



 POLITECNICO DI MILANO



## *Exercises Session on Fault Detection*

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## Exercise 1

*Method: AAKR (Signal Reconstruction)+SPRT (Decision)*

*Component: Gas Turbine*

## Exercise 2

*Method: PCA*

*Component: Gas Turbine*

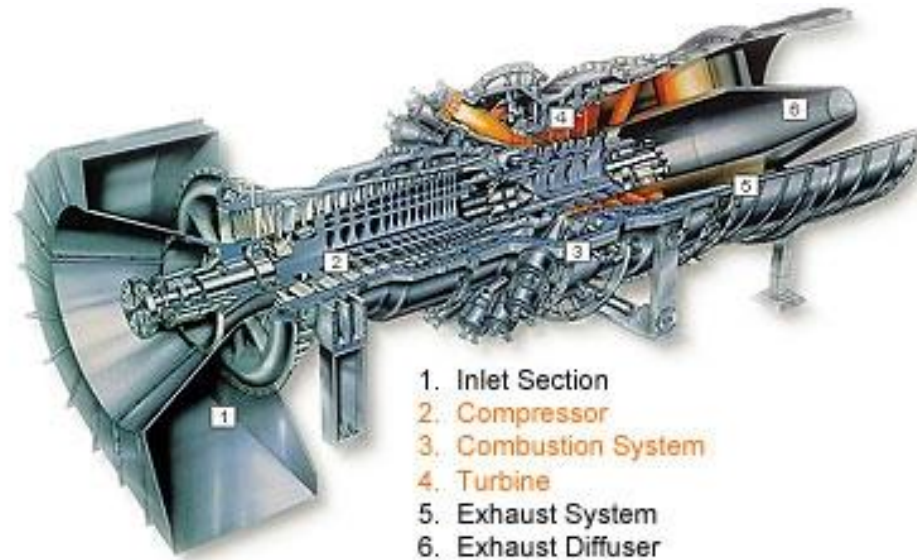
## Exercise 3 (take home)

*Method: you choose*

*Component: Wind Turbine*

# Exercise 1

*Component: Gas Turbine*



1. Inlet Section
2. Compressor
3. Combustion System
4. Turbine
5. Exhaust System
6. Exhaust Diffuser

Courtesy of Siemens Westinghouse

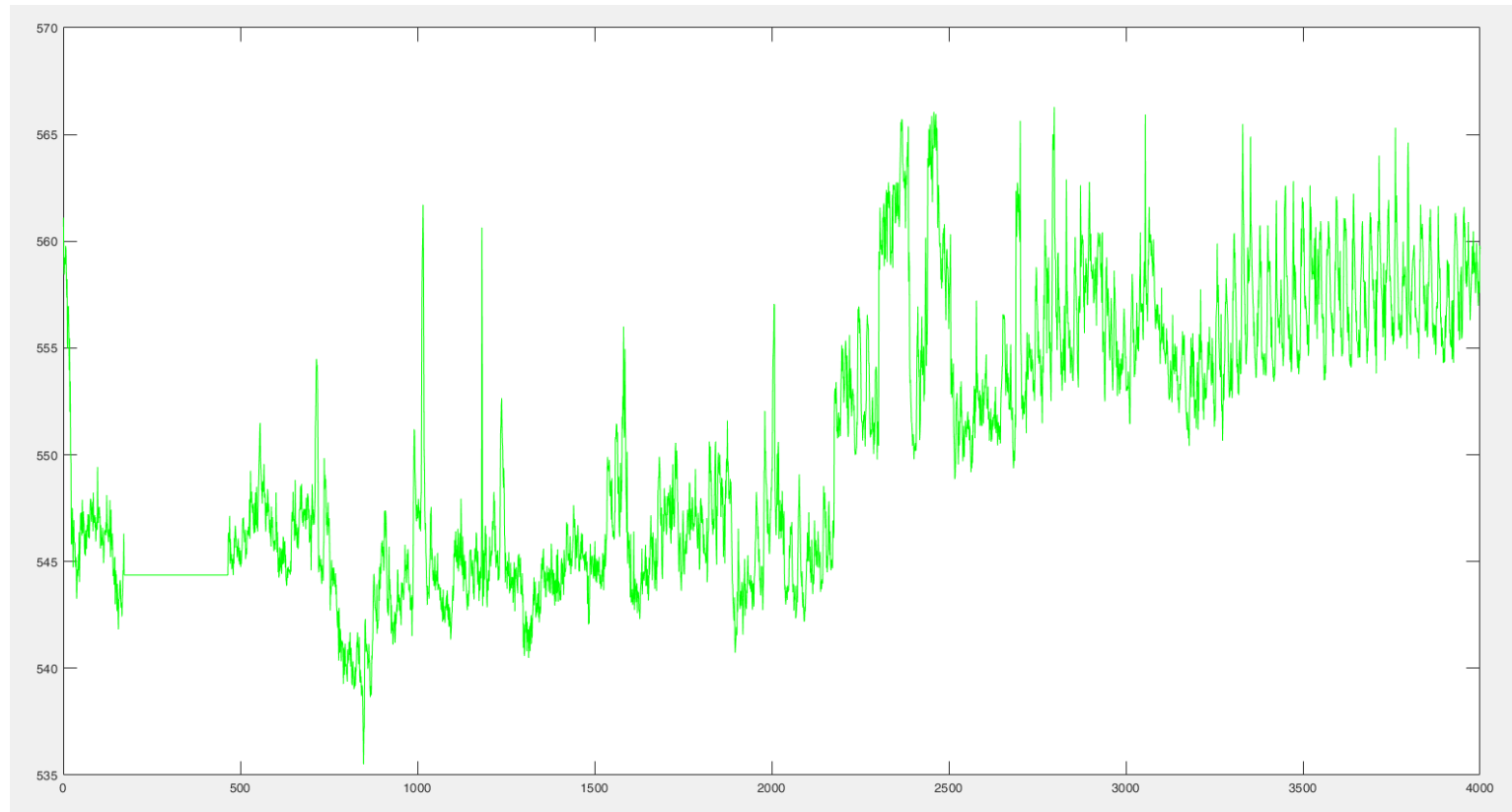


Temperature location 1 (°C)
Temperature location 2 (°C)
Temperature location 3 (°C)
Temperature location 4 (°C)
Temperature location 5 (°C)
Temperature location 6 (°C)



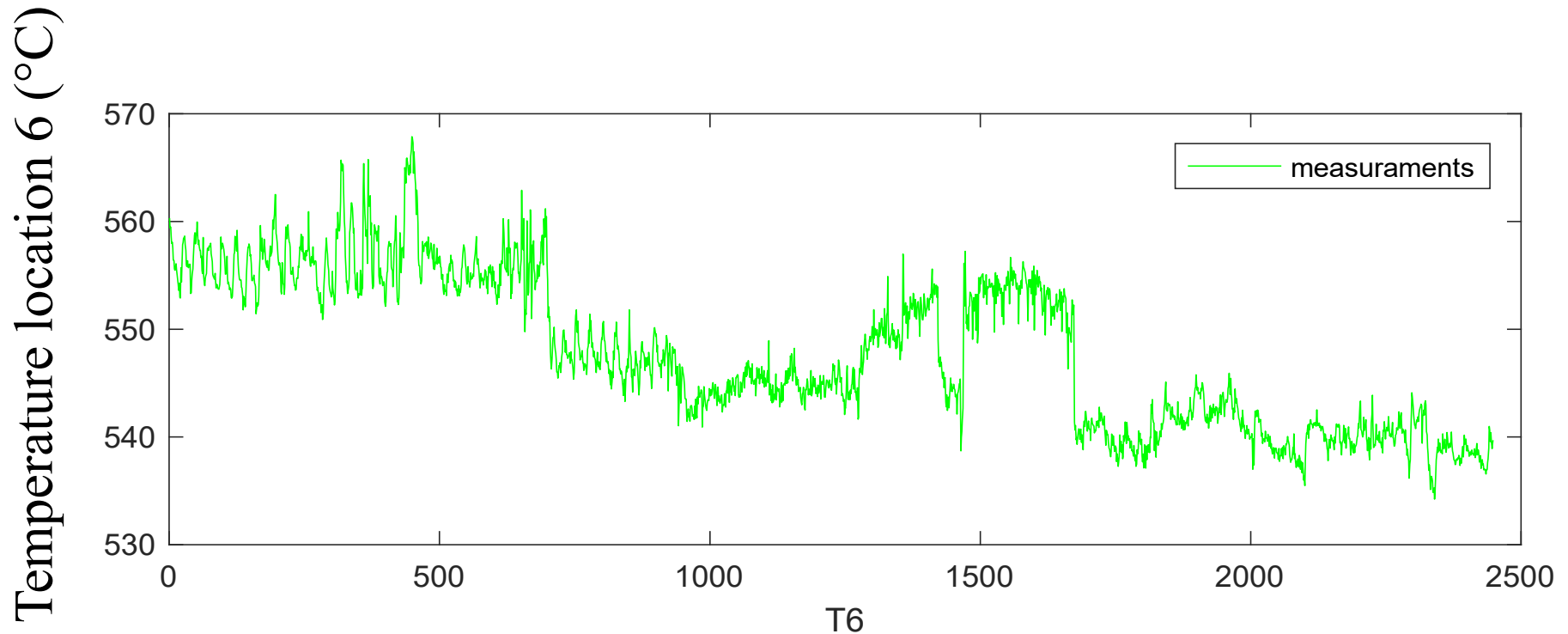
- Train.dat → Normal condition data [6 signals, 4000 data points, frequency: 5200 measurements/year]

Temperature location 6 (°C)





- Validation.dat → Normal condition data [6 signals, 2500 data points, frequency: 5200 measurements/year]





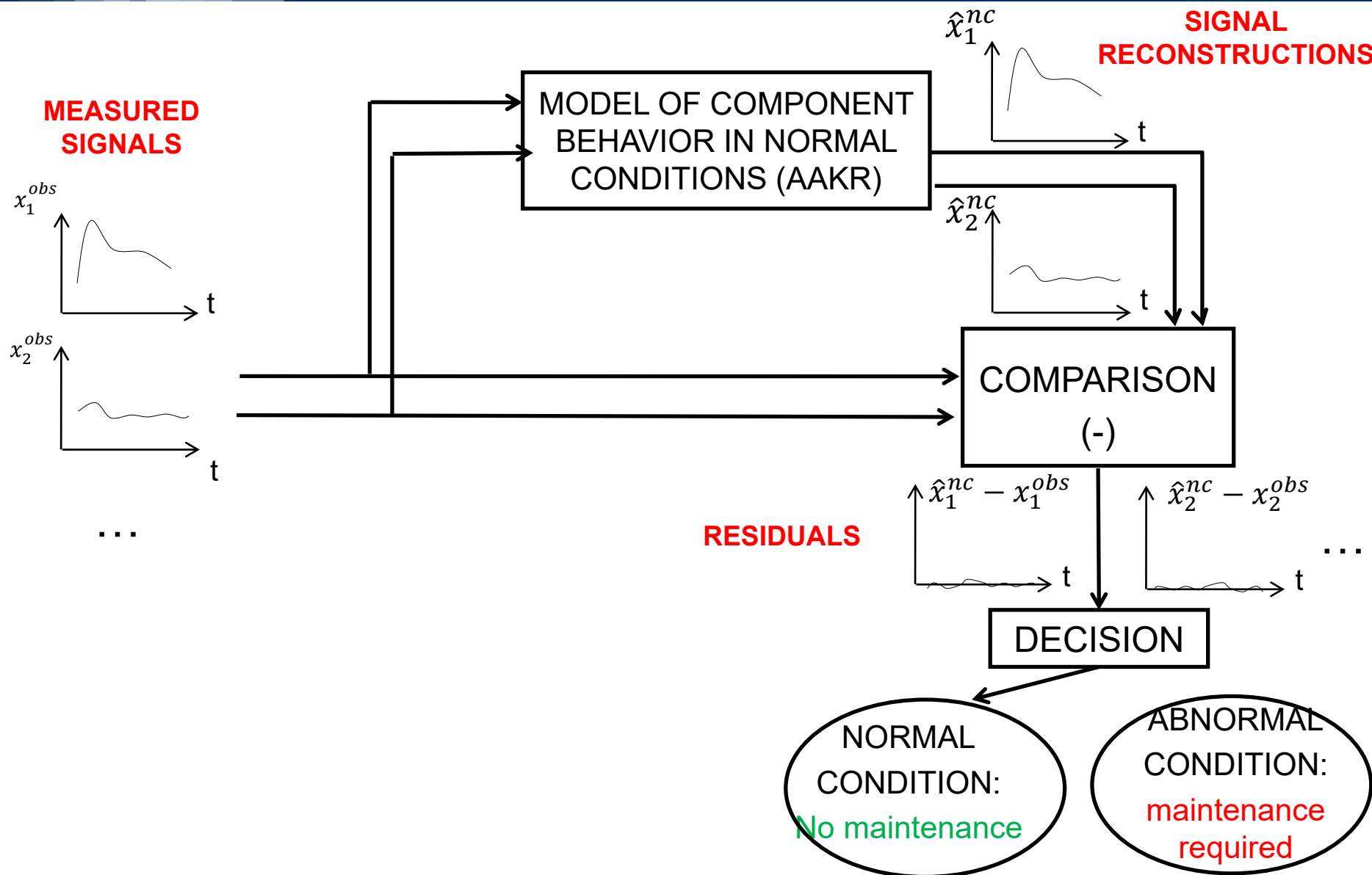
# Exercise 1

*Method: AAKR (Signal Reconstruction)+SPRT (Decision)*



# Fault Detection using AAKR for signal reconstruction

8





# The AAKR code in Matlab: how to run the code?

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Set as the current folder the folder 'AAKR'

The screenshot shows the MATLAB environment. The 'Current Folder' pane on the left lists files including 'AAKR\_reconstruction\_ex.m', 'AAKR\_reconstruction\_ex1.p', 'newforstat.m', and several '.dat' files. The 'Editor' pane shows a script with the following code in the command window:

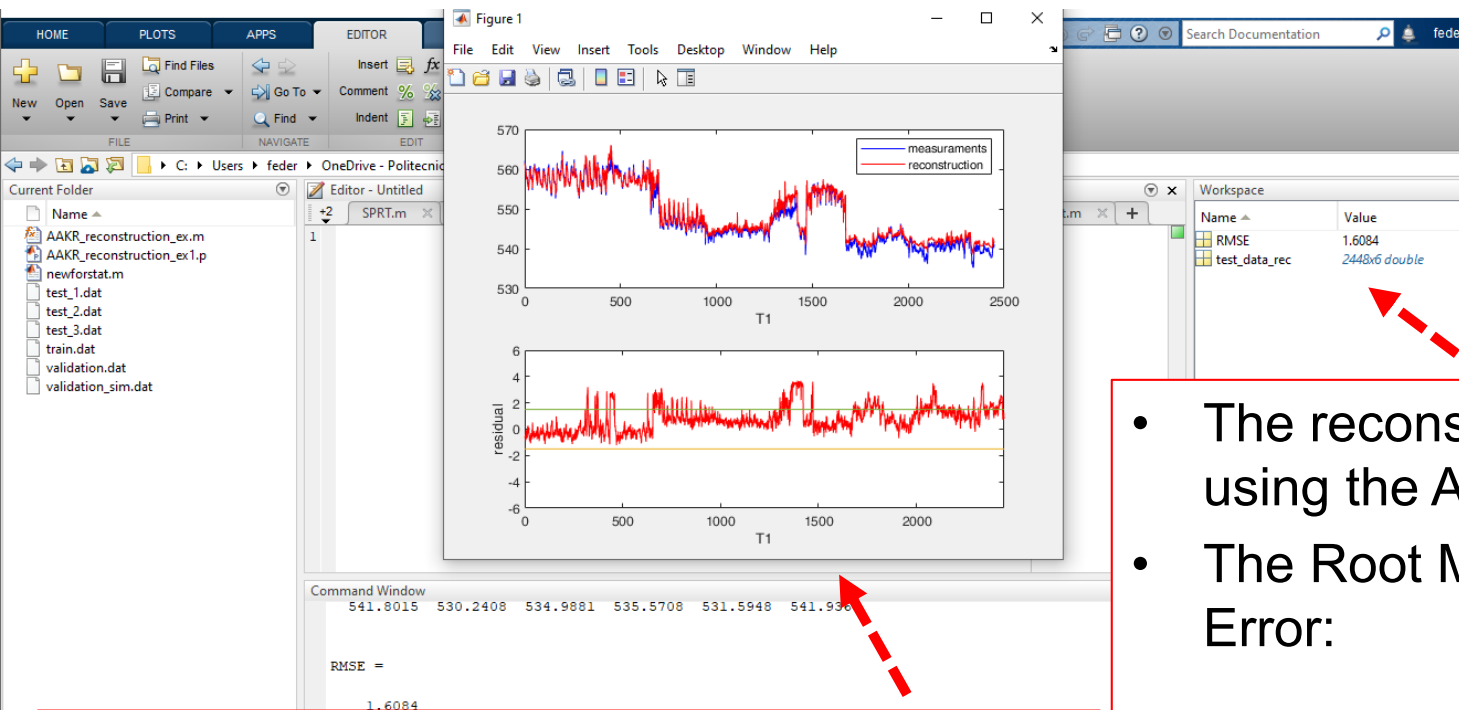
```
fx >> AAKR_reconstruction_ex('train.dat','validation.dat',0.5)
```

A red dashed arrow points from the 'AAKR' folder in the 'Current Folder' pane to the command window. Another red dashed arrow points from the 'Run' button in the 'EDITOR' tab to the command window.

Run the code in the command window:

```
[test_data_rec, RMSE]=AAKR_reconstruction_ex('train.dat','validation.dat',h)
```

h: bandwidth parameter



One figure for each signal representing:

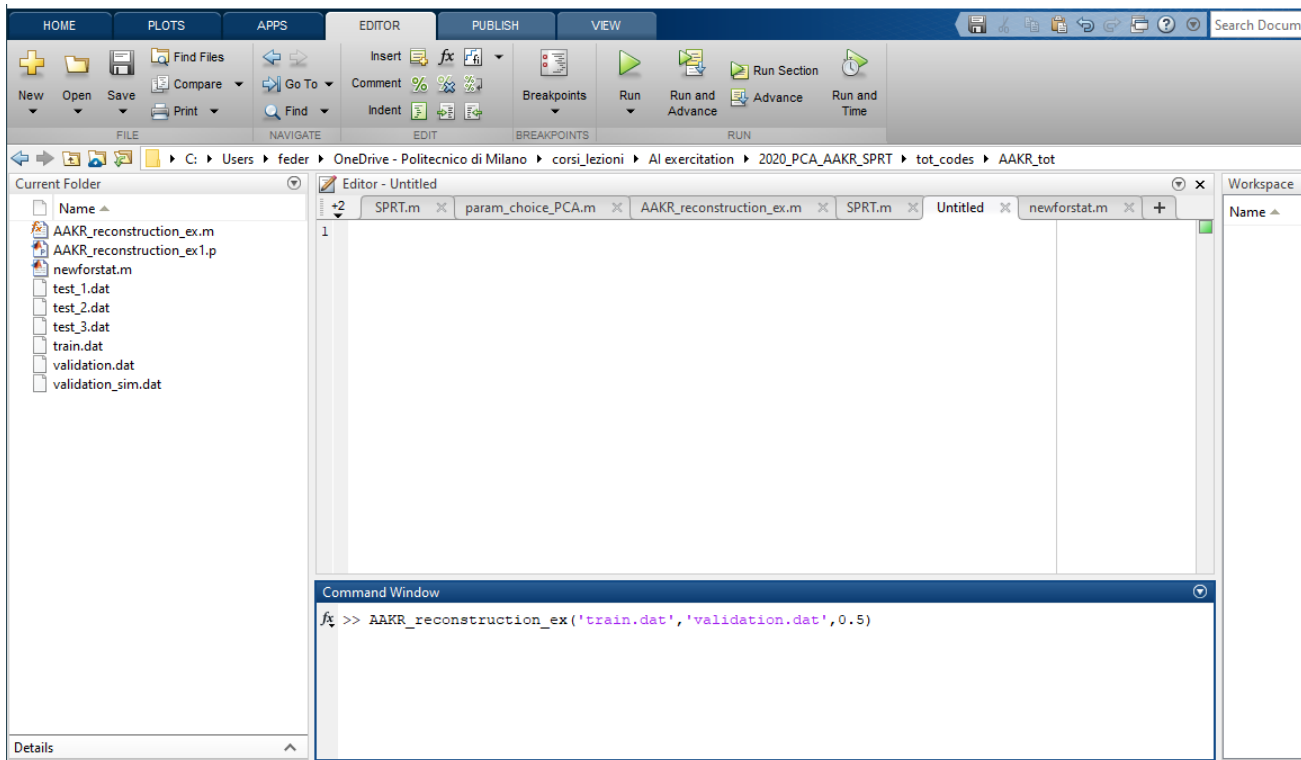
- The original data and the reconstruction obtained using the AAKR algorithm
- The residual for each data point

- The reconstruction obtained using the AAKR algorithm
- The Root Mean Square Error:

$$RMSE = \frac{\sum_{j=1}^n \frac{\sum_{t=1}^{N_{valid}} |\hat{\vec{x}}^{nc}(t,j) - \vec{x}^{obs}(t,j)|^2}{N_{valid}}}{n}$$



5 minutes



**Run the algorithm considering the bandwidth parameter  $h=0.1$**

Execute the code in the command window:

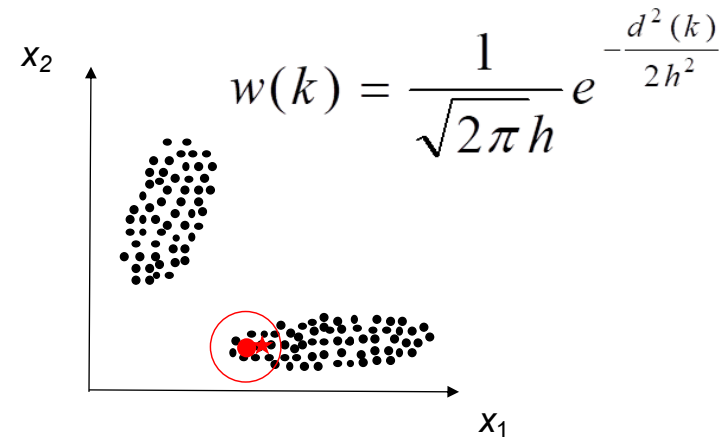
```
[test_data_rec, RMSE]=AAKR_reconstruction_ex('train.dat','validation.dat',h)
```

# Exercise 1

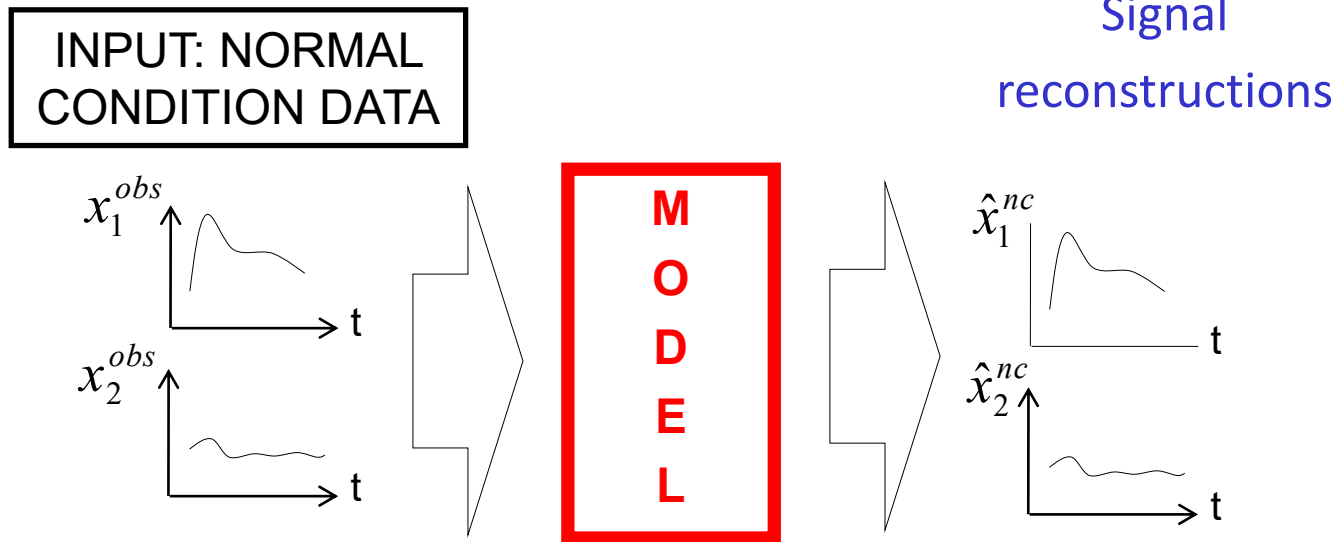
*Method: AAKR*

*Setting the model parameter:*

- *$h$  = bandwidth parameter*



- Objective: ACCURATE and ROBUST reconstruction model

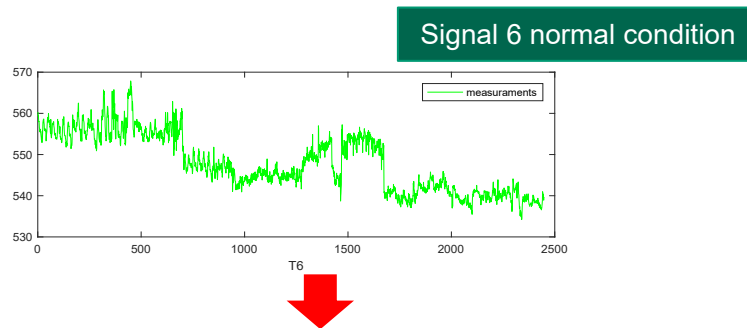
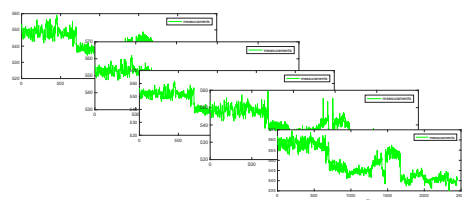


- **Accurate** reconstruction model if:

$$\vec{\hat{x}}^{nc} \cong \vec{x}^{obs}$$

- Metric to measure accuracy: Root Mean Square Error (*RMSE*):

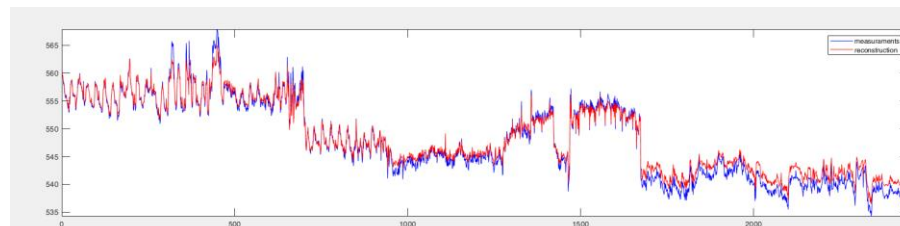
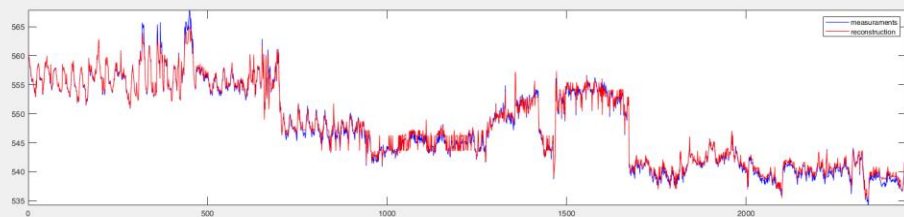
$$RMSE = \frac{\sum_{j=1}^n \frac{\sum_{t=1}^{N_{valid}} |\hat{x}^{nc}(t,j) - x^{obs}(t,j)|^2}{N_{valid}}}{n}$$



$h=0.05$

**AAKR**

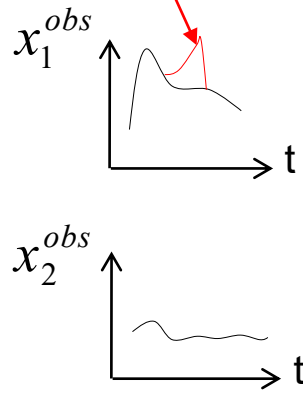
$h=0.4$



$h=0.05$  results in a more accurate model

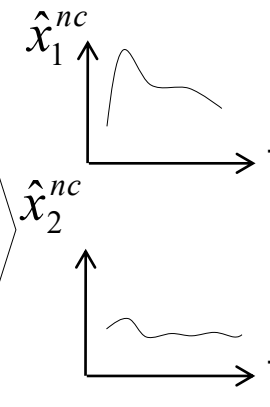


INPUT: ABNORMAL  
CONDITION DATA  
(Simulated)



**M  
O  
D  
E  
L**

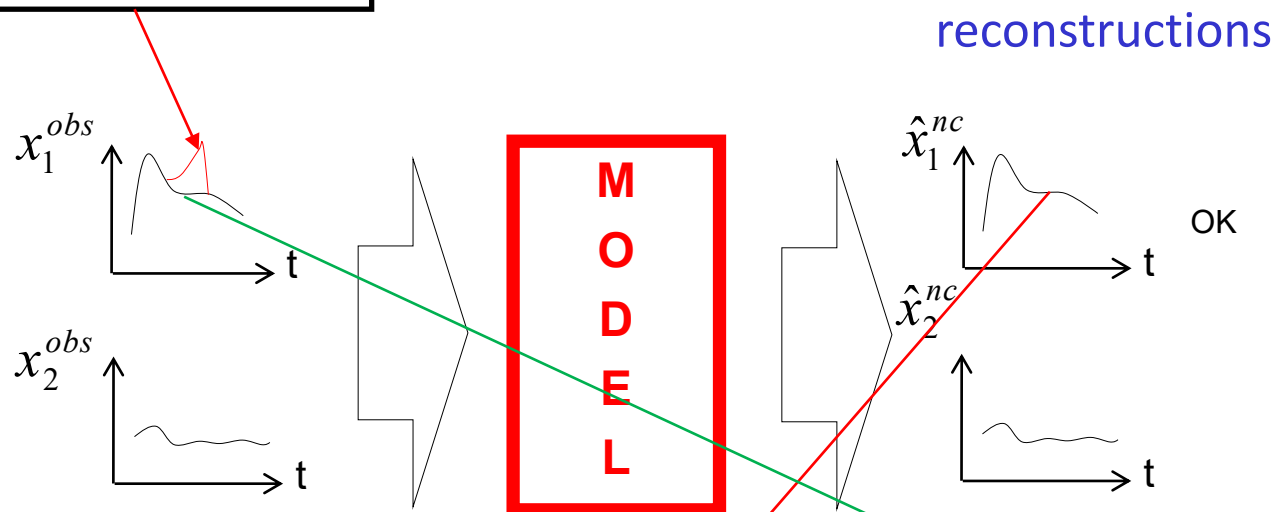
Signal  
reconstructions



- Robust reconstruction if:  $\vec{\hat{x}}^{nc} \cong \vec{x}^{obs-nc}$



INPUT: ABNORMAL  
CONDITION DATA  
(Simulated)



- Robust reconstruction if:  $\vec{\hat{x}}^{nc} \cong \vec{x}^{obs-nc}$

- Metric to measure Robustness:

Reconstruction  
of the anomaly

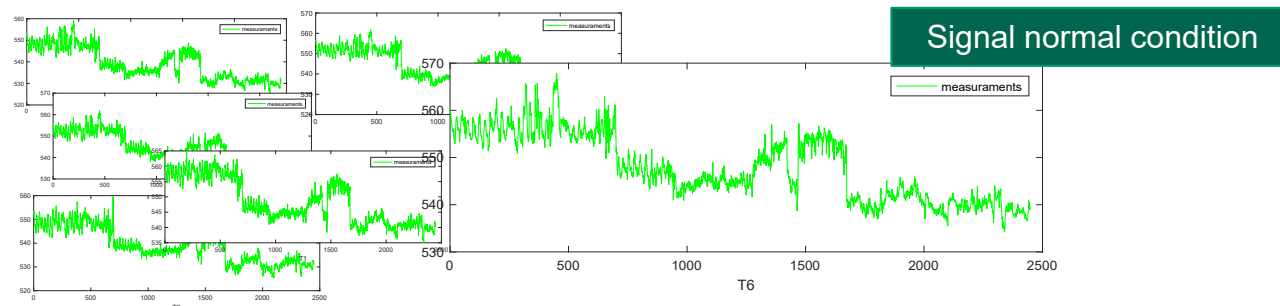
Normal condition data (before  
anomaly simulation)

$$\text{Robustness} = \frac{\sum_{t=1}^{N_{valid}} \left| \hat{x}_1^{nc}(t) - \vec{x}_1^{obs-nc}(t) \right|^2}{N_{valid}}$$



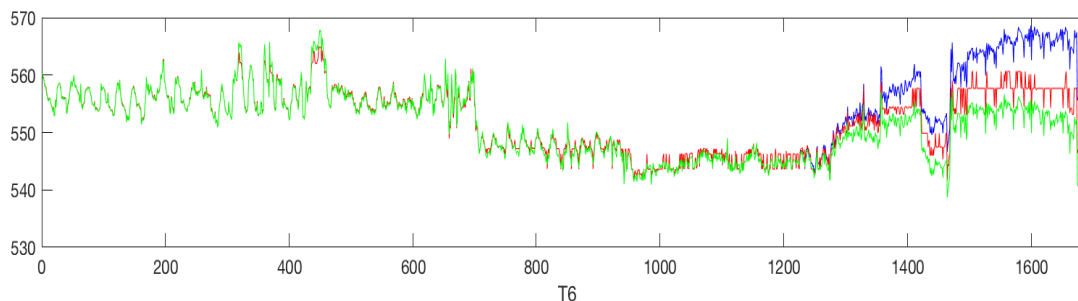
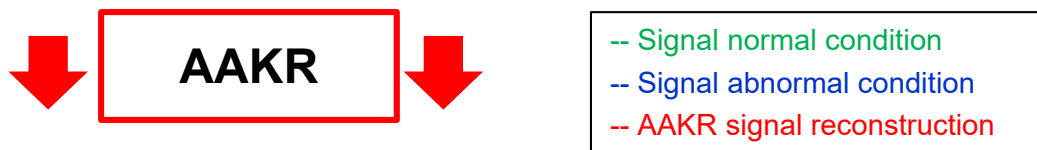
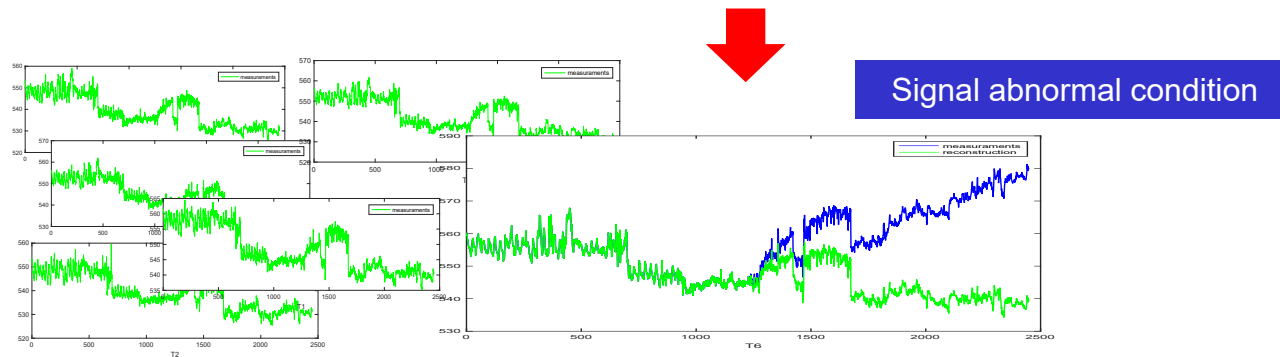
# How to Verify Robustness in Practice?

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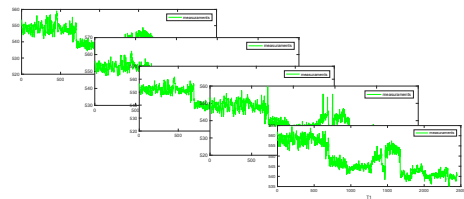
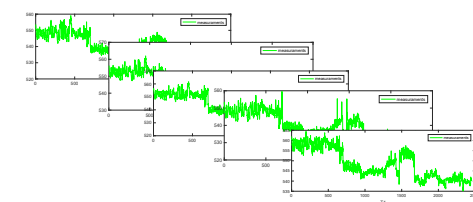


Artificial simulated abnormal condition on signal 6

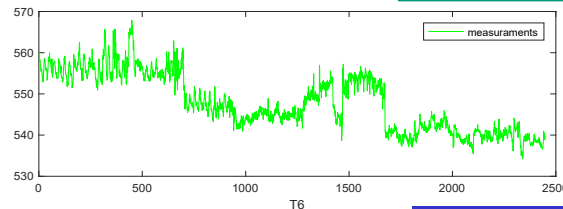
The other signals remain in normal condition



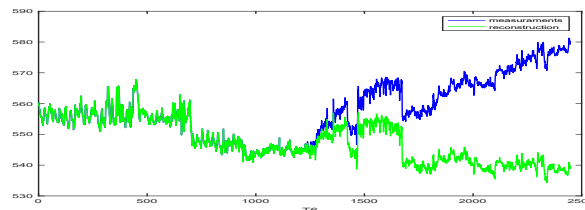
The method is robust if AAKR reconstruction is equal to the signal in normal condition



Signal normal condition



Signal abnormal condition



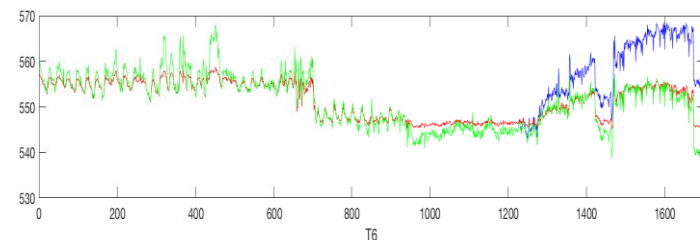
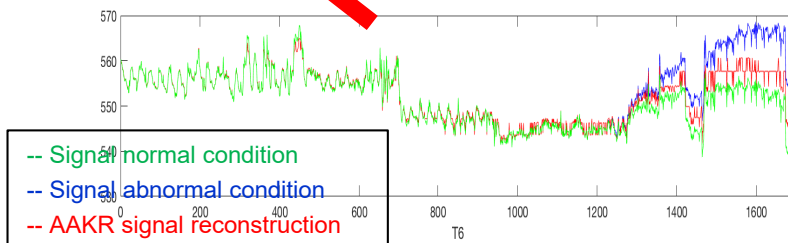
not robust  
but accurate

$h=0.05$

AAKR

$h=0.4$

More robust  
but less  
accurate



Optimal value of  $h$  is a trade off between accuracy and robustness



```
val = load('validation.dat');  
[npat,nsig]=size(val);  
anomaly=load('validation_sim.dat');  
  
[test_reconstruction,rmse]=AAKR_reconstruction('train.dat','validation_sim.dat',0.05);  
  
robustness=sum(((test_reconstruction(:,1)-  
val(:,1)).^2)/npat)^0.5
```



- Set the bandwidth parameter ( $h$ ) of the AAKR-based reconstruction model

You can use the following files:

- Validation.dat → data in normal condition
- Validation\_sim.m → data containing a simulated abnormal condition on **signal 1**

30-40 minutes

When you don't need the plots of the signals, you can add one more parameter to the function and set it to 0:

```
[test_data_rec,rmse]=AAKR_reconstruction_ex(train_data_name,test_data_name,h,0)
```

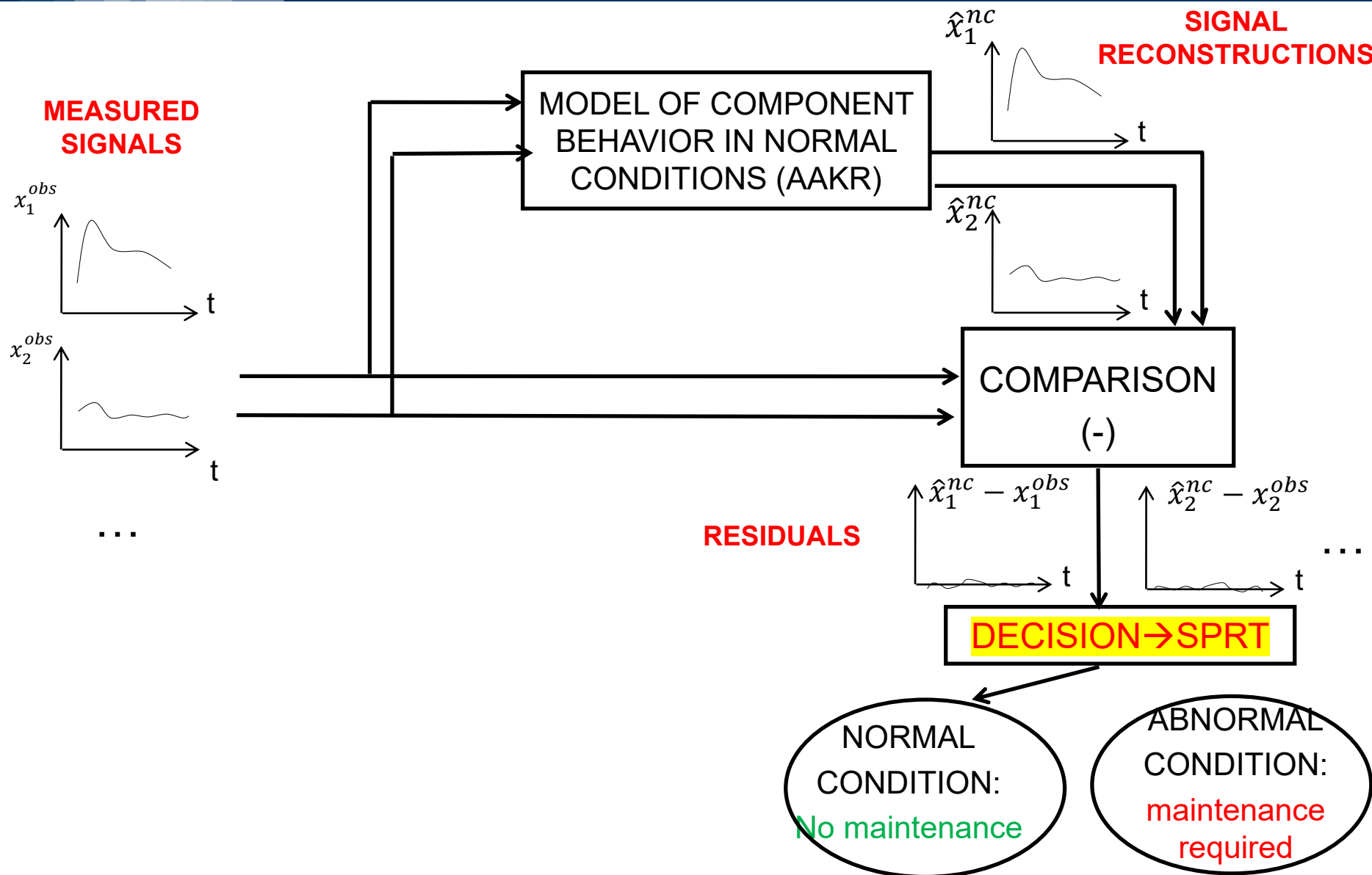


**Parameter to  
avoid plotting**



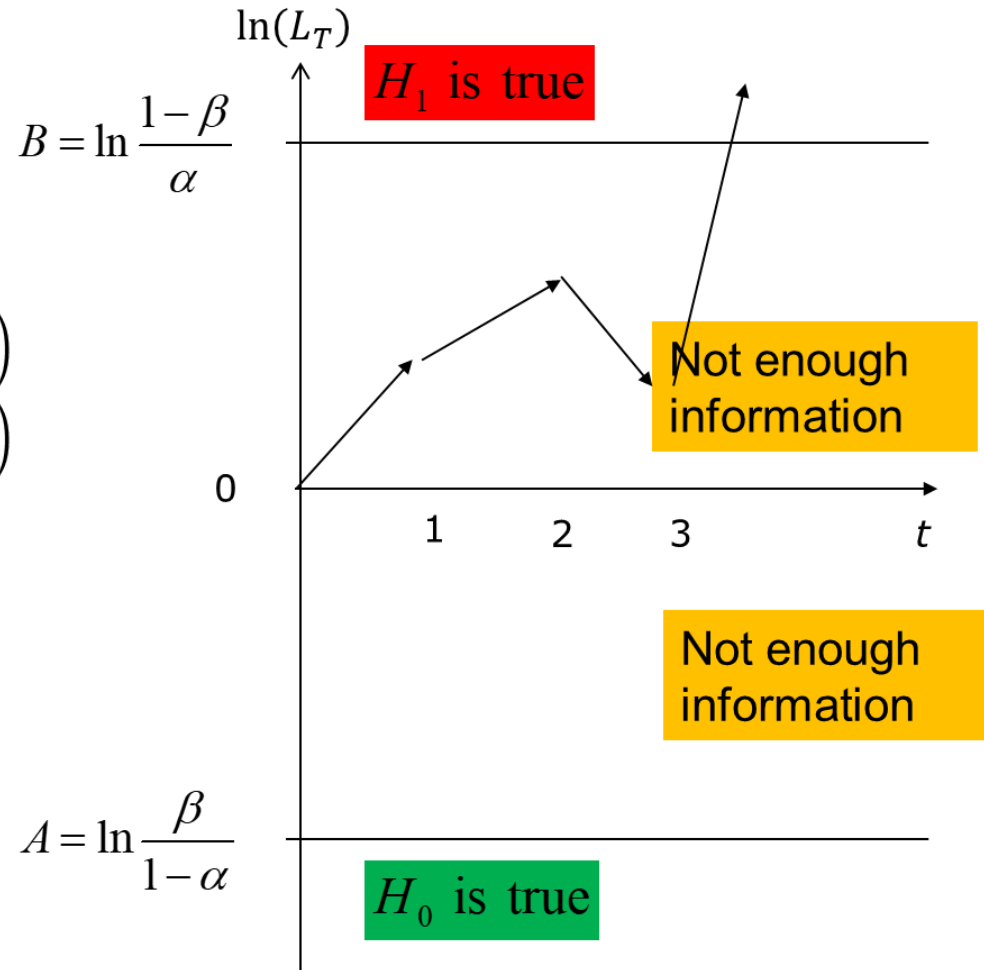
- Perform the reconstruction of the signal measurements in the 4 files test\_1.dat, test\_2.dat, test\_3.dat and test\_4.dat.
- In which files can you detect abnormal conditions? Do you have any hypothesis on the type of abnormal condition?
- Draw your conclusions on the possibility of using the developed model for fault detection.

30 minutes





$$\begin{aligned} L_0 &= 1 \rightarrow \ln(L_0) = 0 \\ \ln(L_1) &= \ln(L_0) + \frac{\mu_1}{\sigma^2} \left( r^{(1)} - \frac{\mu_1}{2} \right) \\ \ln(L_2) &= \ln(L_1) + \frac{\mu_1}{\sigma^2} \left( r^{(2)} - \frac{\mu_1}{2} \right) \end{aligned}$$





$$[SPRT_{Index}] = SPRT(test_{name}, test\ rec_{name}, \alpha, \beta, \mu_{H_0}, \sigma_{H_0}, \mu_{H_1}, \sigma_{H_1})$$

## OUTPUT:

- $SPRT_{Index}$  =  $N$ -by-1 vector containing the  $SPRT$  index

## INPUTS:

- $test_{name}$  =  $N$ -by-1 data matrix containing  $N$  sequential measurements of one signal
- $test\ rec_{name}$  =  $N$ -by-1 data matrix containing the  $N$  reconstructions of the data in  $test_{name}$ .
- $\alpha$  = False alarm rate
- $\beta$  = Missed alarm rate.
- $\mu_{H_0}$  = the mean of the Gaussian distribution of  $H_0$  hypothesis (the component is working in normal conditions).
- $\sigma_{H_0}$  = variance of Gaussian distribution  $H_0$
- $\mu_{H_1}$  = the mean of the Gaussian distribution of  $H_1$  hypothesis (the component is working in abnormal conditions).
- $\sigma_{H_1}$  = variance of Gaussian distribution  $H_1$

Example:

$$[SPRT_{Index}] = SPRT('test\_4A\_meas\_S5.dat', 'test\_4A\_rec\_S5.dat', 0.01, 0.01, 0, 0.15, 1, 0.15)$$



- Apply the SPRT to detect abnormal conditions to the measurements of signal 5 in files “test\_4A.dat” and “test\_4B.dat”. The following setting of the SPRT parameters is suggested:
  - $\alpha = 0.01$
  - $\beta = 0.01$
  - and Gaussian distributions for the two hypothesis  $H_0$  and  $H_1$  with parameters:
    - $\mu_{H_0} \approx 0, \sigma_{H_0} = 0.15$
    - $\mu_{H_1} = 1, \sigma_{H_1} = 0.15$
- Do you detect the occurrence of abnormal conditions? When?
- Compare the results with those of a threshold-based method with threshold = 1.
- Draw your conclusions on the possibility of using the SPRT in decision-making for fault detection.



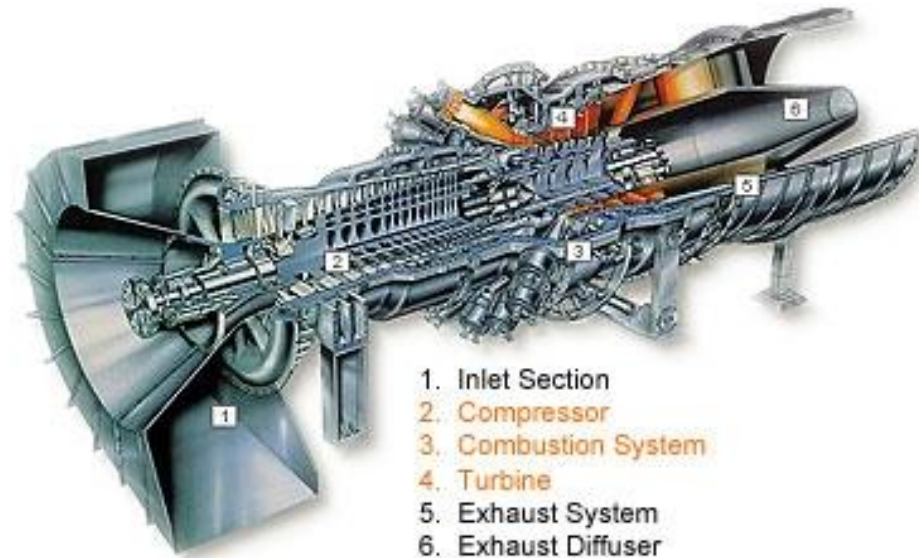
# Exercise 2

*Fault detection with PCA considering turbine data*

1. *Optimize the PCA to achieve:*
  - a. *Accuracy*
  - b. *Robustness*
2. *Apply the PCA model on test datasets*
3. *Compare AAKR and PCA results*

# Exercise 2

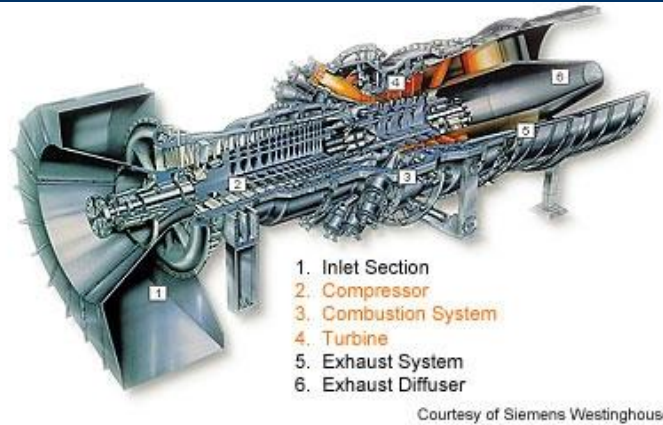
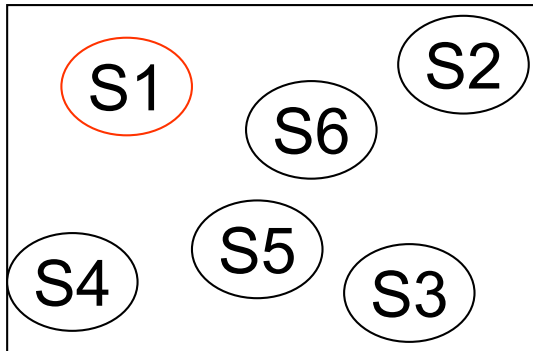
*Component: Gas Turbine*



Courtesy of Siemens Westinghouse



## Component

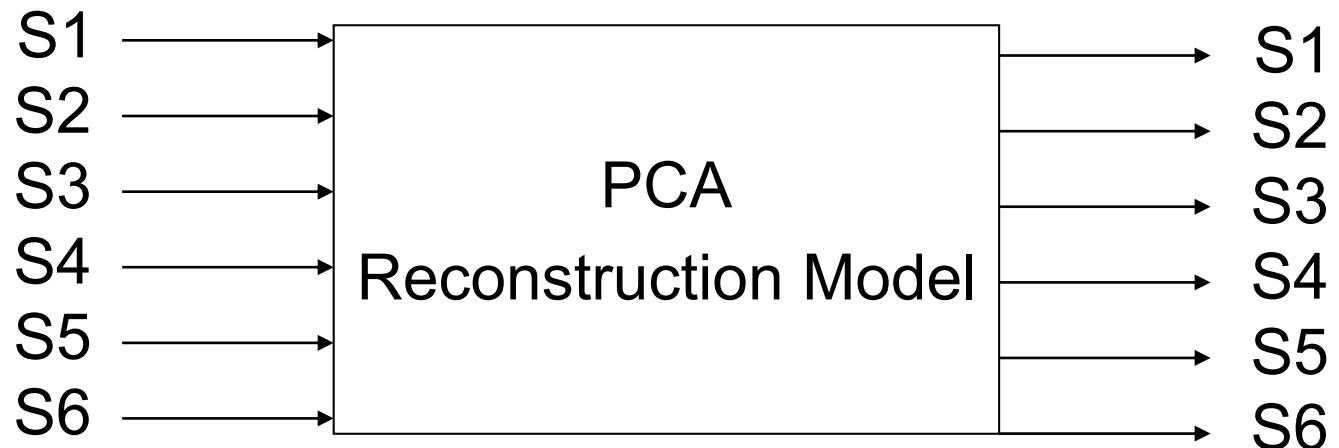


Real  
measurements

Signal  
Reconstructions

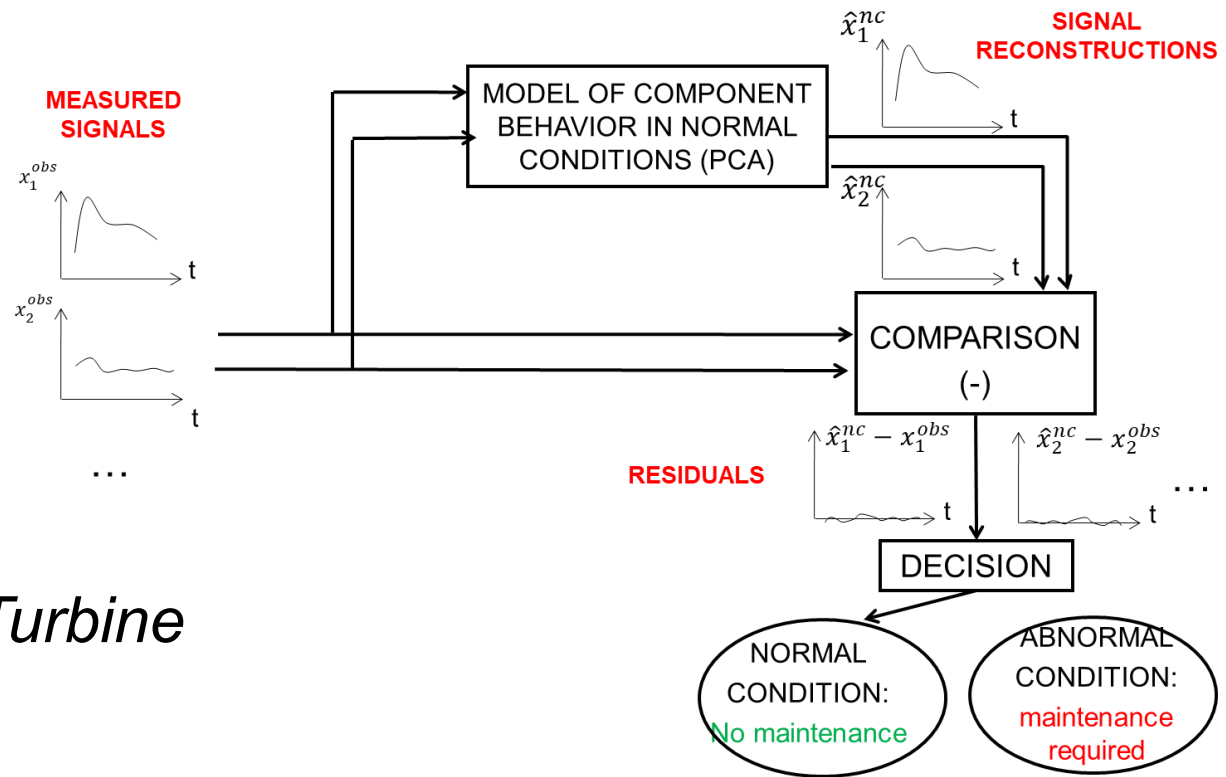
=

Expected signal values in  
normal conditions



# Exercise 2

Method: PCA  
Component: Gas Turbine





# The PCA code in Matlab: how to run the code?

30

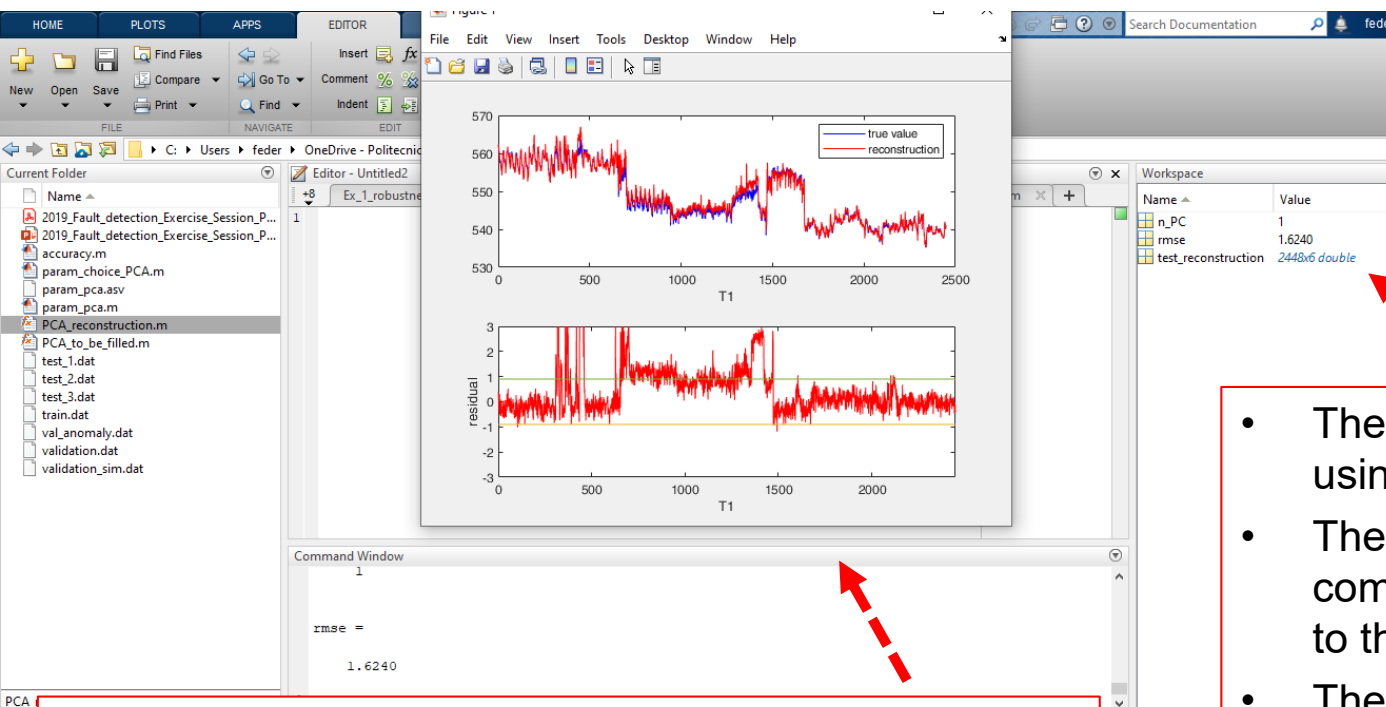
The screenshot shows the MATLAB environment. The 'Current Folder' pane on the left lists files including `PCA_reconstruction.m`. The 'Editor' pane shows a script with the command `PCA_reconstruction('train.dat', 'validation.dat', v(i))`. The 'Command Window' at the bottom is empty. A red dashed arrow points from the text box to the `PCA_reconstruction` function call in the script. Another red dashed arrow points from the text box to the `PCA_reconstruction.m` file in the file list.

Set as the current folder the folder:  
'PCA'

Execute the code in the command window:

`[test_data_rec, RMSE]=PCA_reconstruction('train.dat','validation.dat',v)`

`v=minimum percentage of variance to be represented in the PCA space [e.g., 0.95]`



One figure for each signal representing:

- The original data and the reconstruction obtained using the PCA algorithm
- The residual for each data point

- The reconstruction obtained using the PCA algorithm
- The number of principal components corresponding to the selected variance
- The RMSE



Consider the same data used in Exercise 1

1. Optimize the PCA reconstruction model to achieve:

a. Accuracy

b. Robustness    Validation\_sim.m → data containing a simulated abnormal condition on signal 6

2. Apply the PCA model on test datasets: test\_1.dat, test\_2.dat, test\_3.dat and test\_4.dat.

3. Compare AAKR and PCA results on test\_3.dat

Hints:

- Consider different numbers of Principal Components by trying different values for the parameter  $v = \text{minimal percentage of variance considered in the PCA space}$
- Consider the root mean square error as a performance measure and compute it on test data under normal condition and simulated abnormal condition

```
[test_reconstruction,n_PC, RMSE]=PCA_reconstruction('train.dat','validation.dat',0.95)
```

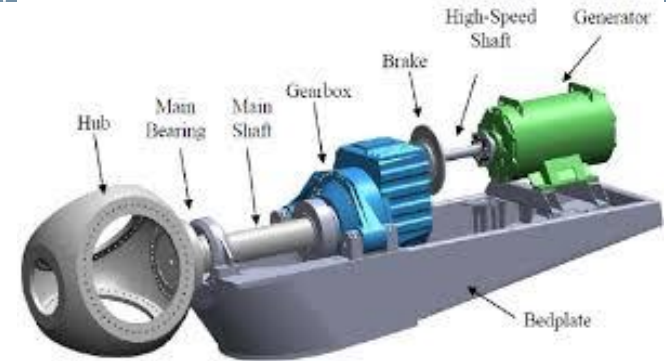
# Exercise 3 (take home)

*Method: you choose*

*Component: Wind Turbine*



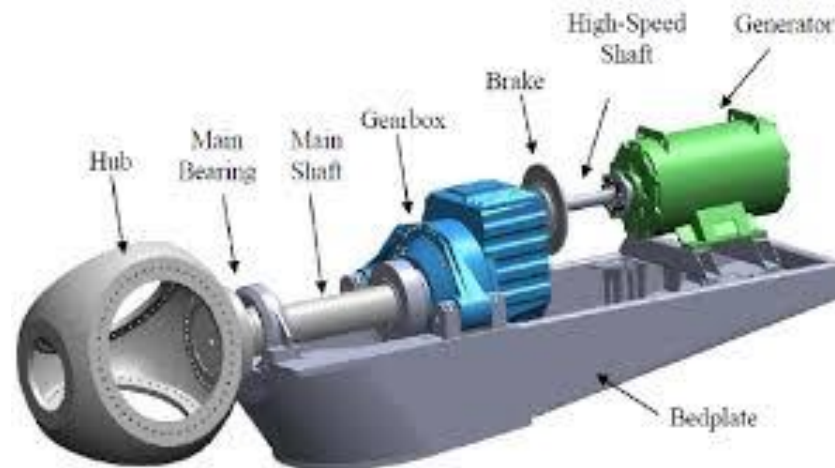
## Wind turbine farm



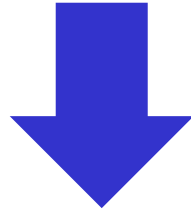
- Main Bearing+ Planetary Gear box + Gearbox + Generator
- Monitoring system: 6 accelerometers and 1 sensor measuring the rotating speed.

# Some possible failures modes

- a) Crashing of the highspeed shaft of the gear box
- b) Breaking of a teeth in the planetary stage
- c) Misalignment between the generator and the gearbox shafts
- d) Wearing of the rear bearing housing of the generator

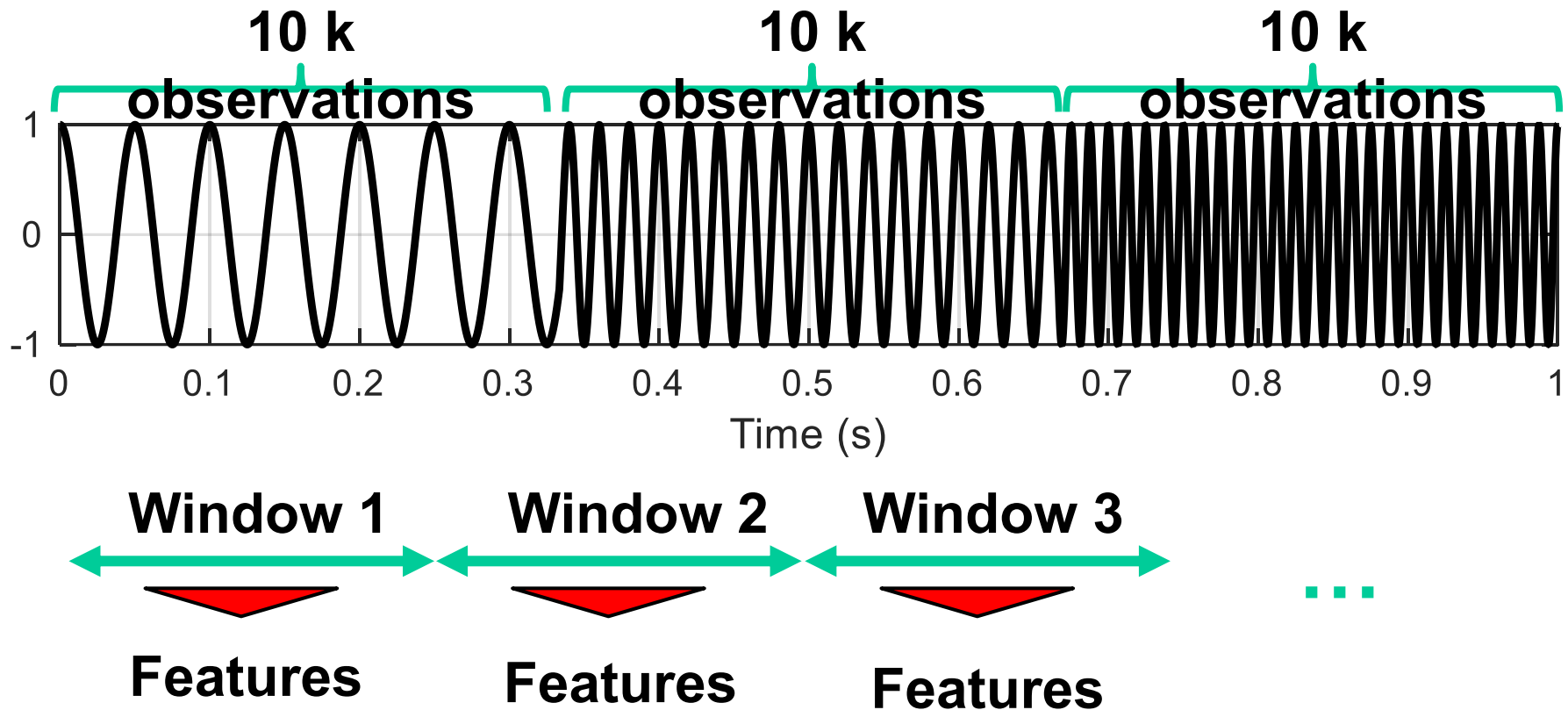


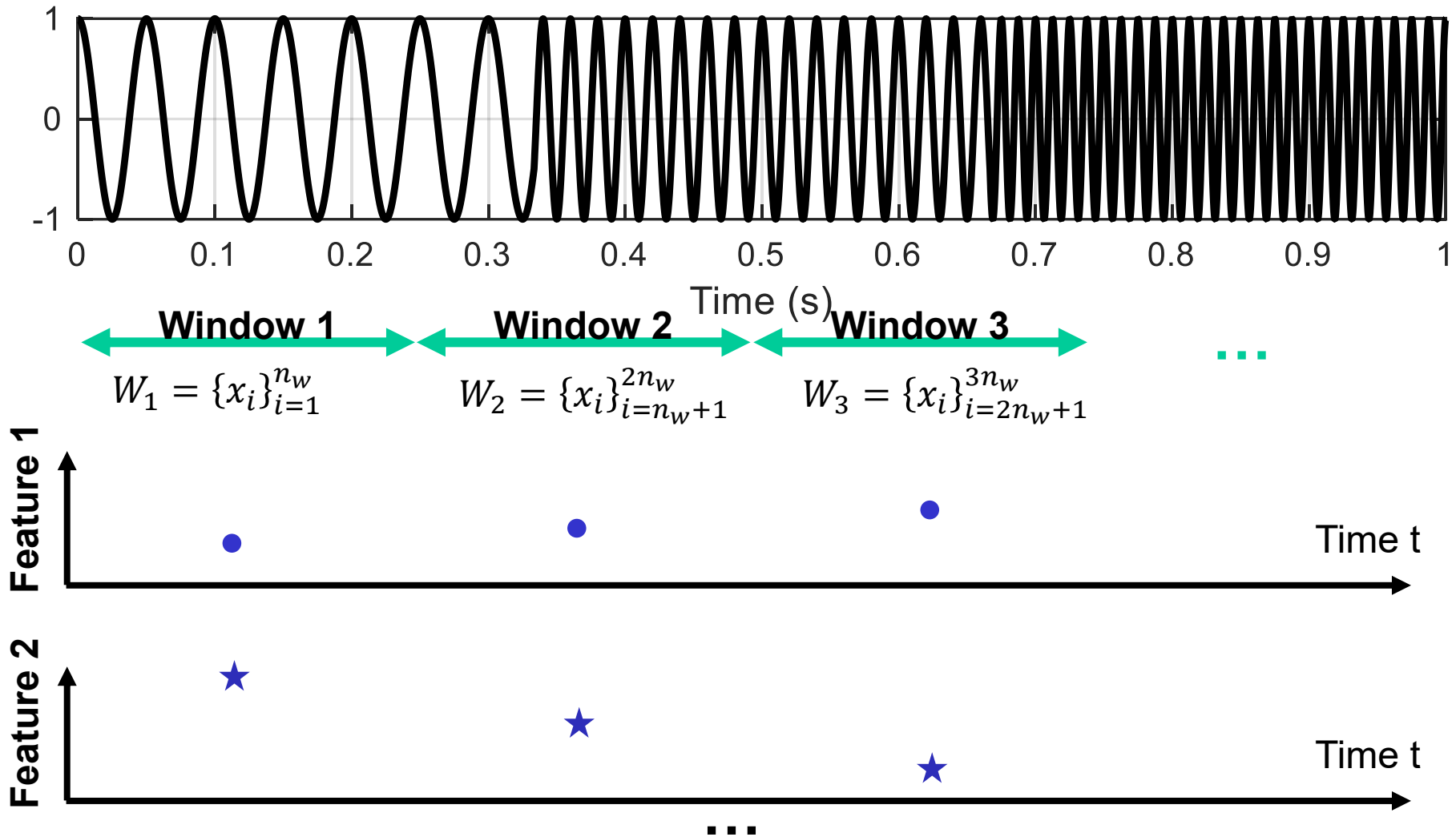
## Gearbox-Generator Failure

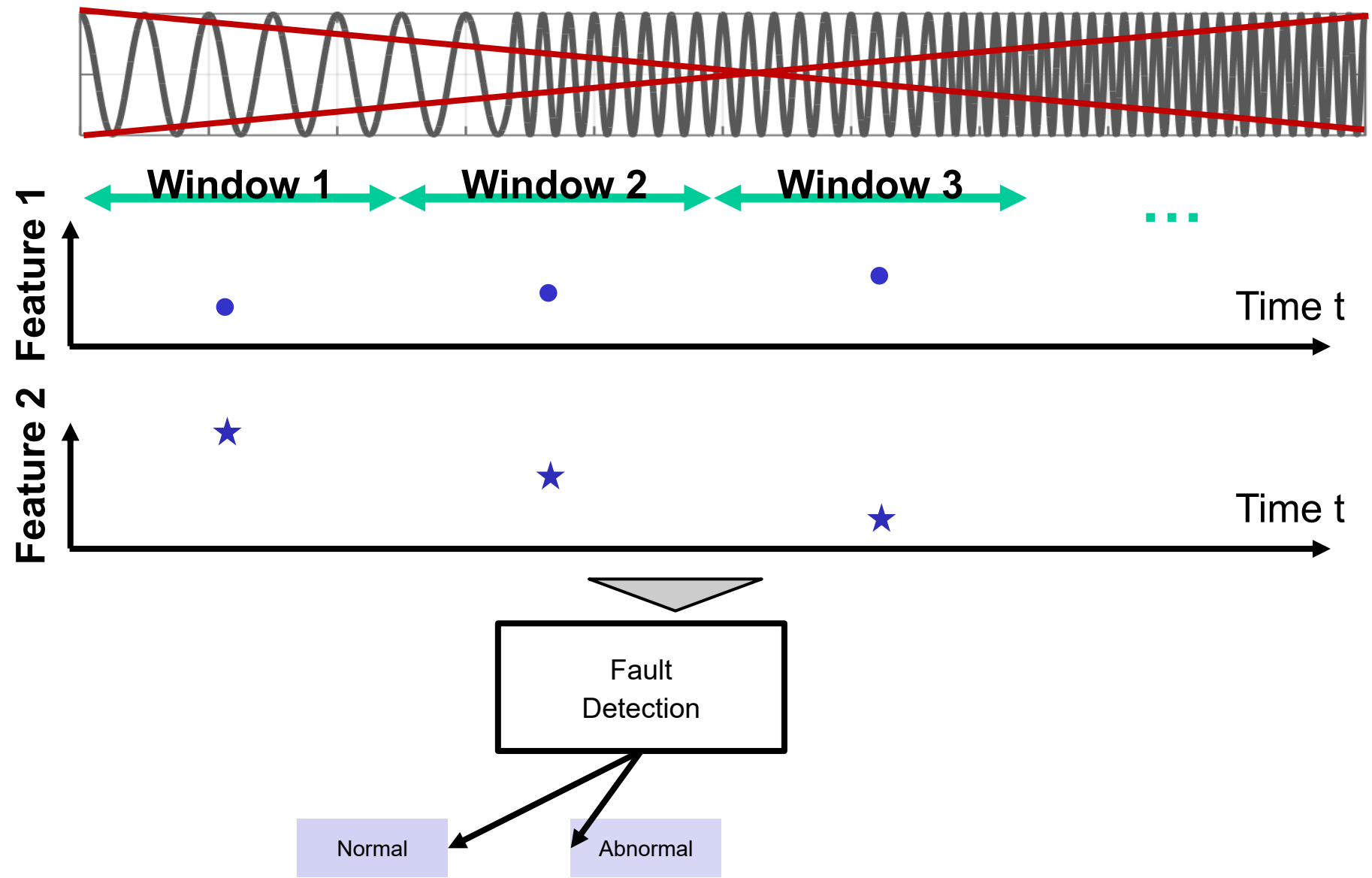


- Long turbine downtime [up to some months]
- Intervention of expensive tools

High frequency sensor:







- Each 5 sec signal data are divided in 40 segments(windows) in order to extract features

- The following statistical features are considered:

Mean, standard deviation, Kurtosis, Skewness, min, max, 2<sup>nd</sup> moment, 3<sup>rd</sup> moment

- A window contains  $N$  values  $W = \{x_i\}_{i=1}^N$ , the corresponding Statistical Features are:

Sample Mean:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

Sample Variance:

$$\sigma^2 \approx s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$$

Standard Deviation:  $\sigma$

$k$ -th Moments:

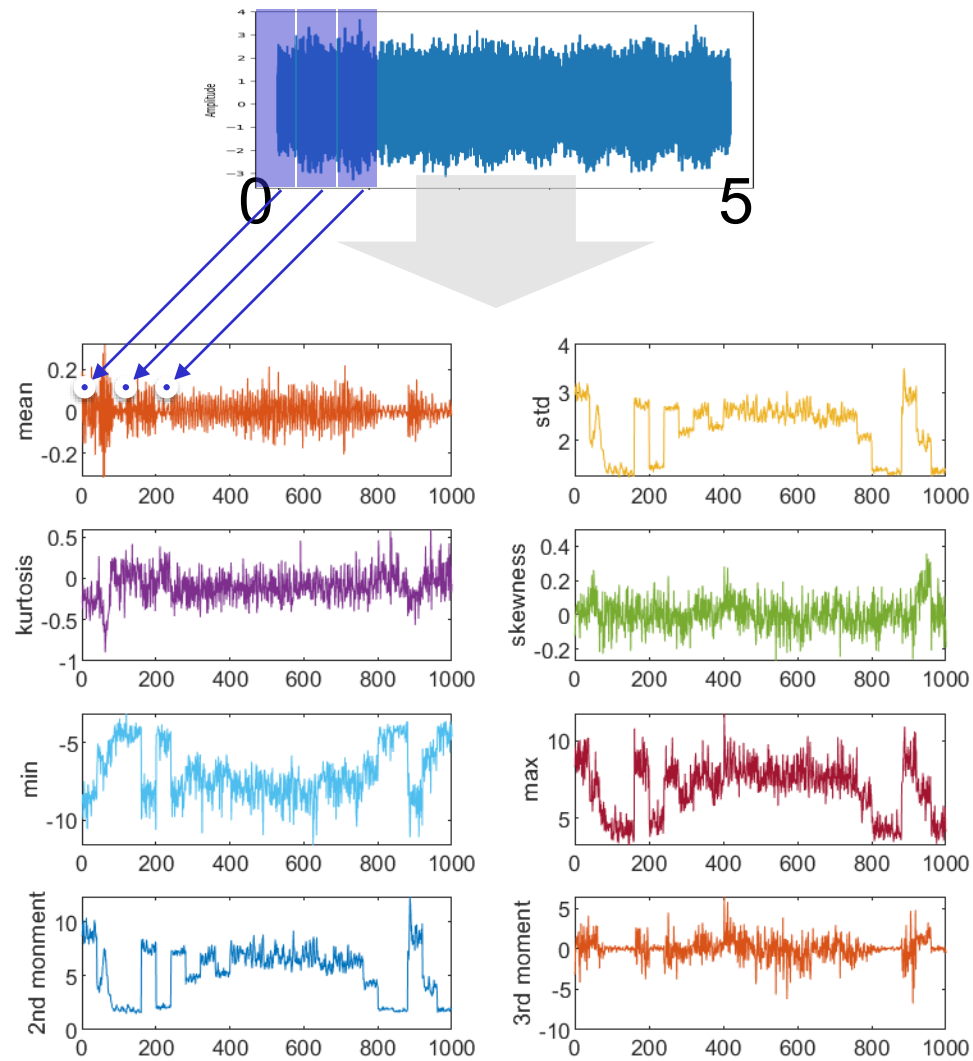
$$\mu_k \approx \frac{1}{N} \sum_{i=1}^N x_i^k$$

Skewness → “asymmetry”  
(the third standardized moment)

$$\frac{\mu_3}{\sigma^3}$$

Kurtosis → “peak”  
(the fourth standardized moment)

$$\frac{\mu_4}{\sigma^4}$$



# Exercise 3: Assignment

## Given:

- 1) The file 'train.dat' which contains the features already extracted in normal condition
- 2) The file 'validation.dat' which contains the features already extracted in normal condition
- 3) The file 'validation\_sim.dat' which contains the features in a simulated abnormal condition

train								
800x8 double								
	1	2	3	4	5	6	7	8
1	0.1918	2.9473	-0.2172	-0.1264	-8.9417	8.3333	8.6760	-3.2251
2	-0.0553	3.1255	-0.3734	0.0047	-8.3208	9.5974	9.7568	0.1444
3	-0.0273	3.0340	-0.0800	-0.0165	-9.9243	8.4762	9.1938	-0.4595
4	-0.1549	3.1874	-0.2684	0.0424	-8.4574	9.1952	10.1471	1.3688
5	0.0066	3.0410	-0.2734	0.0520	-9.3214	8.4448	9.2366	1.4564
6	0.1720	2.8165	-0.1769	0.0581	-7.4707	7.9281	7.9228	1.2936
7	-0.0533	2.9428	-0.1788	0.0175	-9.2303	9.3448	8.6492	0.4444
8	0.1164	2.8412	-0.1314	0.0722	-8.2652	9.2579	8.0627	1.6507
9	-0.0958	2.8827	-0.2671	0.0034	-8.6233	9.9347	8.2996	0.0815
10	-0.1537	2.9199	-0.2698	0.0793	-8.7042	8.0308	8.5154	1.9674
11	0.0615	2.9858	-0.2282	0.0806	-9.8658	9.6261	8.9039	2.1387
12	-0.0193	3.2212	-0.4878	-0.0667	-8.2399	8.1581	10.3633	-2.2198
13	0.0821	3.0265	-0.2508	-0.0710	-8.5418	9.9236	9.1485	-1.9599
14	0.0416	3.1149	-0.1363	-0.1381	-10.4691	10.0679	9.6909	-4.1581
15	0.0044	2.9303	-0.1060	0.0274	-10.5794	8.6044	8.5764	0.6874
16	-0.0010	2.8905	-0.1156	0.0113	-7.7779	8.4754	8.3445	0.2727
17	-0.0743	2.9348	-0.3117	0.1239	-7.6838	9.3669	8.6025	3.1201
18	-0.0671	2.9080	-0.2487	-0.0200	-8.6593	8.2900	8.4459	-0.4903
19	0.0190	2.8127	-0.2573	-0.0495	-8.1409	7.6882	7.9017	-1.0966
20	0.0753	2.9773	-0.3975	0.0638	-8.9418	8.6407	8.8533	1.6776

mean      std      kurtosis      skewness      min      max      2<sup>nd</sup> Moment      3<sup>rd</sup> Moment

## You are required to:

- 1) **Develop** a fault detection tool for the turbine generator [you can use AAKR or PCA, as you prefer];
- 2) **Apply the developed tool** to the data in the file "test.mat" and Identify possible abnormal conditions period.

### **Exercise 3.1**

*Fault detection with AAKR on new data*

1. *Optimize the AAKR to achieve Accuracy and Robustness*
2. *Apply the AAKR model on test data*

Use data in folder 'second case study'

### **Exercise 3.2**

*Fault detection with PCA*

1. *Optimize the PCA to achieve Accuracy and Robustness*
2. *Apply the PCA model on test data*

Use data in folder 'second case study'