

AAKR remarks: Accuracy

- Accuracy:
 - depends on the training set:
 - $\uparrow N \rightarrow \uparrow \text{Accuracy}$



Few patterns and not well
distributed in the training space



Inaccurate reconstruction

Pros:

- No need of hypothesis on data distribution (e.g. linearity)

Cons

- Performance related to number of training observations

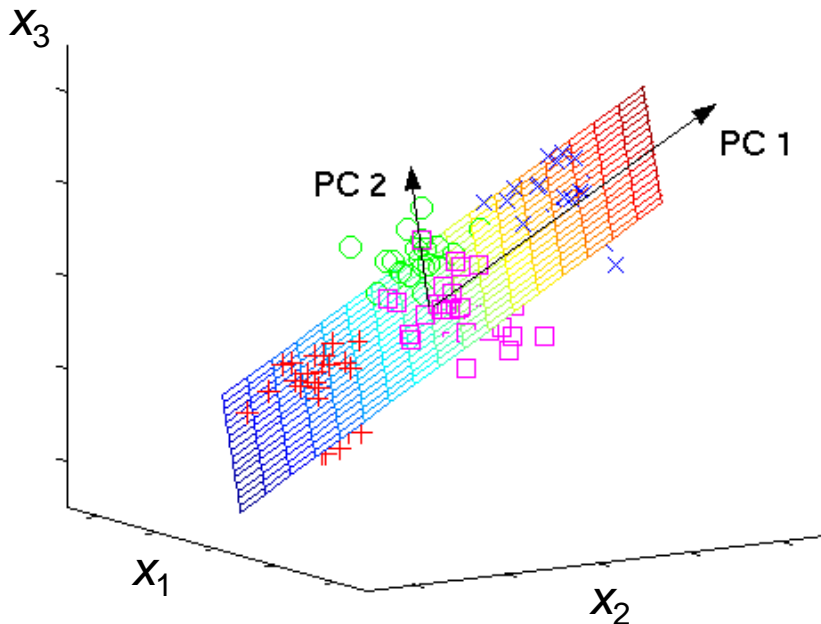
PART 1: Model of the Equipment Behaviour in Normal Condition

- 1A) Auto Associative Kernel Regression (AAKR)
- **1B) Principal Component Analysis (PCA)**

PCA:

- Space transformation
- From an n -dimensional space to a l -dimensional space ($l < n$)
- Retaining most of the information (loosing the least information)

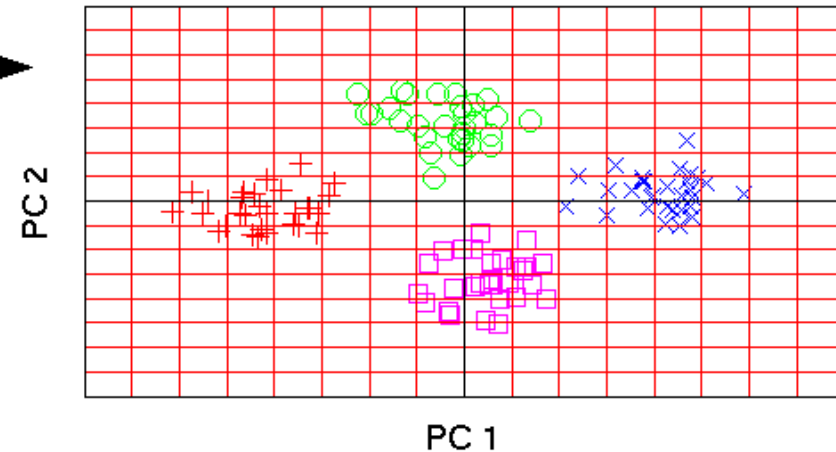
original data space



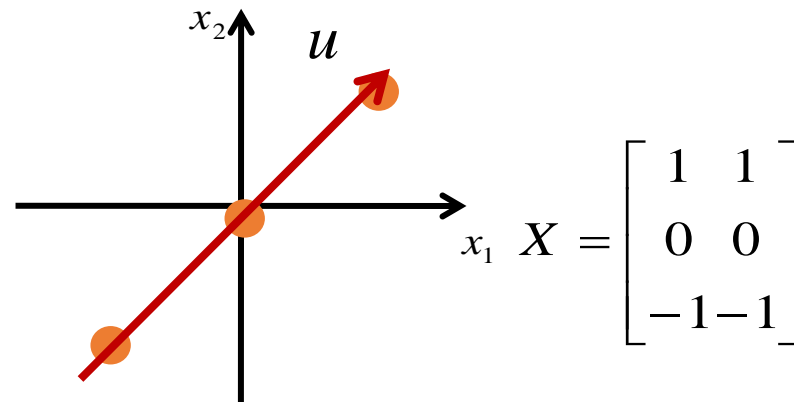
PCA



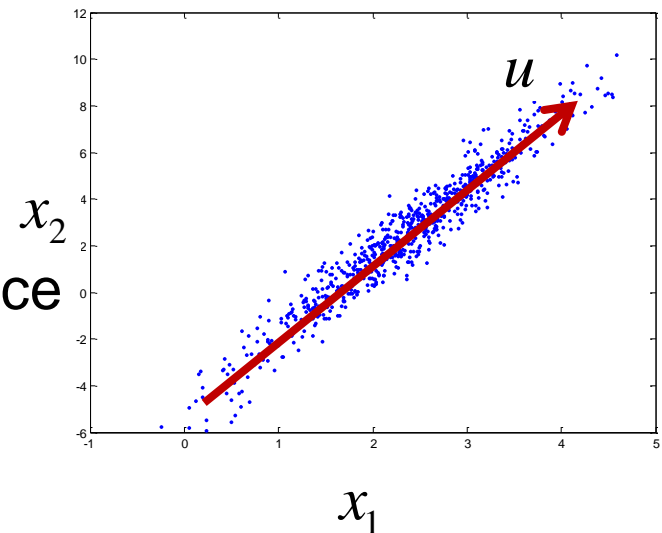
component space



- Two signals are highly correlated or dependent
→ One is enough!



- Key underlying phenomena
→ Areas of variance in data
→ Focus on directions along which the observations have largest variance



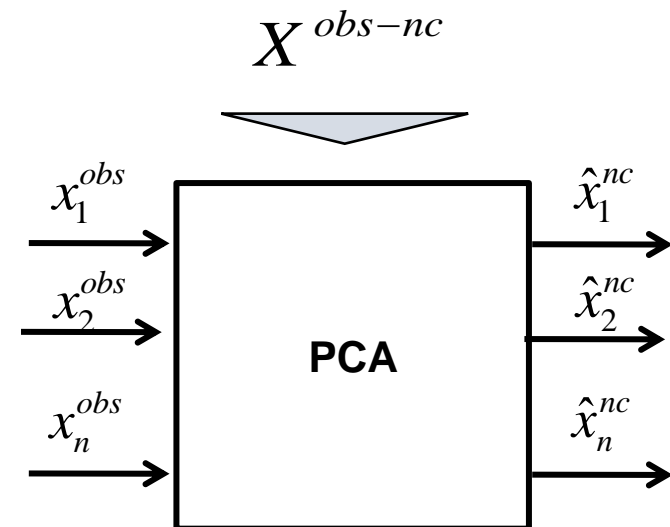
- Training patterns:
Historical signal measurements
in normal condition

$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j}^{obs-nc} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj}^{obs-nc} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$$

Signal

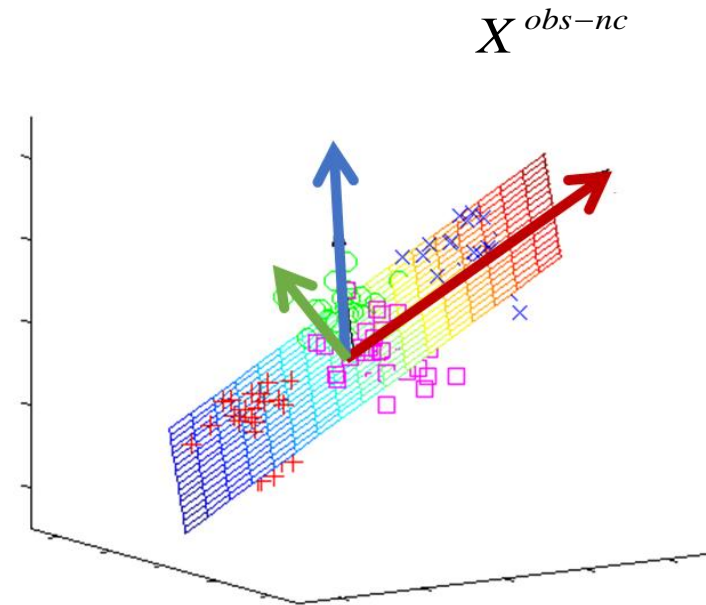
Observation

- Test input: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$
Signals measured at current time
- Test Output: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$
Signal reconstructions
(expected values of the signals
in normal condition)



Step 1: find Principal Components (PCs) in the training set X^{obs-nc} :

- 1) PC1 \rightarrow is the direction of maximum variance
- 2) PC2 \rightarrow is **orthogonal** to PC1 and describes the maximum residual variance
- 3) PC3 \rightarrow is **orthogonal** to PC1 and PC2 and describes the maximum residual variance



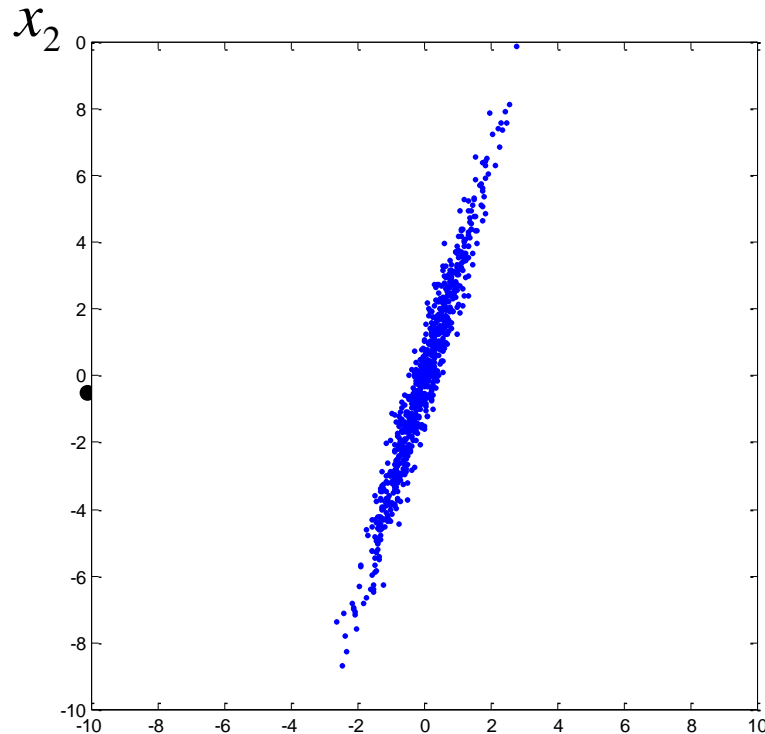
Objective: find principal components

Procedure:

- Compute V = covariance matrix of X^{obs-nc}

$$V = \left(X^{obs-nc} - \bar{X}^{obs-nc} \right)^T \left(X^{obs-nc} - \bar{X}^{obs-nc} \right)$$

Empirical
mean
matrix



$$\sum_{i=1}^N (x_{i1} - \bar{x}_1)^2 \quad \sum_{i=1}^N (x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)$$

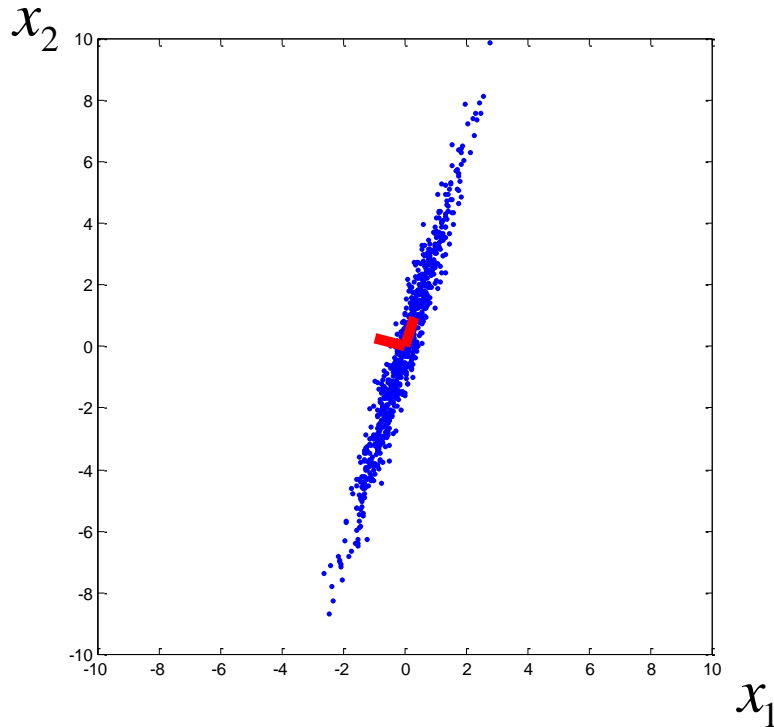
$$V = \begin{pmatrix} 0.79 & 2.55 \\ 2.55 & 8.75 \end{pmatrix}$$

x_1

Objective: find principal components

Procedure:

- Compute V = covariance matrix of X^{obs-nc}
- Find the n eigenvectors $\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n$ of V and the corresponding eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_n$



$$V\vec{p} = \lambda\vec{p}$$



$\lambda_1=9.50$ largest

$\lambda_2=0.04$ smallest



$$\vec{p}_1 = [0.28 \quad 0.96]^T$$

$$\vec{p}_2 = [-0.96 \quad 0.28]^T$$

$$P = [\vec{p}_1, \vec{p}_2] = \begin{bmatrix} 0.28 & -0.96 \\ 0.96 & 0.28 \end{bmatrix}$$

➤ P is an orthonormal basis:

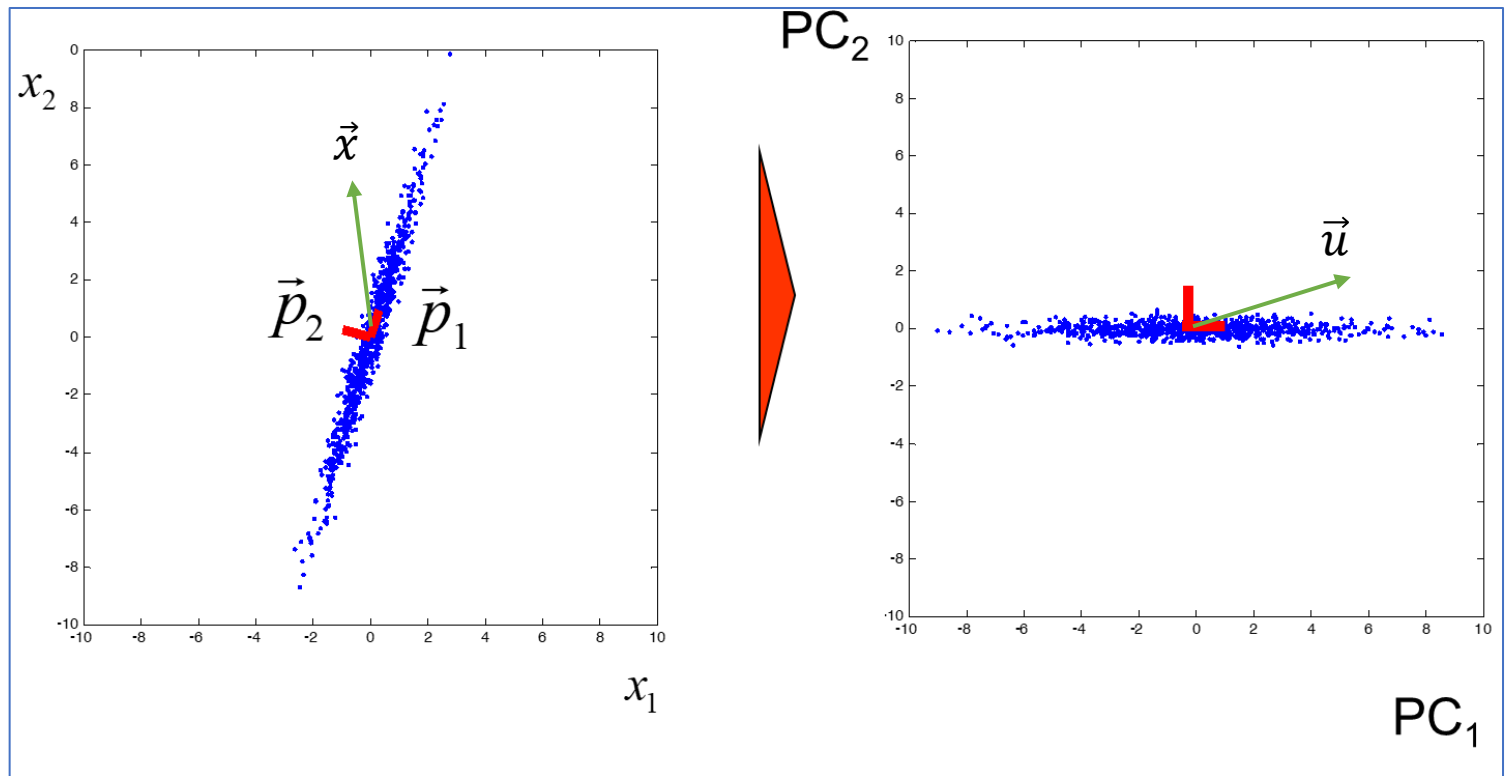


$$\vec{p}_i \perp \vec{p}_j \quad \forall i \neq j$$

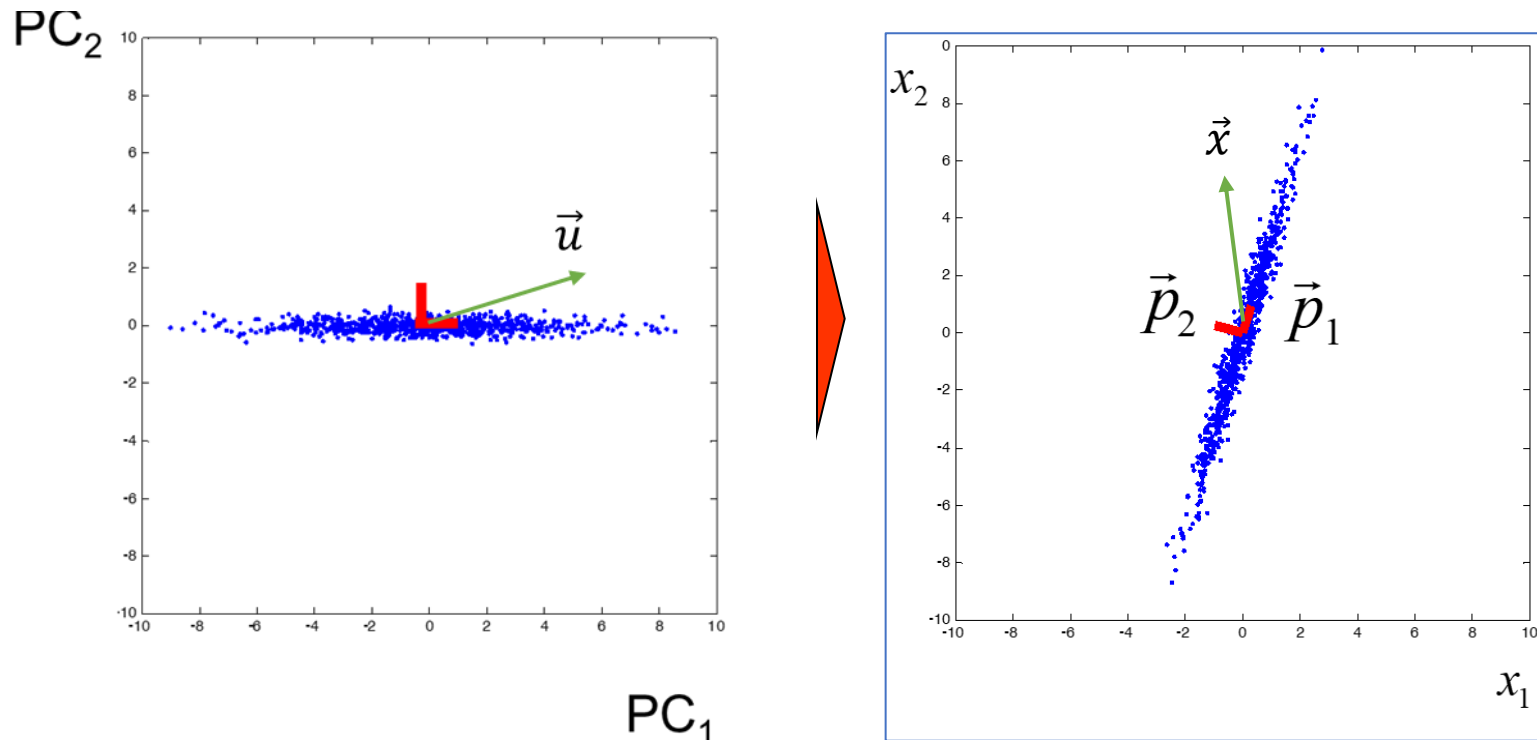
$$|\vec{p}_i| = 1$$

$$PP^T = I$$

- P is an orthonormal basis:
- Data can be transformed from the original to the transformed bases and viceversa without any loss of information (multiplication for P and P^T)
 - \vec{u} = the projection of \vec{x} on the new basis is given by: $\vec{u} = \vec{x} \cdot P$



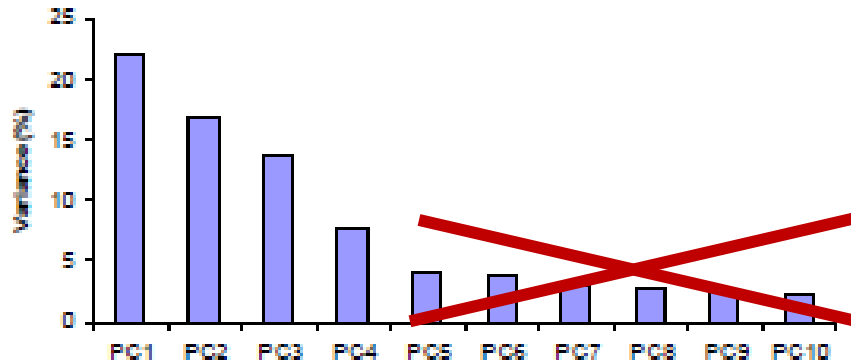
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 - \vec{x} can be obtained from \vec{u} by: $\vec{x} = \vec{u} \cdot P^T$
- The percentage of variance retained by the i -th principal component is:

$$\%Var(PC_i) = \frac{\lambda_i}{\sum_{i=1, \dots, n} \lambda_i}$$

Step 2 [PCA approximation]: ignore the PCs of lower significance.



- Lost small information
- Reduce the number of dimensions from $n=10$ to $= 4$

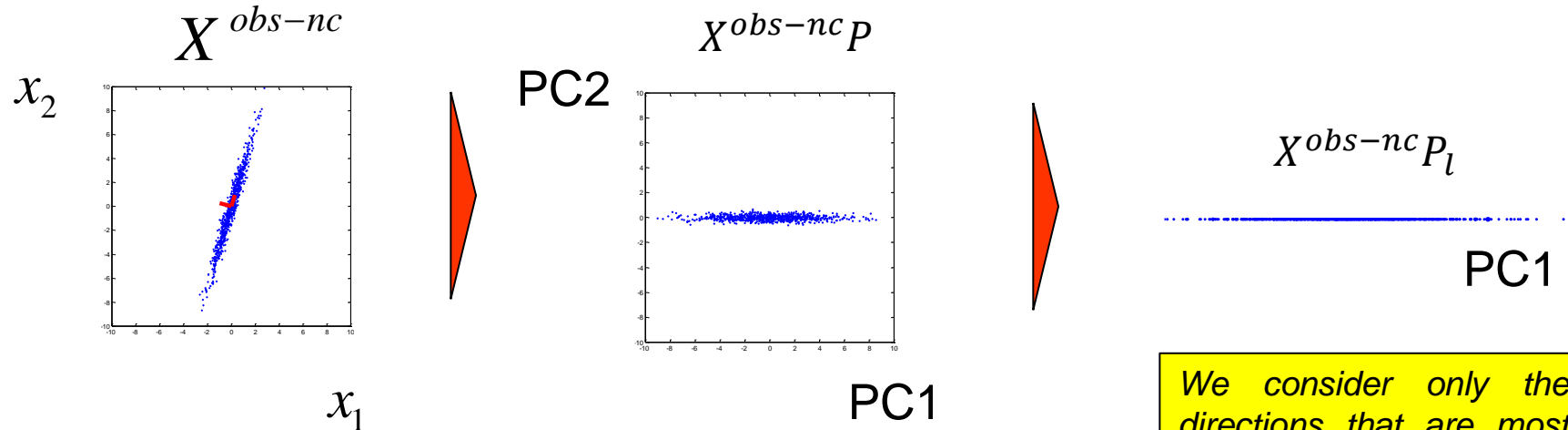
PCA for fault detection: operational steps (2)

- Step 2 [PCA approximation]: ignore the PCs of lower significance.

map the observation \vec{x}^{obs} in a subspace $\mathfrak{R}^l \subset \mathfrak{R}^n$ identified by the first $l < n$ eigenvectors $\vec{p}_1, \dots, \vec{p}_l$:

$$\vec{x}^{obs} P_l \quad \text{with } P_l = [\vec{p}_1, \dots, \vec{p}_l]$$

Example of application to the normal condition data: X^{obs-nc}



We consider only the directions that are most meaningful in normal condition (directions of maximum variance)

PCA for fault detection: operational steps (3)

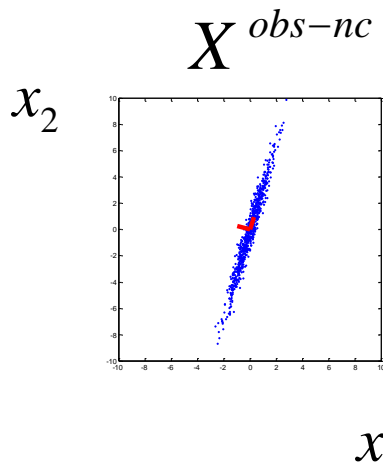
- Step 2 [PCA approximation]: ignore the PCs of lower significance.

map the observation \vec{x}^{obs} in a subspace $\mathfrak{R}^l \subset \mathfrak{R}^n$ identified by the first $l < n$ eigenvectors $\vec{p}_1, \dots, \vec{p}_l$:

$$\vec{x}^{obs} P_l \quad \text{with } P_l = [\vec{p}_1, \dots, \vec{p}_l]$$

- Step 3: [Antitransformation]: signal reconstructions $\vec{x}^{nc} = \vec{x}^{obs} P_l P_l^T$

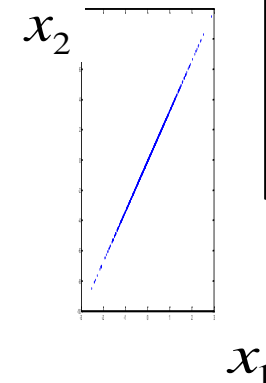
Example of application to the normal condition data: X^{obs-nc}



$$P_\lambda = [\vec{p}_1, \dots, \vec{p}_\lambda]$$


$$X^{obs-nc} P_\lambda$$

PC1




We loose the noise in the space of the measured signals

PCA for fault detection: Summary

- Historical data $X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$  Find P_l from X^{obs-nc}

PCA for fault detection: Summary

- Historical data $X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$  Find P_l from X^{obs-nc}
- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$

- Transform and project $\vec{x}^{obs} P_l$

I'm looking at the measurements considering only the directions that are most meaningful in normal condition (directions of maximum variance)

- Antitransform $\hat{\vec{x}}^{nc} = \vec{x}^{obs} P_l P_l^T$ Signal reconstructions

$\hat{\vec{x}}^{nc} \cong \vec{x}^{obs} \rightarrow$ normal condition

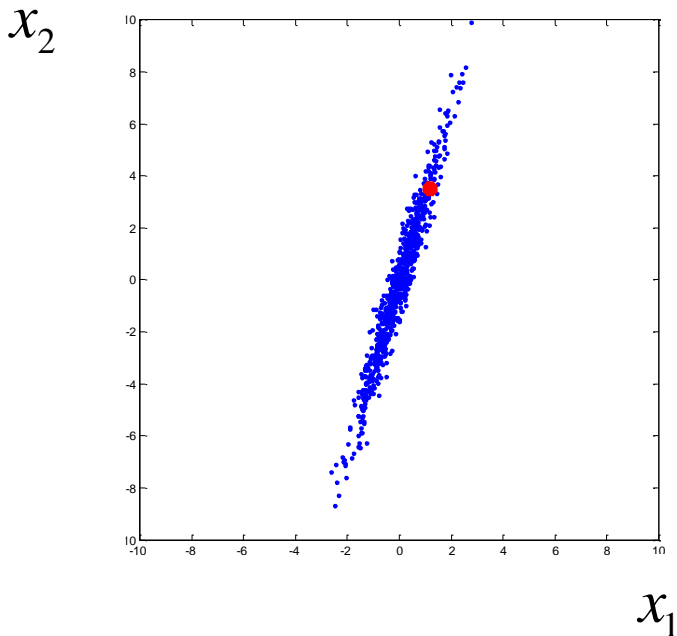
I loose only the irrelevant noise

$\hat{\vec{x}}^{nc} \neq \vec{x}^{obs} \rightarrow$ abnormal condition

The process is changed

Exercise 1

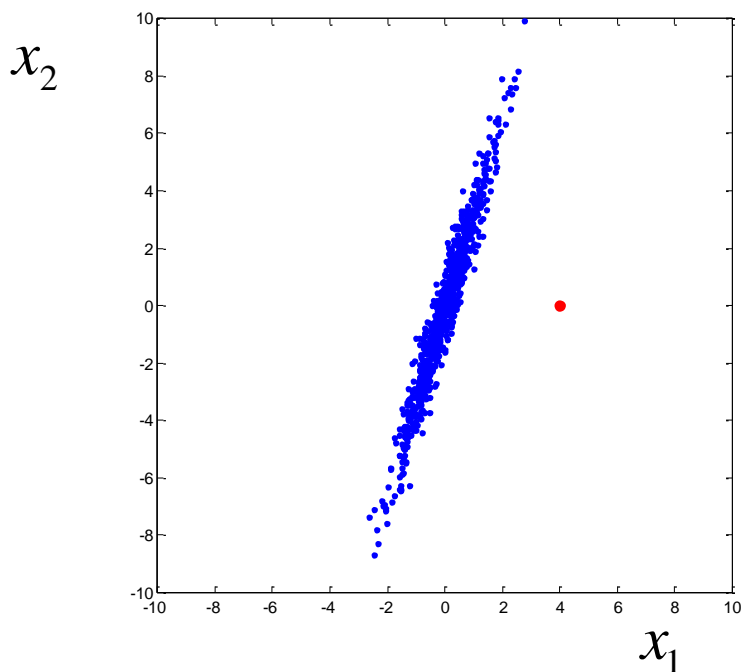
- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ •
- Signal reconstructions?
- Normal or abnormal condition?



- available historical signal measurements in normal plant condition

Exercise 2

- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ •
- Signal reconstructions?
- Normal or abnormal condition?



- available historical signal measurements in normal plant condition

Computational time:

- Training time = computational time necessary to find the Principal Components is proportional to the number of measured signals n
- Execution time: very short (only 2 matrix multiplications)
→ OK for online applications

Performance:

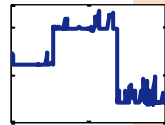
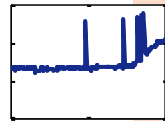
Unsatisfactory for dataset characterized by highly non-linear relationships



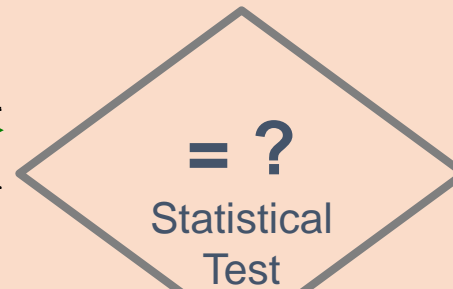
Part 1

MODEL OF THE
EQUIPMENT BEHAVIOR
IN NORMAL CONDITION

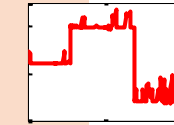
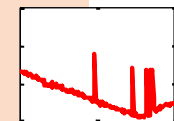
Signal
reconstructions



Part 2

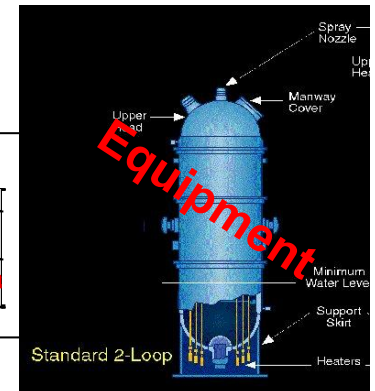


Real
measurements



Normal
Condition

Abnormal
Condition



- Part 1: Model of the Equipment Behavior in Normal Condition
 - 1A) Auto Associative Kernel Regression (AAKR)
 - 1B) Principal Component Analysis (PCA)
- **Part 2: Statistical Test**
 - 2A) Thresholds-Based
 - 2B) Q-Statistics
 - 2C) Sequential Probability Ratio Test (SPRT)

PART 2: Statistical Test

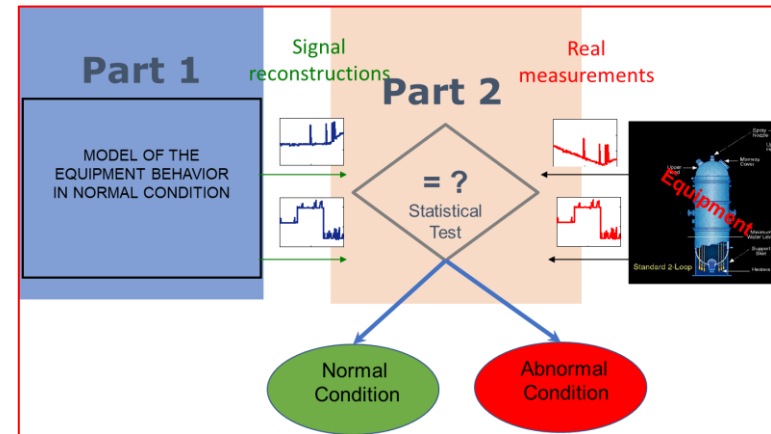
- Thresholds-based
- Q Statistics
- Sequential Probability Ratio Test (SPRT)

- Basics of the decision: residual analysis

$$\vec{r} = \vec{x}^{obs} - \vec{\hat{x}}^{nc} \quad \left\{ \begin{array}{l} \vec{r} \approx 0 \rightarrow \text{Normal condition} \\ \vec{r} \neq 0 \rightarrow \text{Abnormal condition} \end{array} \right.$$

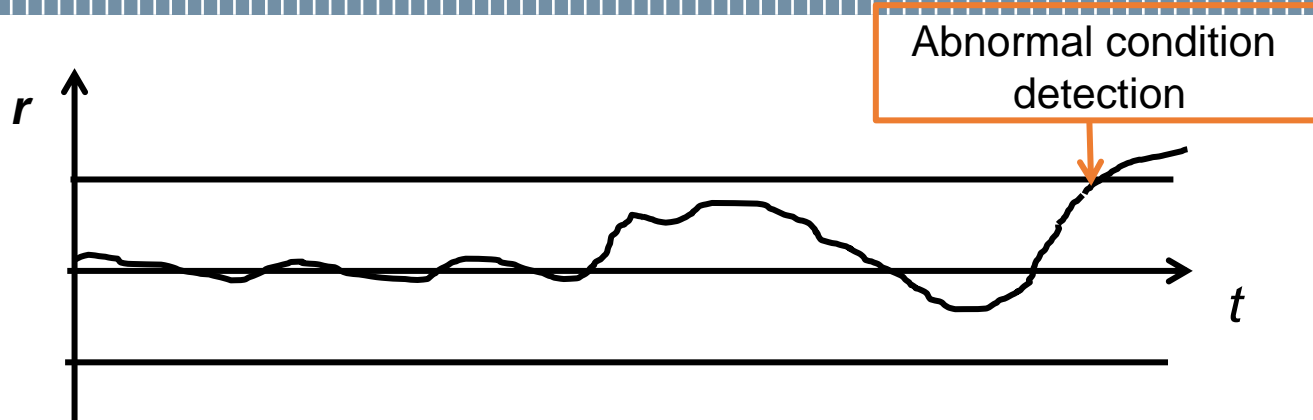
- Methods

- Thresholds-based approach
- Stochastic approaches:
 - Q Statistics
 - Sequential Probability Ratio Test (SPRT)



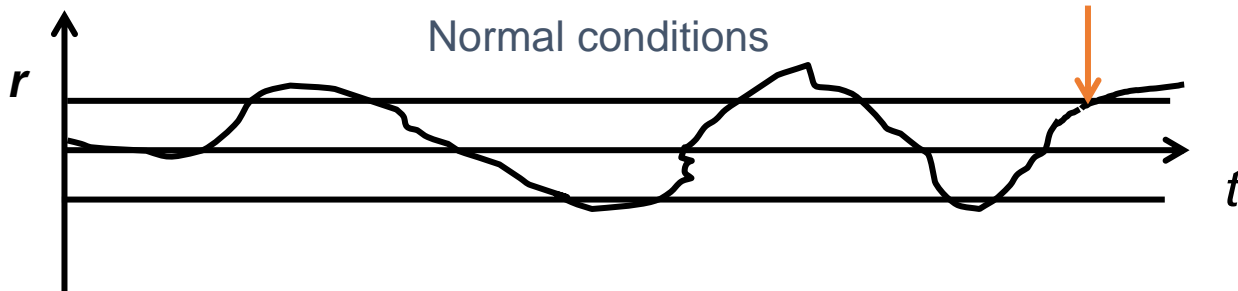
PART 2 A: Statistical Test

- **Thresholds-based**
- Q Statistics
- Sequential Probability Ratio Test (SPRT)

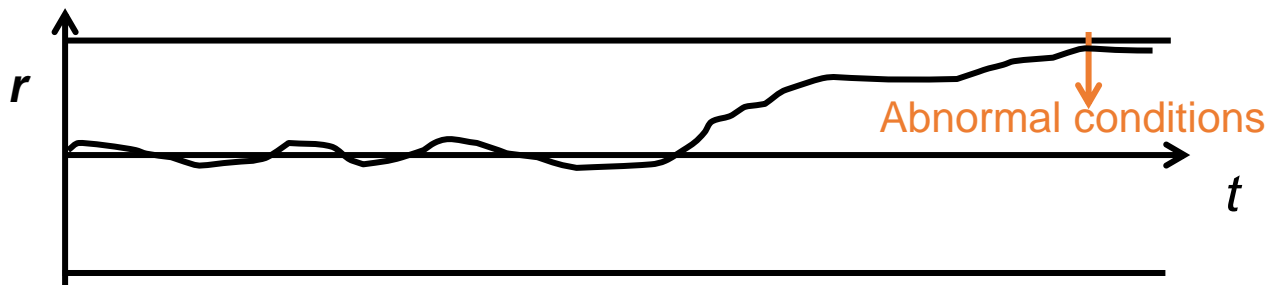


- Easy to apply
- Thresholds setting is difficult and error-prone

Too small thresholds \rightarrow high false alarm rates (α)



Too large thresholds \rightarrow high missing alarm rates (β)

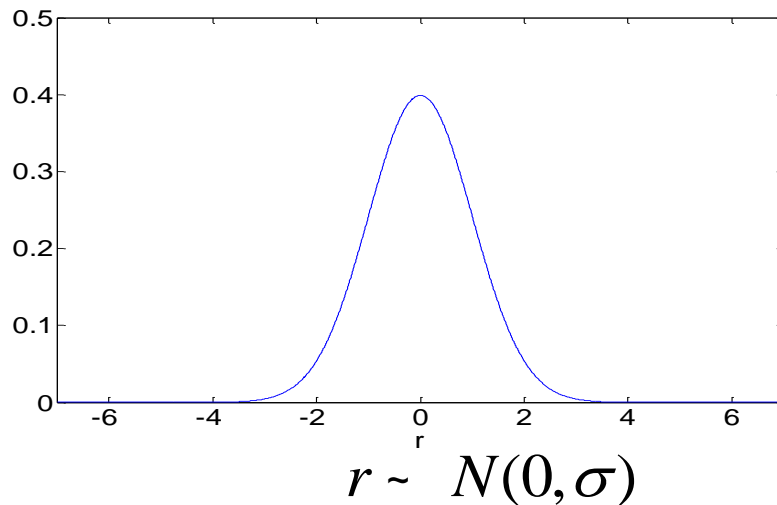


PART 2 B: Statistical Test

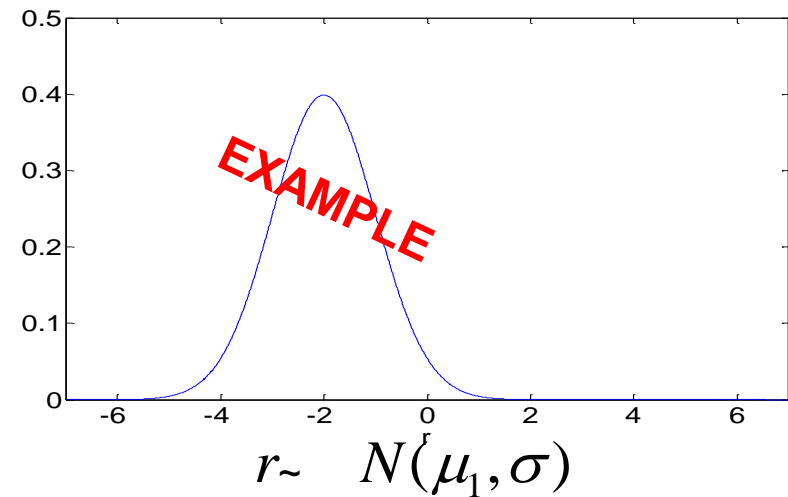
- Thresholds-based
- **Q-Statistics**
- Sequential Probability Ratio Test (SPRT)

- Residual (r)= **random variable** described by a probability law
- The probability law is different in case of normal/abnormal condition

Normal condition



Abnormal condition



- Assuming the signal reconstructions at time t are:

$$\vec{\hat{x}}^{nc}(t) = (\hat{x}_1(t), \hat{x}_2(t), \dots, \hat{x}_n(t)),$$

then the Q-stat at time t is:

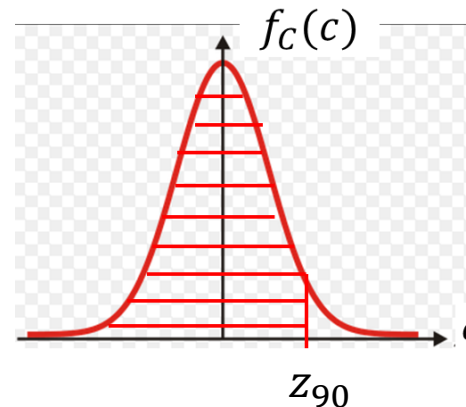
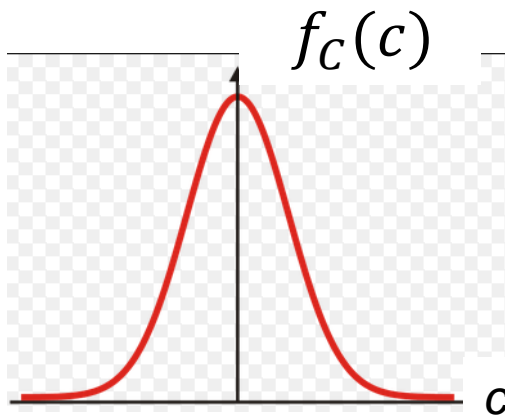
$$\begin{aligned} Q(t) = \vec{r}(t)^T \cdot \vec{r}(t) &= (\vec{x}^{obs}(t) - \vec{\hat{x}}^{nc}(t)) (\vec{x}^{obs}(t) - \vec{\hat{x}}^{nc}(t))^T = \\ &= \sum_{i=1}^n (x_i^{obs}(t) - \hat{x}_i^{nc}(t))^2 \end{aligned}$$

The Q-statistics (squared prediction error) is a metric that accounts for the amount of variance that is not captured by the **chosen** l - dimensional PCA model, which represents the “normal behaviour” of the signals.

- Let $\vartheta_i = \sum_{k=l+1}^n \lambda_k^i, i = 1,2,3; h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}$

Theorem ,see Appendix 1, J.E. Jackson and G.S. Mudholkar, Technometrics, 21(3), (1979), 341-349

- $c = \frac{\theta_1 \left[\left(\frac{Q}{\theta_1} \right)^{h_0} - 1 - \frac{\theta_2 h_0 (h_0 - 1)}{\theta_1^2} \right]}{\sqrt{2\theta_2 h_0^2}}$ is approximately $N(0,1)$

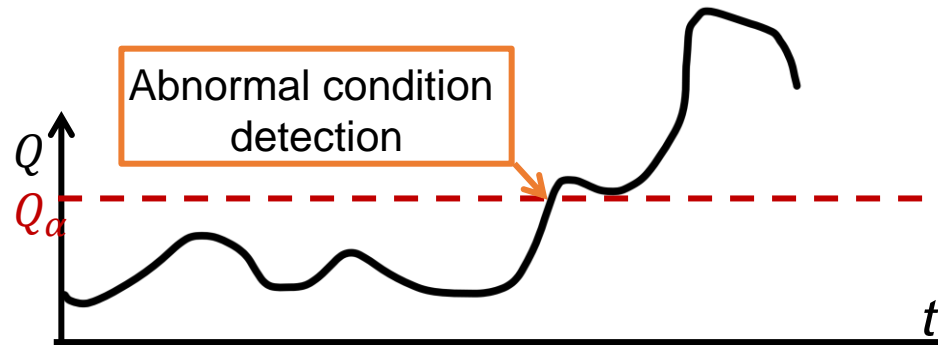


$$F_C(c) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^c e^{-\frac{1}{2}\xi^2} d\xi$$

from tables

Equipment in normal condition $\rightarrow P(c \leq z_{90})=0.9$

$$Q_\alpha = \vartheta_1 \left(\frac{h_0 z_\alpha \sqrt{2\theta_2}}{\theta_1} + 1 + \frac{h_0 \theta_2 (h_0 - 1)}{\theta_1^2} \right)^{1/h_0} \quad \rightarrow P(Q \leq Q_{90}) = 0.9$$



$$P(\text{abnormal condition} | \text{normal condition}) = 0.1$$

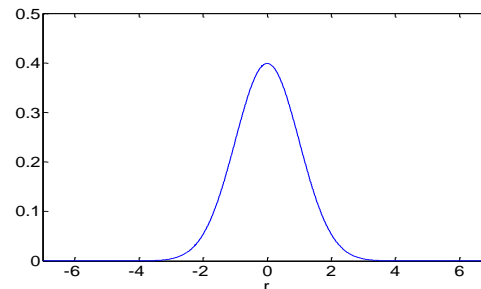
FALSE POSITIVE

PART 2 C: Statistical Test

- General Idea
- Q Statistics
- **Sequential Probability Ratio Test (SPRT)**

- $R_T = \{r^{(1)}, \dots, r^{(T)}\}$ sequence of residuals at time $t = 1, \dots, T$, where $r^{(t)} = x^{obs}(t) - \hat{x}^{nc}(t)$
- Binary hypothesis test:
 - Null hypothesis (H_0) \equiv Normal condition

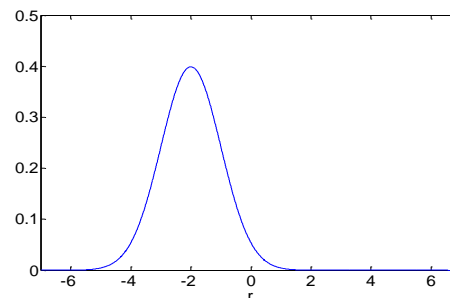
$$r^{(t)} \sim \mathcal{N}(0, \sigma), \forall t$$



$$f_0(r) \equiv \mathcal{N}(0, \sigma)$$

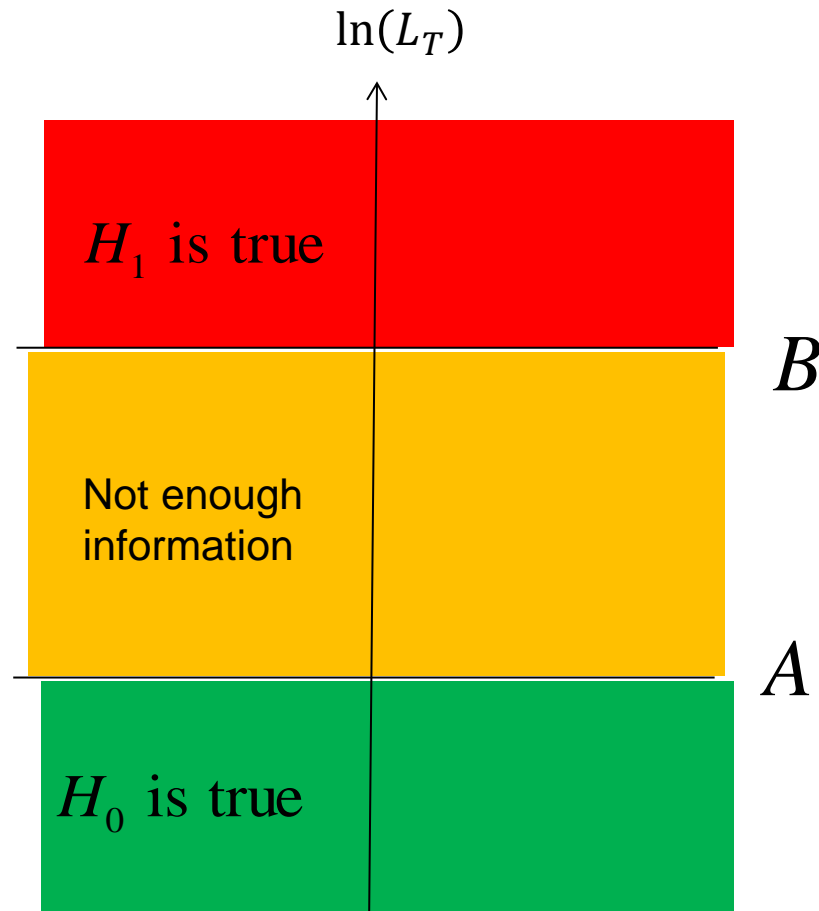
- Alternative hypothesis (H_1) \equiv Abnormal condition

$$r^{(t)} \sim \mathcal{N}(\mu_1, \sigma), \forall t$$



$$f_1(r) \equiv \mathcal{N}(\mu_1, \sigma)$$

$$L_T = \frac{P\{R_T|H_1 \text{ is true}\}}{P\{R_T|H_0 \text{ is true}\}} = \frac{f_1(r^{(1)}) \cdot f_1(r^{(2)}) \cdot \dots \cdot f_1(r^{(T)})}{f_0(r^{(1)}) \cdot f_0(r^{(2)}) \cdot \dots \cdot f_0(r^{(T)})}$$



$$B = \ln \frac{1 - \beta}{\alpha}$$

$$A = \ln \frac{\beta}{1 - \alpha}$$



False alarm

$$P\{D_1 \mid H_0 \text{ is true}\} \leq \alpha$$
$$P\{D_0 \mid H_1 \text{ is true}\} \leq \beta$$

Missing alarm

- Null hypothesis (H_0) \equiv Normal condition $r^{(t)} \sim \mathcal{N}(0, \sigma)$
- Alternative hypothesis (H_1) \equiv Abnormal condition $r^{(t)} \sim \mathcal{N}(\mu_1, \sigma)$

$$L_T = \frac{P(r^{(1)}, \dots, r^{(T)} | H_1)}{P(r^{(1)}, \dots, r^{(T)} | H_0)} = e^{-\frac{1}{2\sigma^2} \sum_{t=1}^T \mu_1(\mu_1 - 2r^{(t)})} = e^{\frac{\mu_1}{\sigma^2} \sum_{t=1}^T (r^{(t)} - \frac{\mu_1}{2})}$$



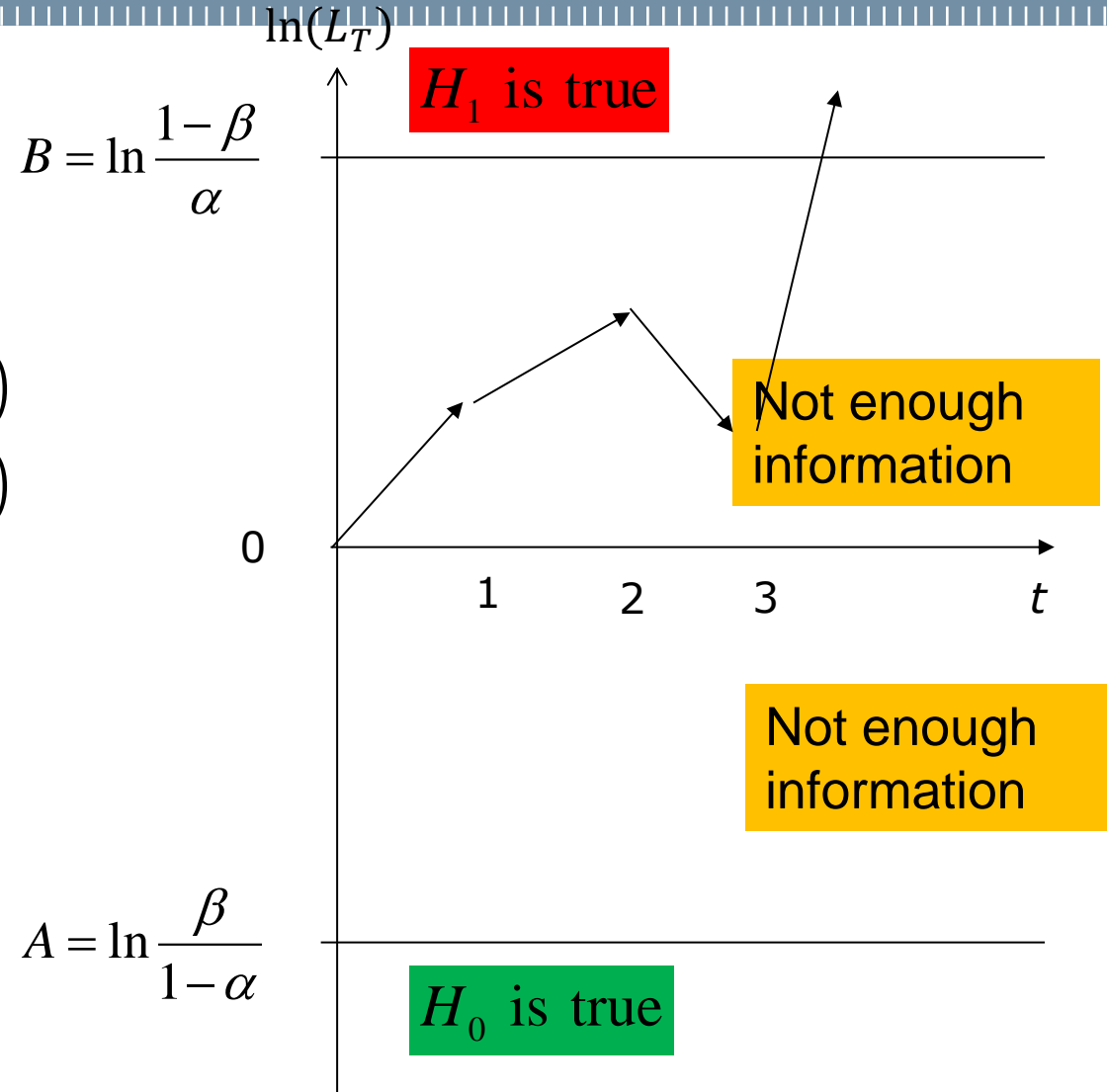
$$\begin{aligned} \ln(L_T) &= \frac{\mu_1}{\sigma^2} \sum_{t=1}^T \left(r^{(t)} - \frac{\mu_1}{2} \right) = \frac{\mu_1}{\sigma^2} \sum_{t=1}^{T-1} \left(r^{(k)} - \frac{\mu_1}{2} \right) + \frac{\mu_1}{\sigma^2} \left(r^T - \frac{\mu_1}{2} \right) \\ &= \ln(L_{T-1}) + \frac{\mu_1}{\sigma^2} \left(r^{(T)} - \frac{\mu_1}{2} \right) \end{aligned}$$

Sequential
Formula!

$$L_0 = 1 \rightarrow \ln(L_0) = 0$$

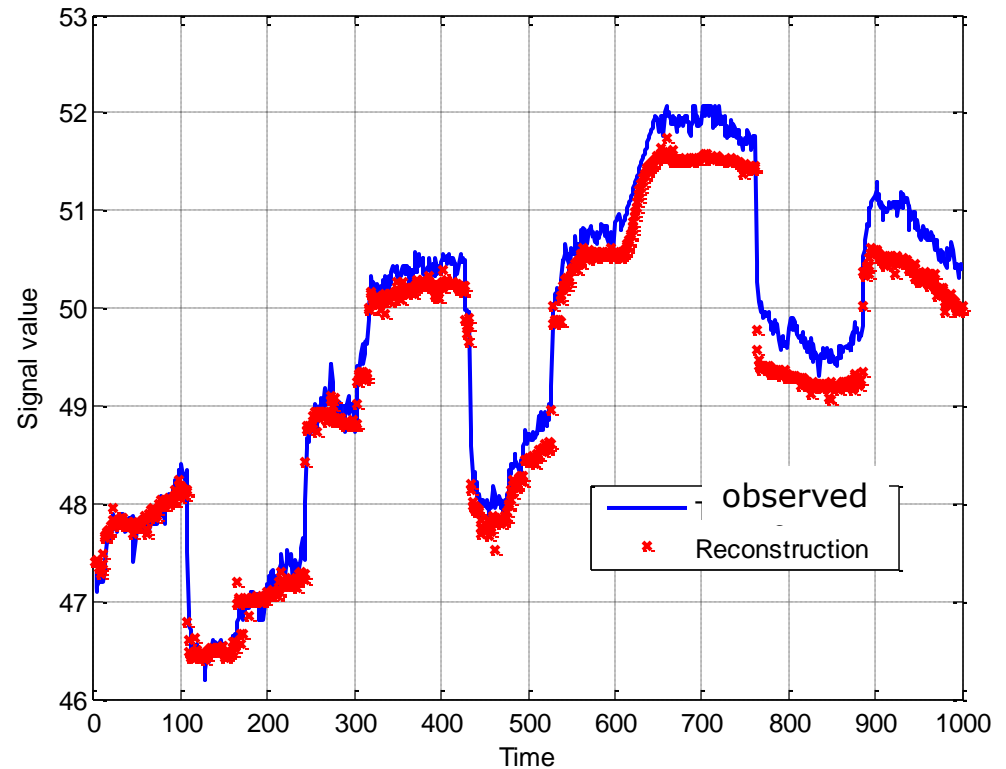
$$\ln(L_1) = \ln(L_0) + \frac{\mu_1}{\sigma^2} \left(r^{(1)} - \frac{\mu_1}{2} \right)$$

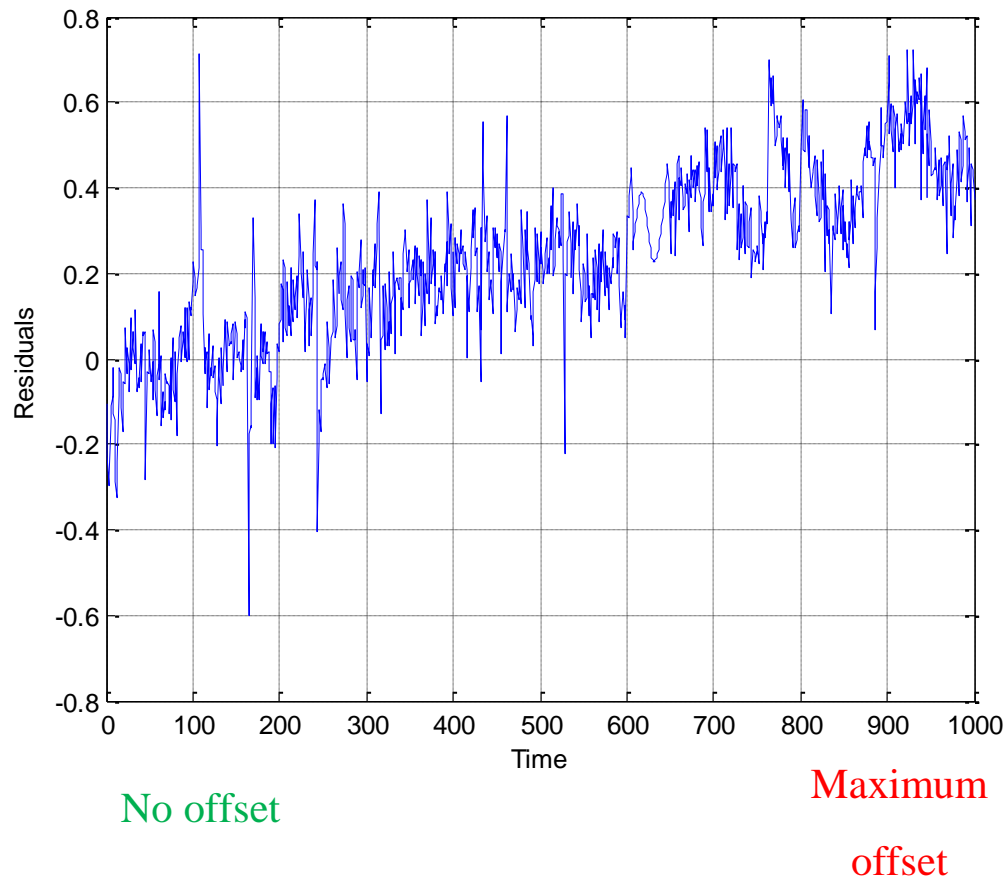
$$\ln(L_2) = \ln(L_1) + \frac{\mu_1}{\sigma^2} \left(r^{(2)} - \frac{\mu_1}{2} \right)$$



- the residual variance in normal condition (σ^2)
- the expected offset amplitude (μ_1)
- the maximum acceptable false alarm rate (α)
- the maximum acceptable missing alarm rate (β)

Time interval	Simulated Offset
[0-200]	No
[201-400]	Yes (amplitude = 0.11)
[401-600]	Yes (amplitude = 0.23)
[601-800]	Yes (amplitude = 0.34)
[801-1000]	Yes (amplitude = 0.46)





H_1 is true

Parameter	Value
α	0.01
B	0.01
μ_0	0
μ_1	0.46
σ^2	0.12

$$B = \ln \frac{1-\beta}{\alpha}$$

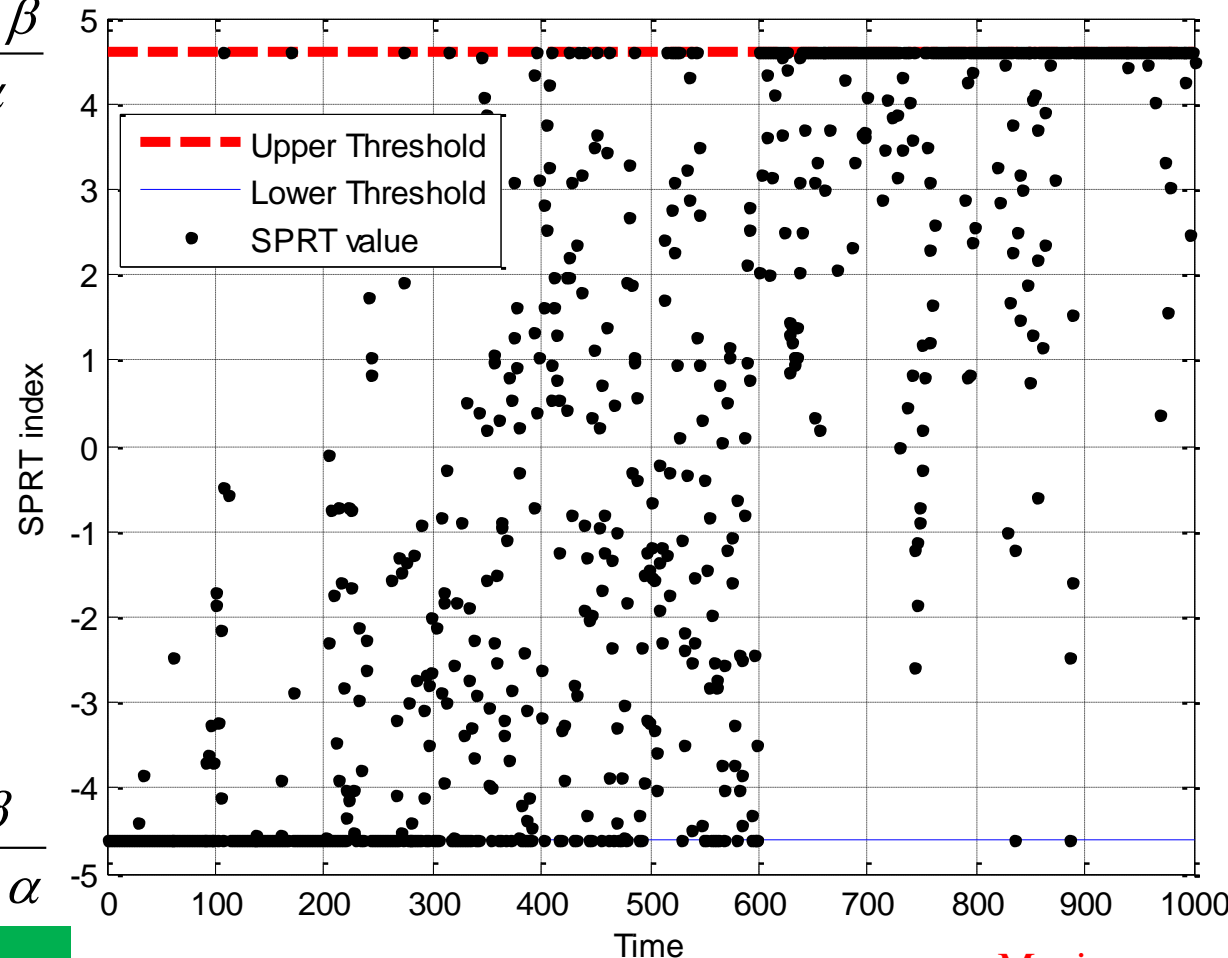
$$L_0 = 1 \rightarrow \ln(L_0) = 0$$

$$\ln(L_1) = \ln(L_0) + \frac{\mu_1}{\sigma^2} \left(r^{(1)} - \frac{\mu_1}{2} \right)$$

$$\ln(L_2) = \ln(L_1) + \frac{\mu_1}{\sigma^2} \left(r^{(2)} - \frac{\mu_1}{2} \right)$$

$$A = \ln \frac{\beta}{1-\alpha}$$

H_0 is true



No offset

Maximum
offset

Average Sample Number (ASN) needed to deliver a decision



Time interval	Offset	Estimated ASN	Number of times in which a normal condition has been detected	Number of times in which an abnormal condition has been detected
[0-200]	No	1.2	150	2
[201-400]	amplitude = 0.11	1.9	70	5
[401-600]	amplitude = 0.23	2.4	15	17
[601-800]	amplitude = 0.34	2.1	0	94
[801-1000]	amplitude = 0.46	1.2	2	142

Challenges?

- Hundreds of Signals are Monitored
- Evolving Environment
- Robustness

Data-Driven Fault Detection: Challenge I

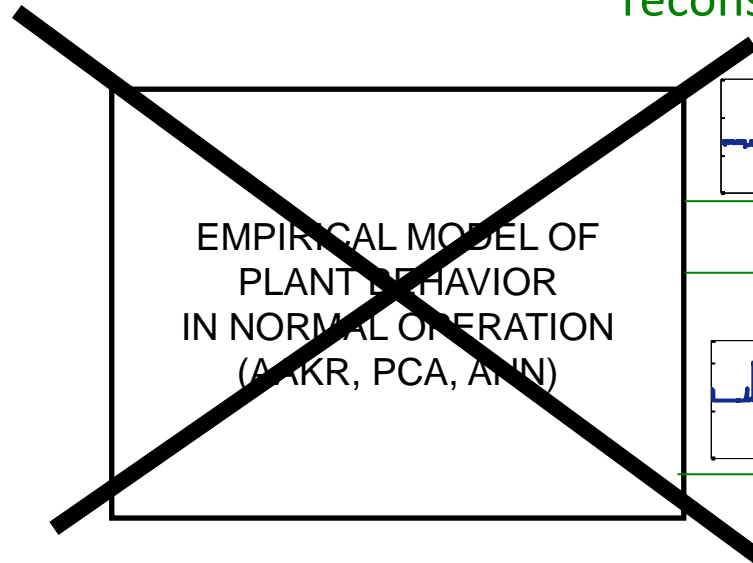
96

1	#312KA031 VALUE	kg/s	sum feedwater flows
2	#312KA032 VALUE	kg/s	sum feedwater flows
3	#312KA033 VALUE	kg/s	sum feedwater flows
4	#312KA201 VALUE	MPa	pressure difference over VA23
5	#312KA301 VALUE	kg/s	flow train A
6	#312KA302 VALUE	kg/s	flow train A
7	#312KA303 VALUE	kg/s	flow train A
8	#312KA306 VALUE	kg/s	flow low power (blade laglast)
9	#312KA502 VALUE	C	feedwater temp train A
10	#312KA503 VALUE	C	temp after VA8
11	#312KC301 VALUE	kg/s	flow train C
12	#312KC302 VALUE	kg/s	flow train C
13	#312KC303 VALUE	kg/s	flow train C
14	#312KC502 VALUE	C	feedwater temp line C
15	#313KA511 VALUE	C	temp PA1 (main circulation pump)
16	#313KA512 VALUE	C	temp PA2 (main circulation pump)
17	#313KA711 VALUE	mm/s	vibration PA1 radial
18	#313KA712 VALUE	mm/s	vibration PA2 radial
19	#313KA721 VALUE	mm/s	vibration PA1 tangential
20	#313KA722 VALUE	mm/s	vibration PA2 tangential
21	#313KA731 VALUE	rpm	rotation speed PA1
22	#313KA732 VALUE	rpm	rotation speed PA2
23	#313KB511 VALUE	C	temp PB1 (main circulation pump)
24	#313KB512 VALUE	C	temp PB2 (main circulation pump)
25	#313KB711 VALUE	mm/s	vibration PB1 radial
26	#313KB712 VALUE	mm/s	vibration PB2 radial
27	#313KB721 VALUE	mm/s	vibration PB1 tangential
28	#313KB722 VALUE	mm/s	vibration PB2 tangential
29	#313KB731 VALUE	rpm	rotation speed PB1
...
123	#313KC511 VALUE	C	temp PC1 (main circulation pump)

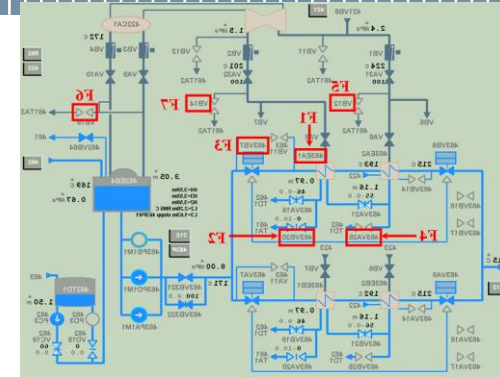
Hundreds of signals are monitored



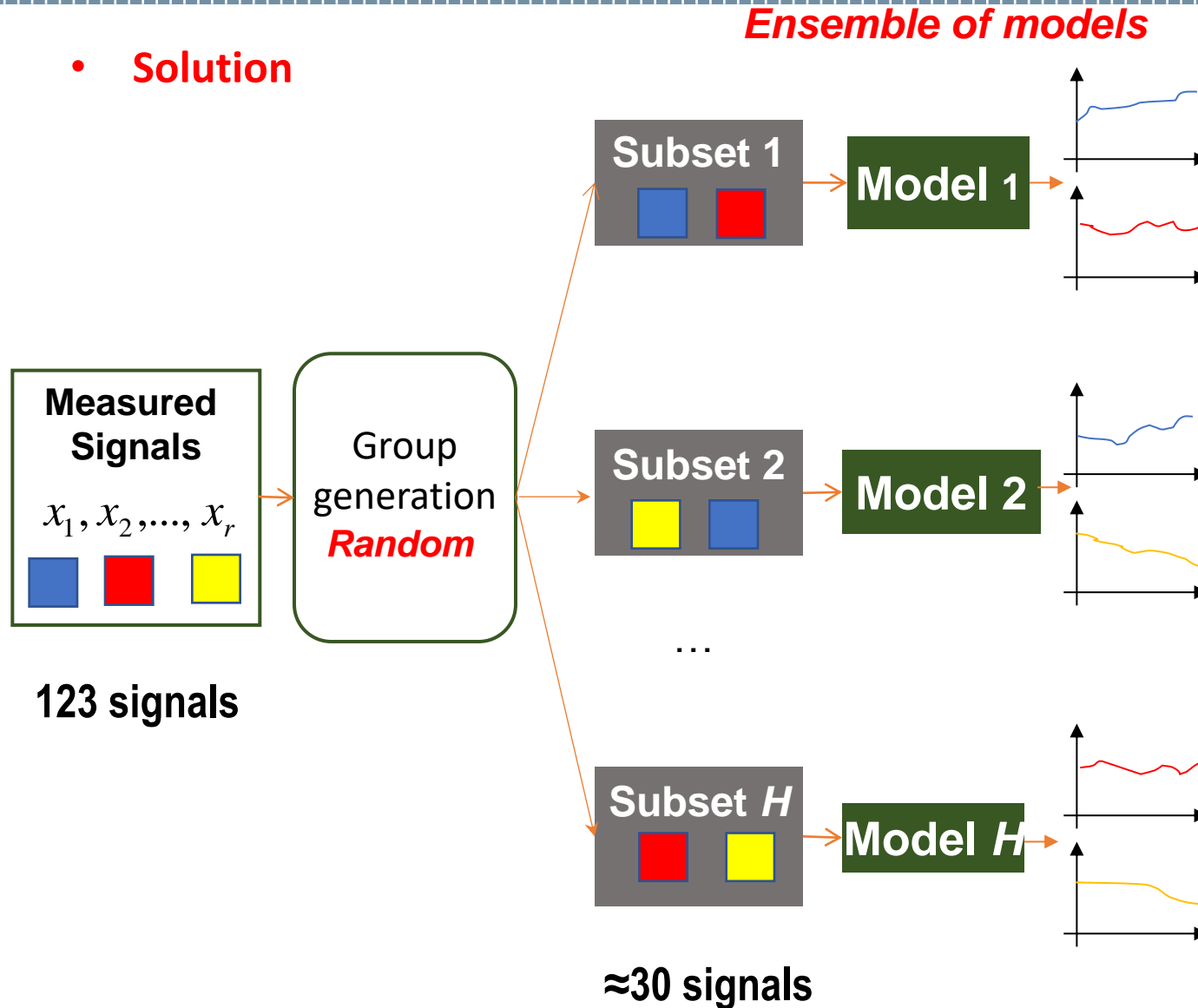
Signal
reconstructions



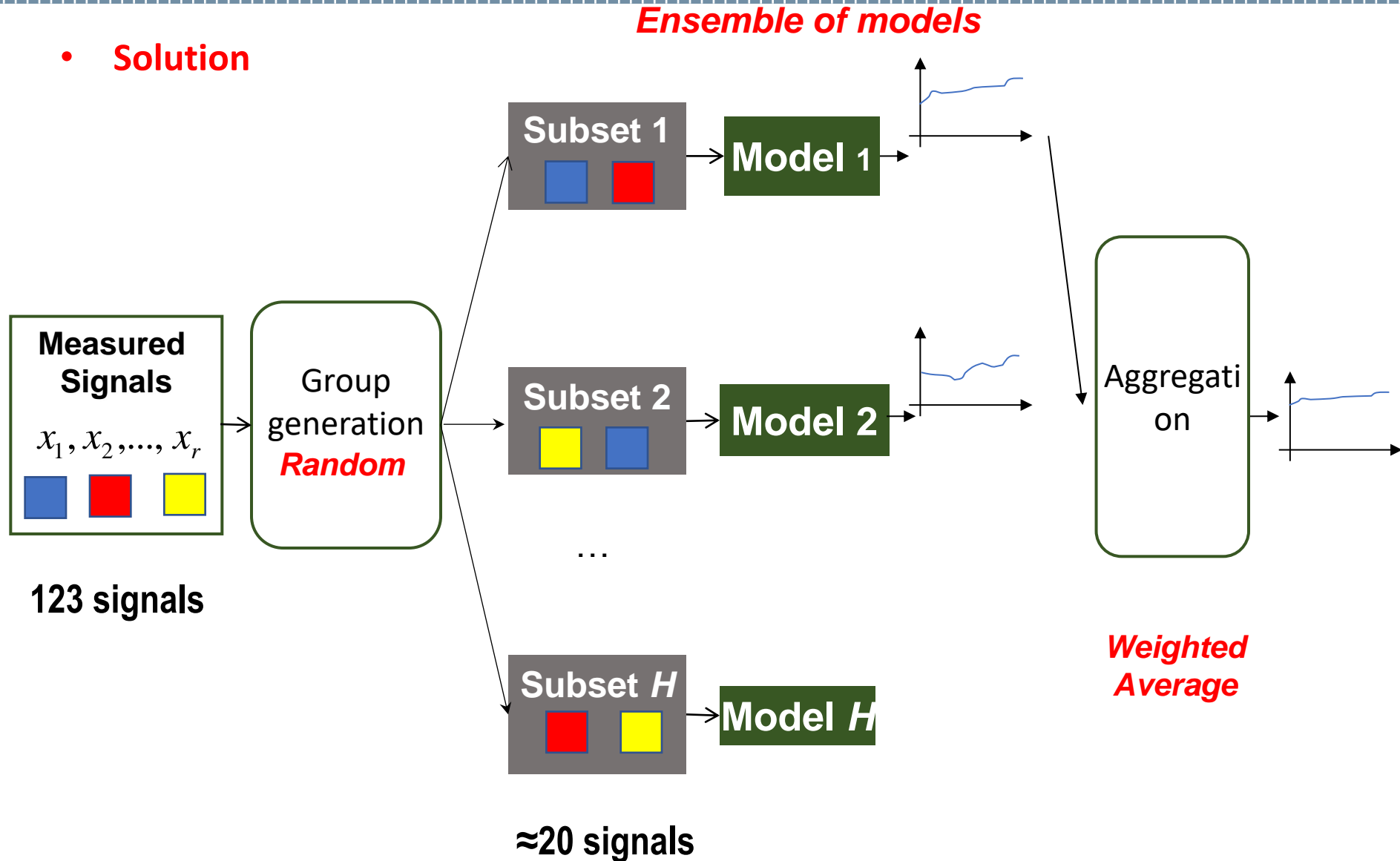
DIFFICULT TO DEVELOP



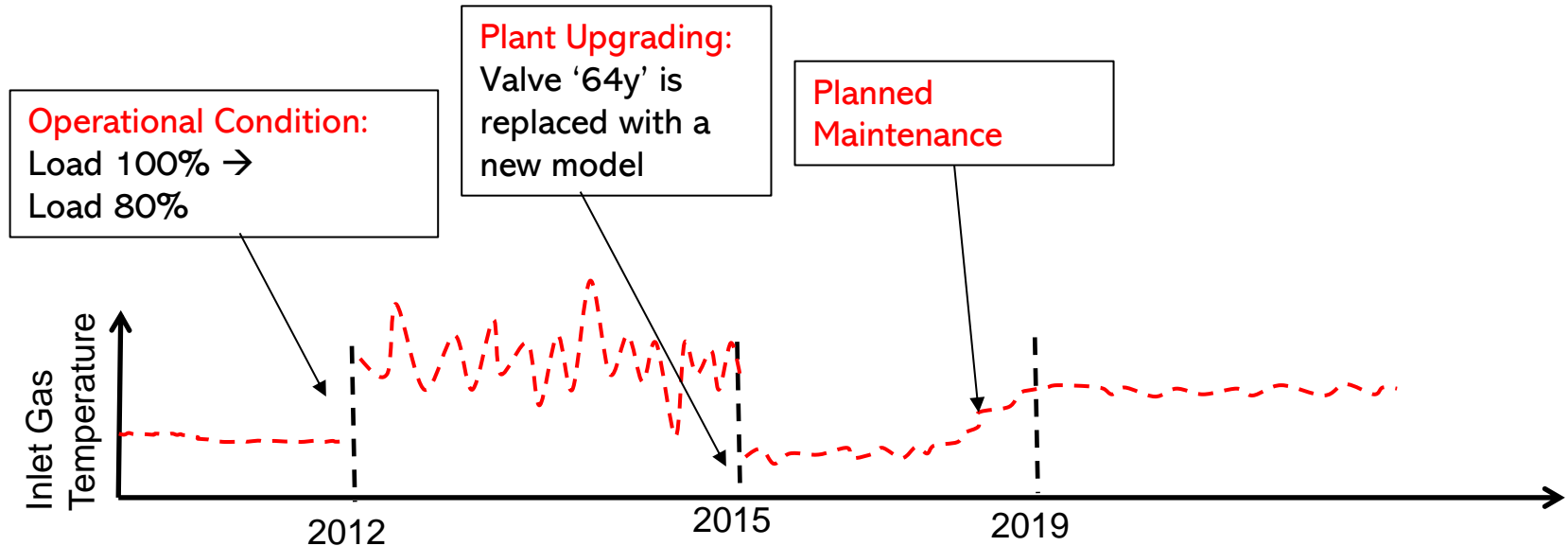
- Solution**



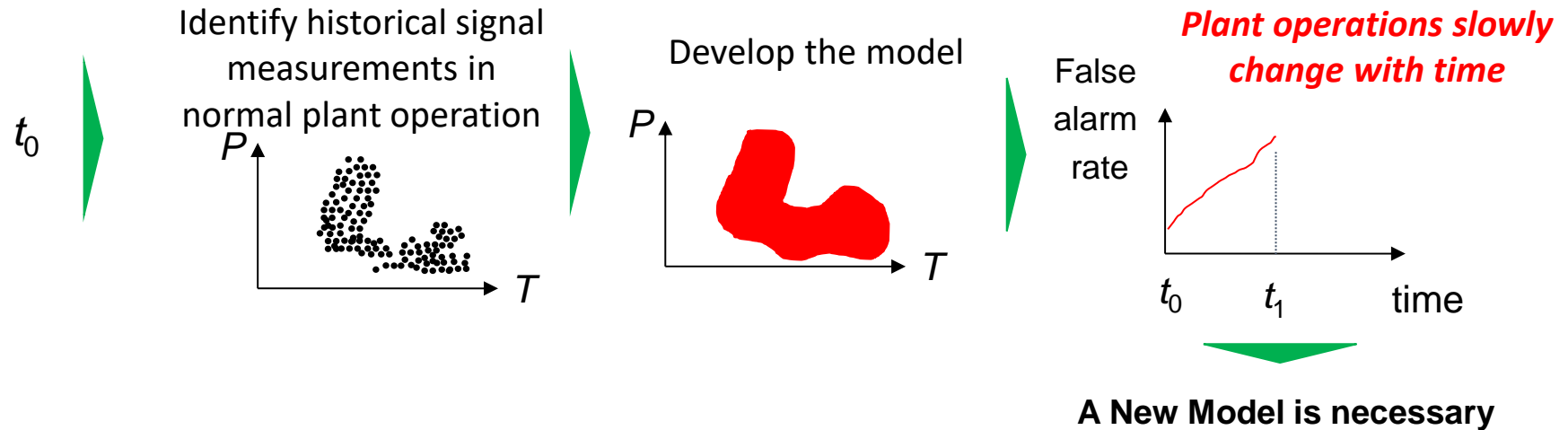
- Solution**



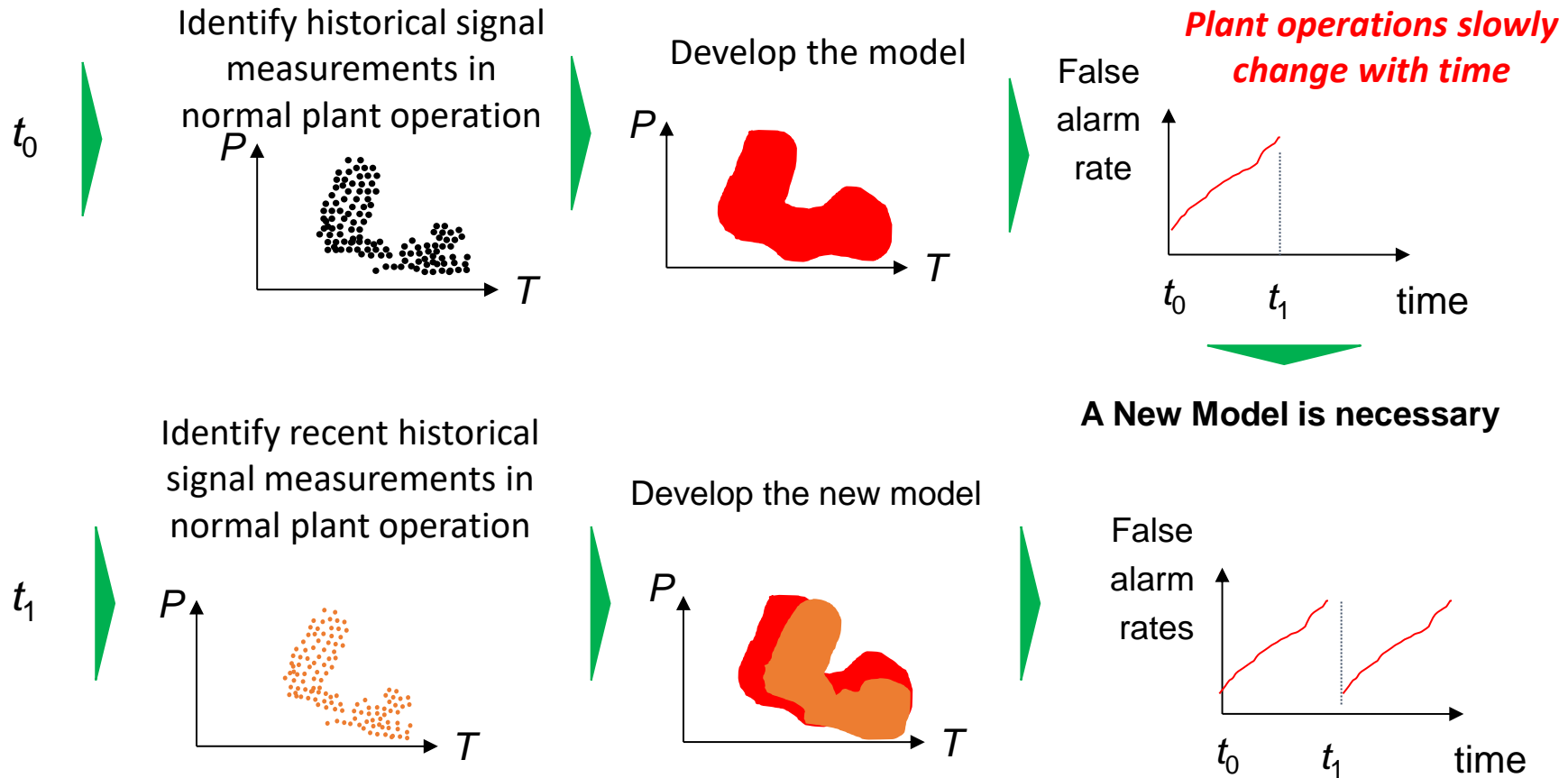
Evolving Environment



Example: monitoring the turbine of an electric power plant

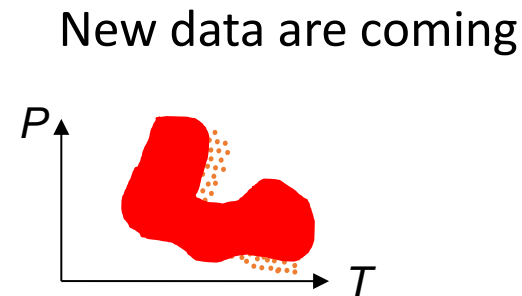
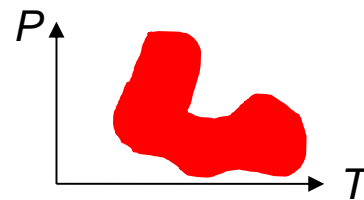


Example: monitoring the turbine of an electric power plant



Periodic Human Interventions for developing new models! → high costs!

- The detection model should be able to follow the process changes:
- Incremental learning of the new data that gradually becomes available
- No necessity of human intervention for:
 - selecting recent normal operation data
 - building the new model

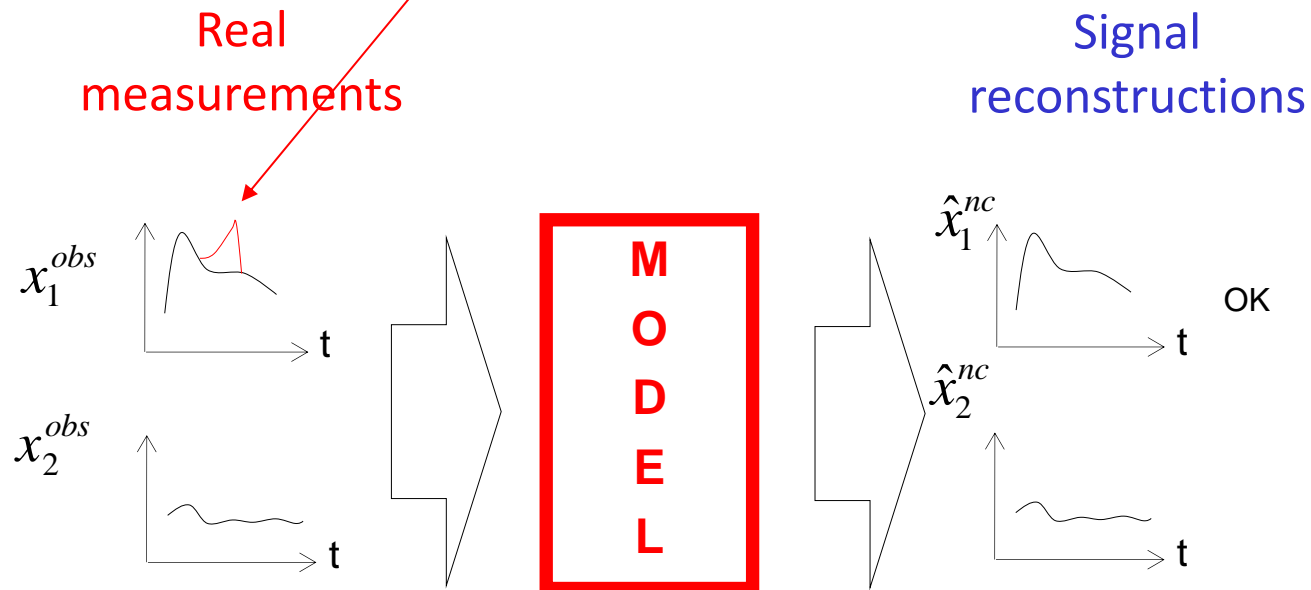


Automatic updating of the model



Robustness of the Reconstruction

ABNORMAL CONDITION



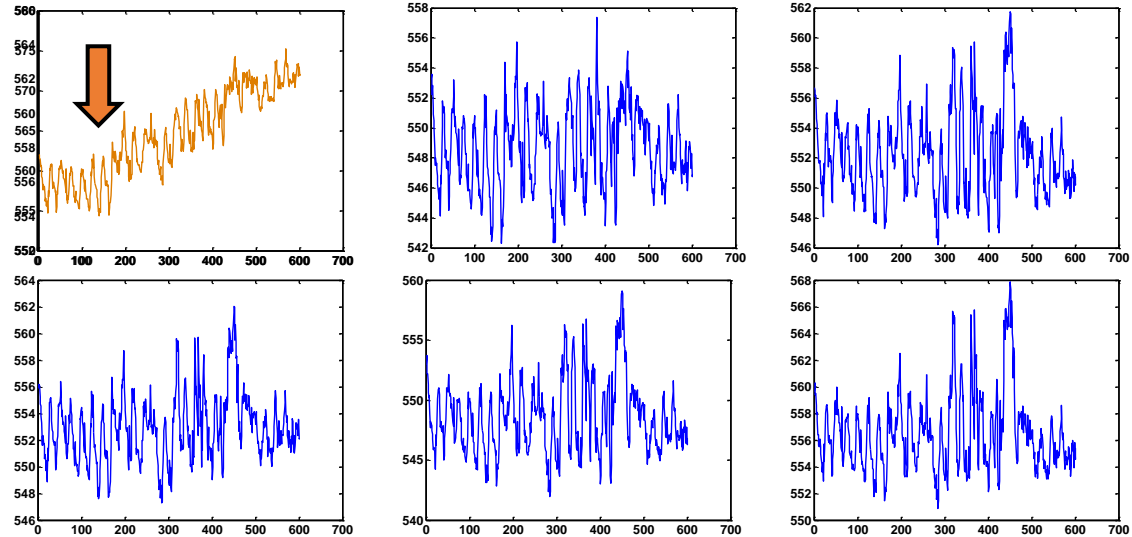
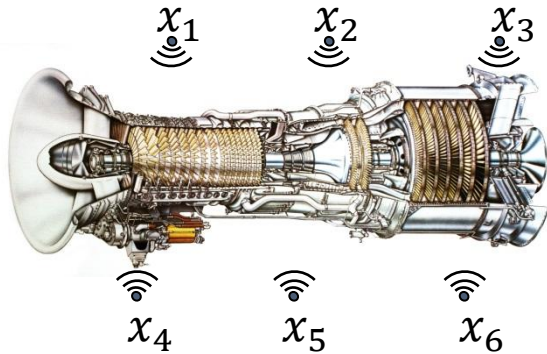
ROBUSTNESS:

$$\vec{\hat{x}}^{nc} \cong \vec{x}^{obs-nc}$$

Example1: Monitoring a Turbine for Energy Production

104

6 Temperature Sensors in different position



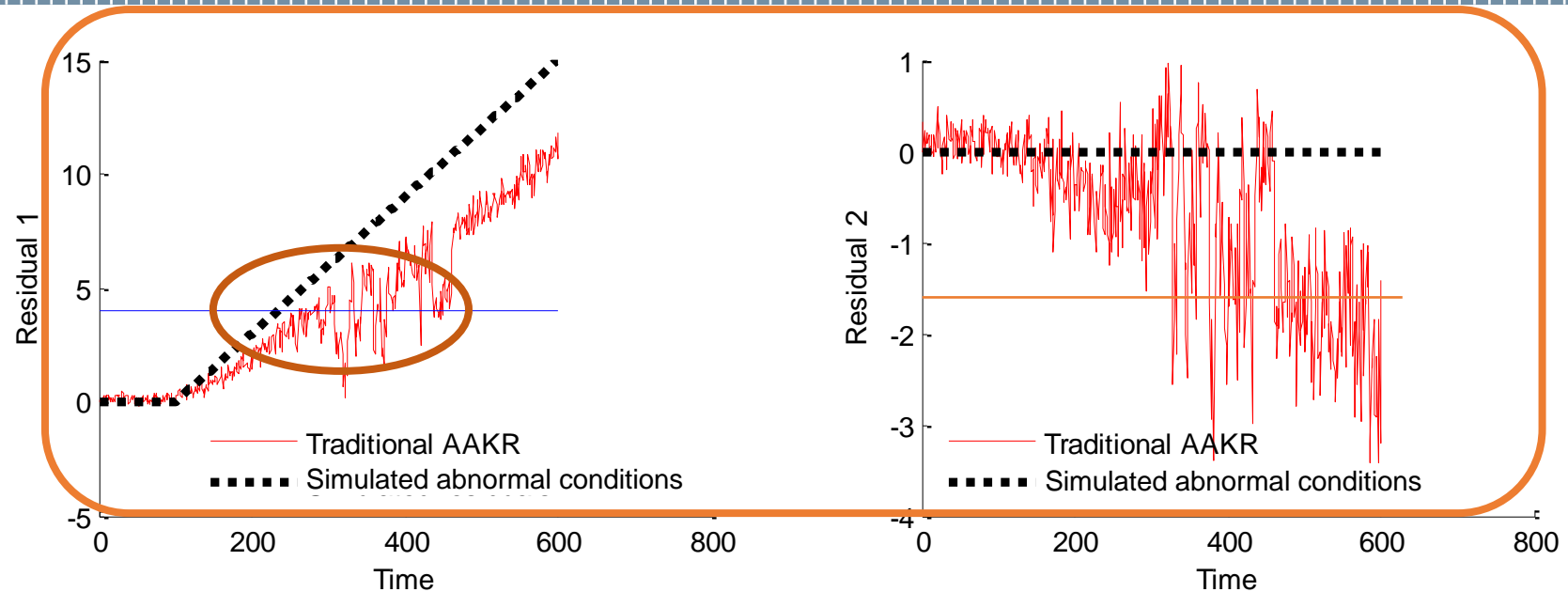
Abnormal condition of the first signal

Highly Correlated Signals

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1	0.97	0.98	0.98	0.99	0.98
x_2	0.97	1	0.95	0.99	0.98	0.96
x_3	0.98	0.95	1	0.96	0.99	0.99
x_4	0.98	0.99	0.96	1	0.98	0.97
x_5	0.99	0.98	0.99	0.98	1	0.99
x_6	0.98	0.96	0.99	0.97	0.99	1

Example1: Traditional AAKR

105

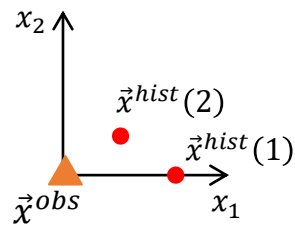
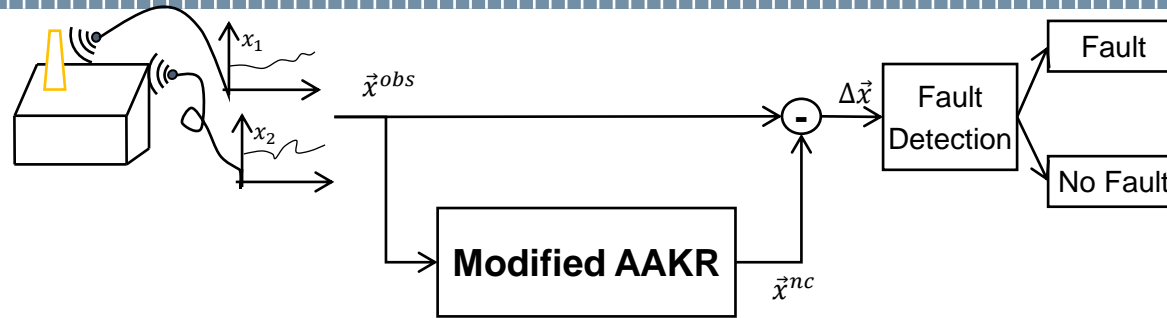


► **DELAY IN THE DETECTION**

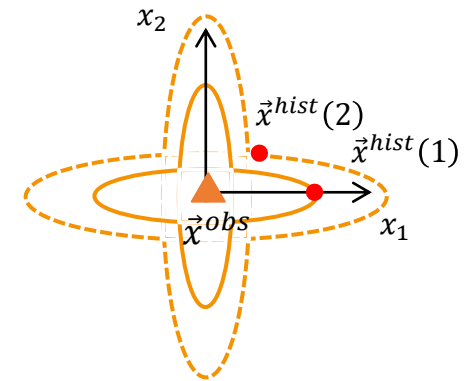
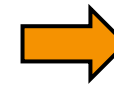
► **IMPOSSIBILITY TO IDENTIFY THE SIGNALS
TRIGGERING THE ABNORMAL BEHAVIOR**

Our Contribution: A modified AAKR method

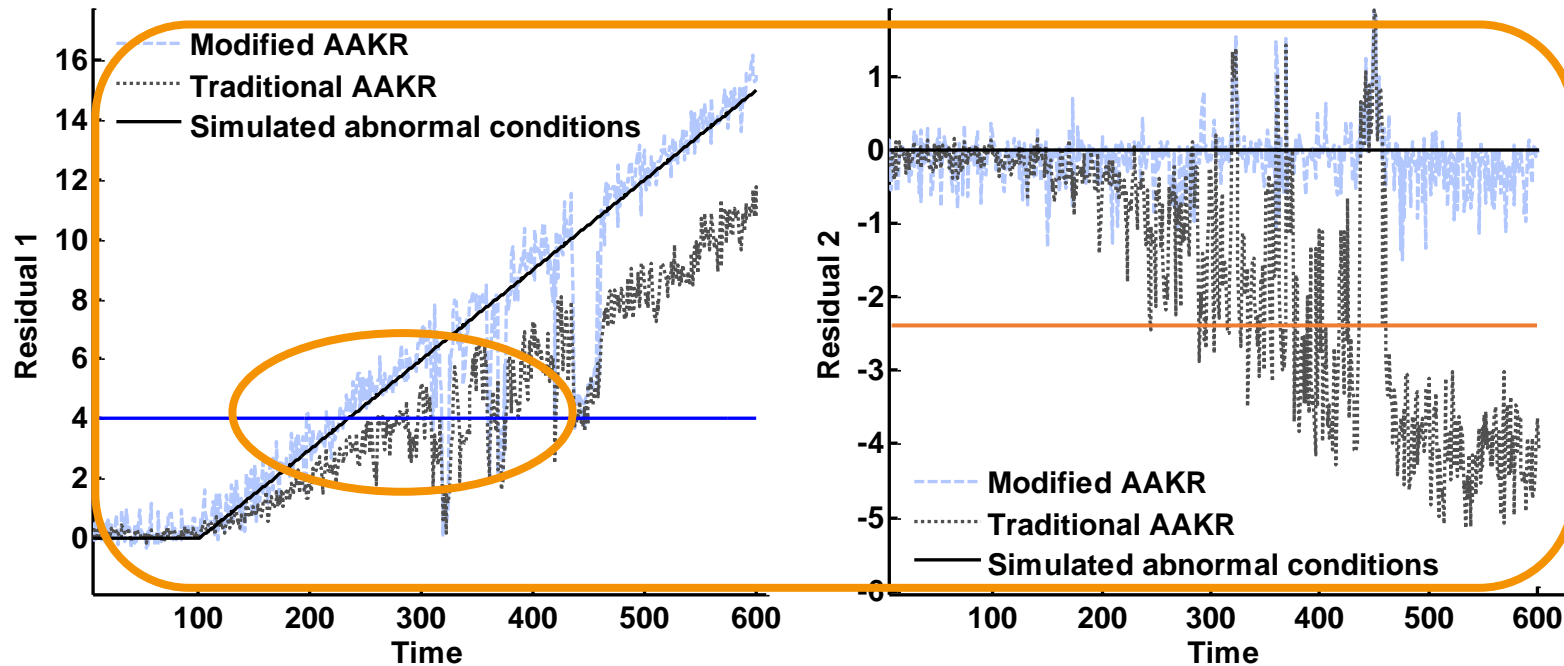
106



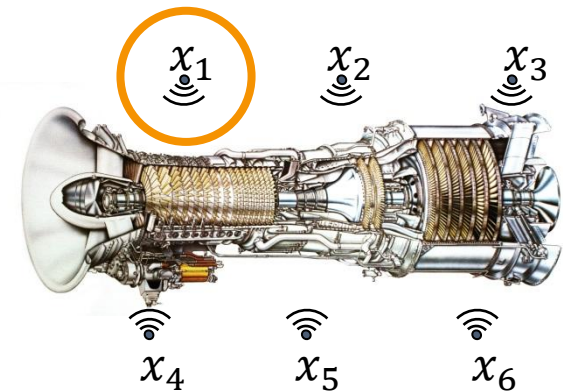
Malfunctions causing variations of a small number of signals are more frequent than those causing variations of a large number of signals

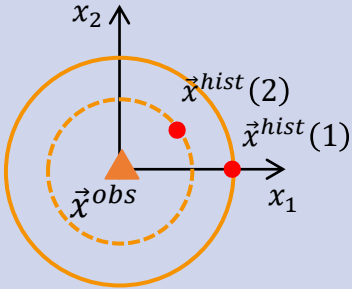
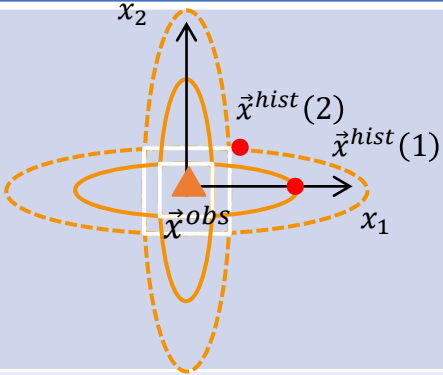


Modification of the loci of equisimilarity points



- **Early Detection**
- **Correct Diagnosis of the signal that triggers the alarms**
- **More Accurate**



	Traditional AAKR	Modify AAKR
Loci of equisimilarity points		
Accuracy	OK!	OK!
Robustness	NO! Especially with correlated signals	<ol style="list-style-type: none"> 1. Robust reconstruction of the values expected in normal conditions 2. Correct identification of signals affected by abnormal condition 3. Good performance with correlated signals

Example2: Monitoring Reactor coolant System of a Nuclear Power Plant

109

6 Sensors of reactor coolant system (RCS) measured during startup transient

S1 (Cold leg temperature)

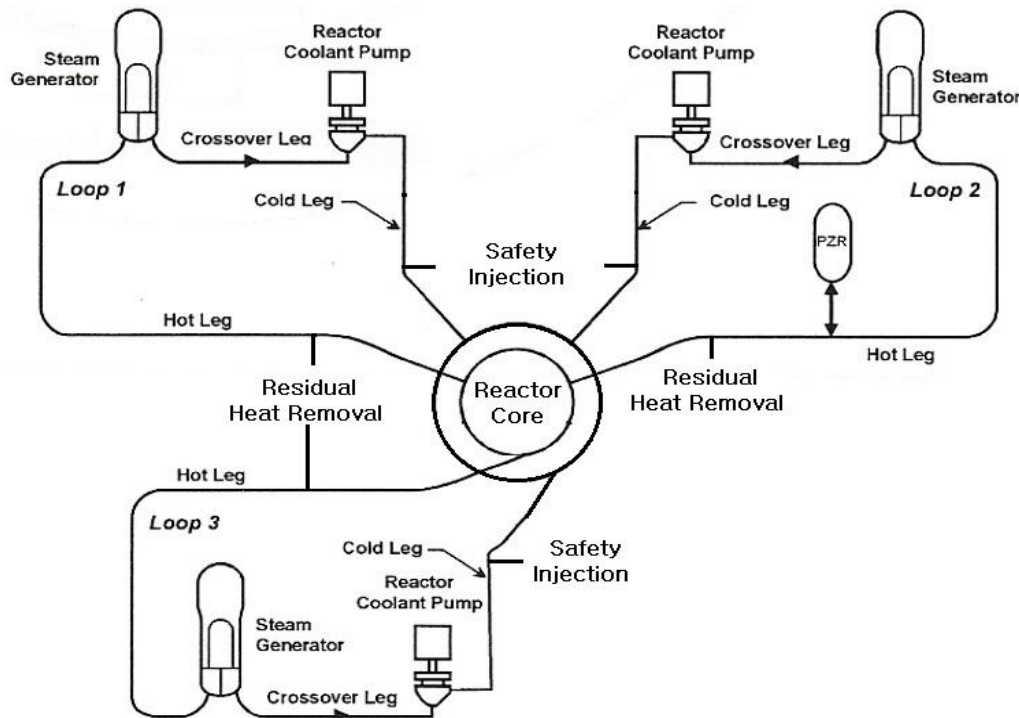
S2 (Core exit temperature)

S3 (Hot leg temperature)

S4 (Safety injection flow)

S5 (Residual heat removal flow)

S6 (Sub-cooling margin temperature)

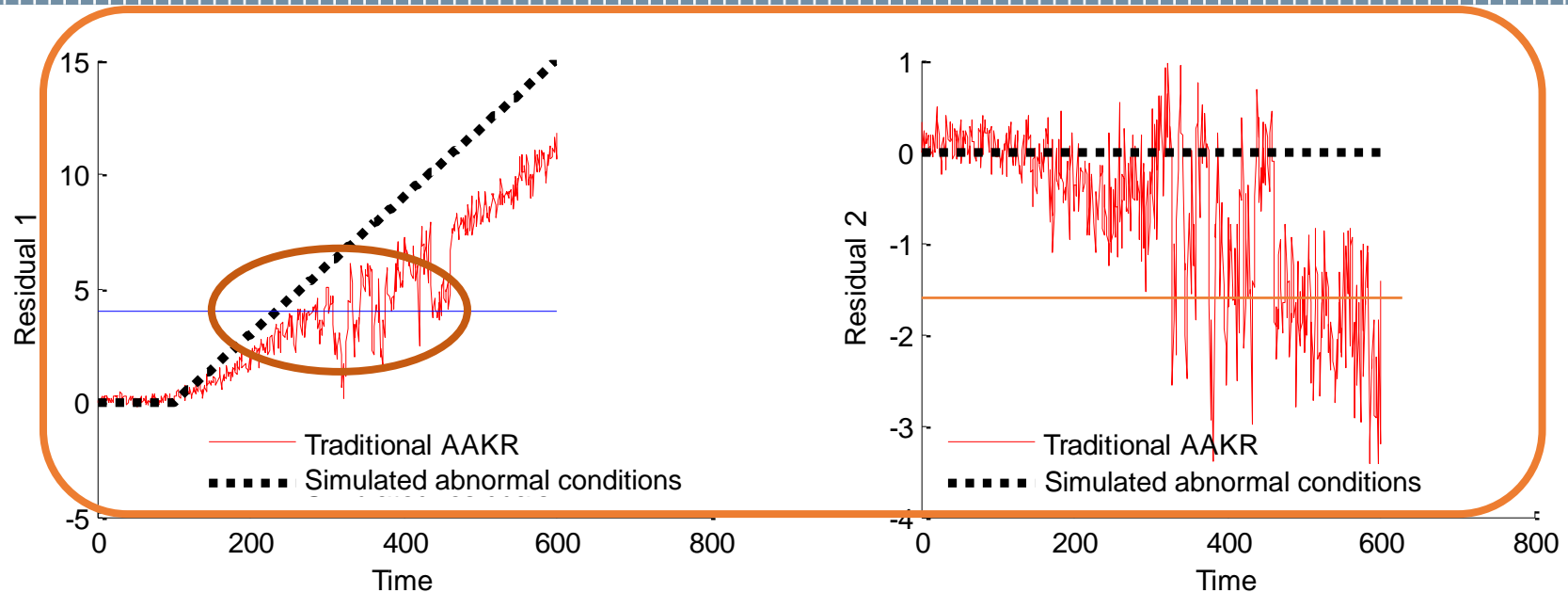


Correlations

	S1	S2	S3	S4	S5	S6
S1	1	0.99	0.99	-0.16	0.39	-0.98
S2	0.99	1	0.99	-0.18	0.39	-0.98
S3	0.99	0.99	1	-0.12	0.42	-0.97
S4	-0.16	-0.18	-0.12	1	0.65	0.31
S5	0.39	0.39	0.42	0.65	1	-0.23
S6	-0.98	-0.98	-0.97	0.31	-0.23	1

Example2: Traditional AAKR

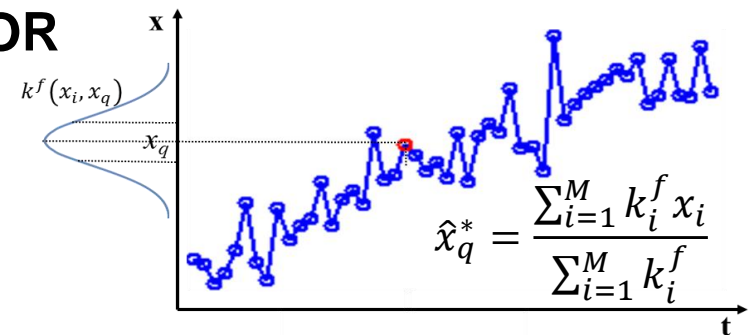
110



▶ **DELAY IN THE DETECTION**

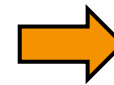
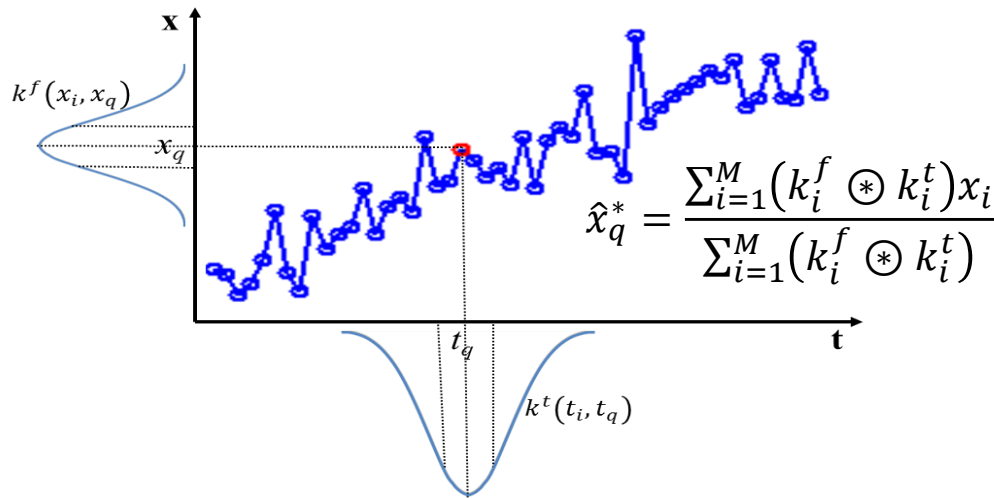
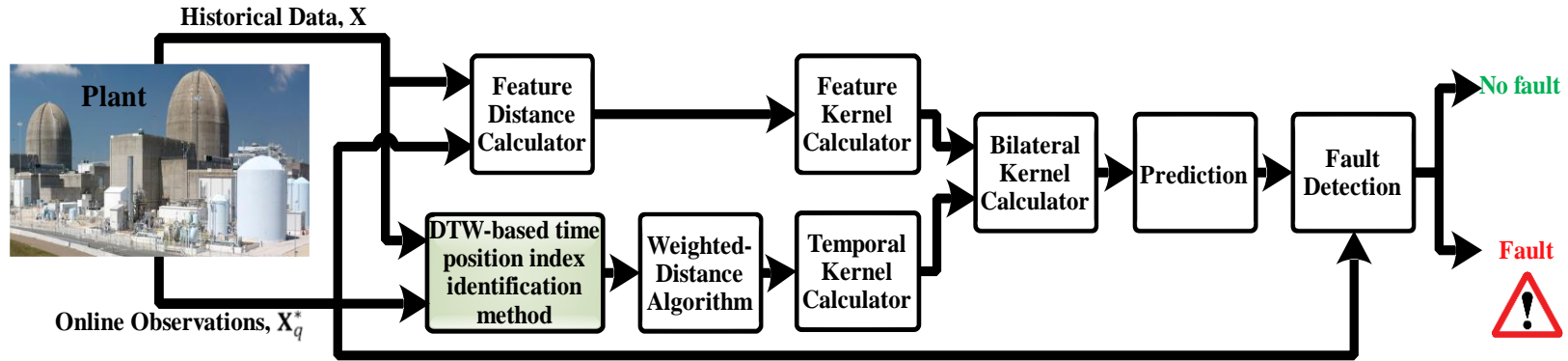
▶ **IMPOSSIBILITY TO IDENTIFY THE SIGNALS
TRIGGERING THE ABNORMAL BEHAVIOR**

▶ **LACKS TEMPORAL INFORMATION**

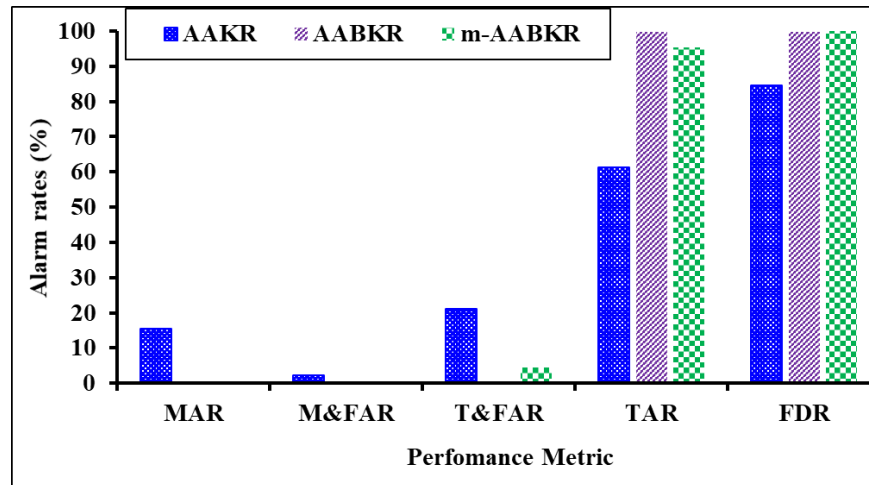


Our Contribution: AABKR method – Aggregating Bilateral Directions

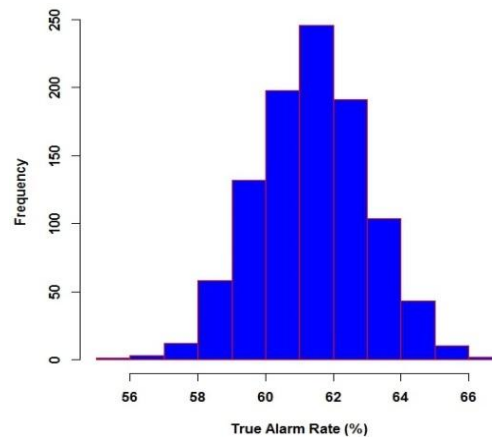
111



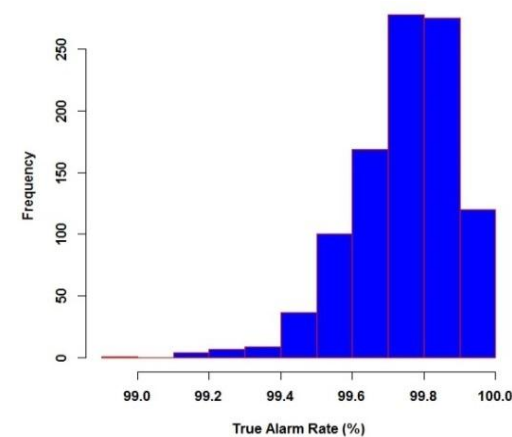
Aggregating bilateral directions capturing both spatial and temporal dependencies



Means of the alarm rates in start-up process operating condition

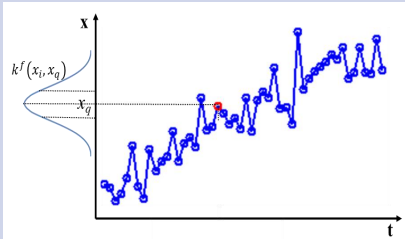
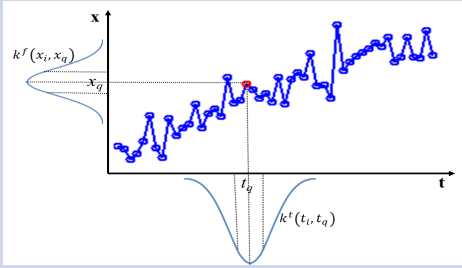


(a) AAKR



(b) AABKR

Distributions of the TARs for a thousand-run Monte Carlo in start-up

	Traditional AAKR	Modify AAKR (AABKR)
Information captured		
Accuracy	OK!	OK!
Robustness	NO! Especially with correlated signals and normal transient data	<ol style="list-style-type: none"> 1. Robust reconstruction of the values expected in normal conditions 2. Correct identification of signals affected by abnormal condition 3. Good performance with normal transient monitoring