# Reliability and Availability of simple systems

## Exercise lesson

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The two components of the system shown in Figure 1 have the same cost, and their reliabilities are $R_1 = 0.7, R_2 = 0.95$, respectively. If you can add two components to the system, would it be preferable

a)  to replace component 1 by three components in parallel (i.e., by adding two additional components of the same type)? or

b)  to replace both components 1 and 2 by simple parallel systems (i.e., by adding one additional component of each type)?
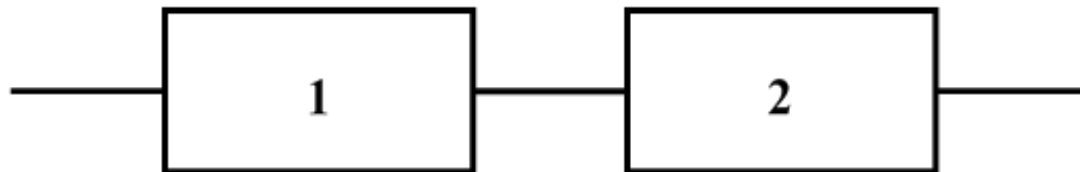
Figure 1: The system

- Consider a satellite transmission system made by two identical transmitters in a cold standby configuration. Loss of transmission can occur if either the transmission system has an internal failure or solar disturbances permanently interfere with transmission. If the rate of failure of the on-line transmitter is $\lambda$ and the rate of solar disturbances is $\lambda_{cm}$, find:

  1. The reliability of transmission
  2. The mean time to transmission failure.

A main coolant pump in a light water reactor has a hazard rate of $10^{-7}/\mathrm{hr}$. A backup pump that automatically starts upon failure of the main pump possibly can be used. The hazard rate for the backup pump is $10^{-6}/\mathrm{hr}$ during operation and $10^{-8}/\mathrm{hr}$ during standby. Calculate the reliability of the system after 2100 hr

(a) with the backup pump and

(b) without the backup pump.

Consider a two-engine plane (engines A and B) in a one-out-of-two logic configuration. When both engines A and B are fully energized they share the total load and the failure densities are $f_A(t)$ and $f_B(t)$. If either one fails, the survivor must carry the full load and its failure density becomes $g_A(t)$ and $g_B(t)$.

1. Derive an expression for the reliability $R(t)$ of the system.
2. Find the reliability if

$$f_A(t) = f_B(t) = \lambda e^{-\lambda t} \quad \text{and} \quad g_A(t) = g_B(t) = k\lambda e^{-\lambda k t}, \quad k > 1$$

A compressor is designed for $T_D = 5$ years of operation. There are two significant contributions to the failure. The first is due to wear (W) of the thrust bearing and is described by a Weibull distribution

$$f(t) = \frac{m}{\vartheta} \left(\frac{t}{\vartheta}\right)^{m-1} e^{\left[-\left(\frac{t}{\vartheta}\right)^m\right]}$$

with $\vartheta = 7.5$ year and $m=2.5$. The second, which includes other causes (O) is described by a constant failure rate $\lambda_o = 0.0013 \ (years)^{-1}$

1. What is the reliability if no preventive maintenance is performed over the 5-year design life?
2. If the reliability of the 5-year design life is to be increased to at least 0.9 by periodically replacing the thrust bearing, how frequently must it be replaced?
3. Suppose that the probability of fault bearing replacement causing failure of the compressor is $p = 0.02$ . What will the design-life reliability be with the replacement program decided in 2)?

The mean time to failure of a component of a safety system is 1000 days. Testing the component requires 6 hours, whereas the time to repair is negligible. You are required to:

1. Compute the average unavailability of the component, assuming that the time interval, $T$, between the end of the previous test and the beginning of the next one is 50 days.

2. Consider the test period as a quantity to be optimized. Which is the value of $T$ that minimizes the average unavailability of the component?

*Hints:*

*a) neglect the contribution of the testing time on the time period of the maintenance cycle*

*b) you can use the approximation:*

$$1 - e^{-\lambda x} \cong \lambda x$$

Consider a one-out-of two system of identical components with constant failure rate $\lambda$. The testing and repair of each component last for $\tau_r$ hours.

1.  In the sequential maintenance scheme, the two components are tested one after the other, $\tau$ being the time between the end of the previous maintenance of the second component and the beginning of the next maintenance of the first one (in other words, every $\tau$ hours we test both components in sequence). Find the average unavailability of the system.
2.  In the staggered maintenance scheme, the first component maintenance starts at $k\tau$, $k < 1$, where $\tau$ is the time between the end of the previous maintenance of the second component and the beginning of the next maintenance of the same second component. Find the average unavailability of the system.

Consider a safety system made by two components (A and B) in parallel with constant failure rates equal to $\lambda_A$ and $\lambda_B$. The testing and repair of each component lasts for $\tau_r$ and a staggered maintenance scheme is applied to the system:

- $\tau$ is the time interval between the end of the previous maintenance of component B and the beginning of the next maintenance of component B.

- maintenance of component A starts after a time interval of $\frac{\tau}{4}$ from the end of the maintenance of component B.

You are required to:

a. Evaluate the average system unavailability over the maintenance cycle (you can assume that $\tau \gg \tau_r$, $\lambda_A \tau \ll 1$ and $\lambda_B \tau \ll 1$)

b. Plot the time evolution of the system instantaneous unavailability (you can assume $\lambda_A = \lambda_B$)