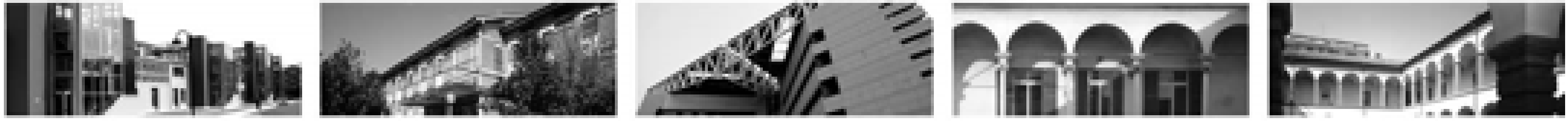
lasar
laboratory of signal and risk analysis

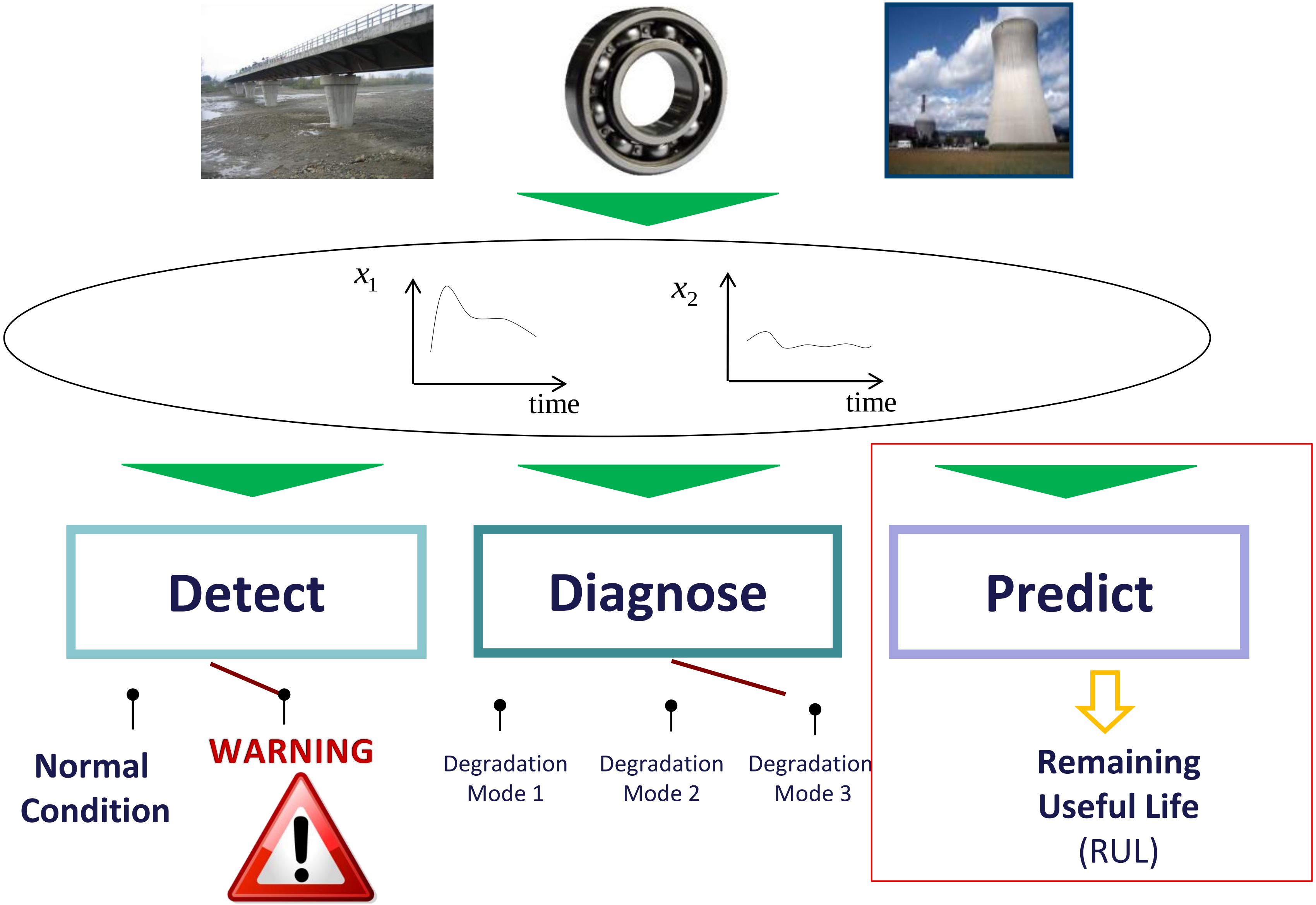
POLITECNICO DI MILANO



Fault Prognostics

Prof. Piero Baraldi

Prognostics and Health Management (PHM): what is it?



Detect

Normal Condition

WARNING

Diagnose

Degradation Mode 1

Degradation Mode 2

Degradation Mode 3

Predict

Remaining Useful Life (RUL)

In This Lecture

- Introduction to prognostics
- Model-based prognostics
  - Particle filtering for RUL estimate
- Applications
  - Maintenance planning
  - Prediction of the remaining useful life of electrolyte capacitors
  - Prediction of the remaining useful life of batteries

Outline

- Introduction to prognostics
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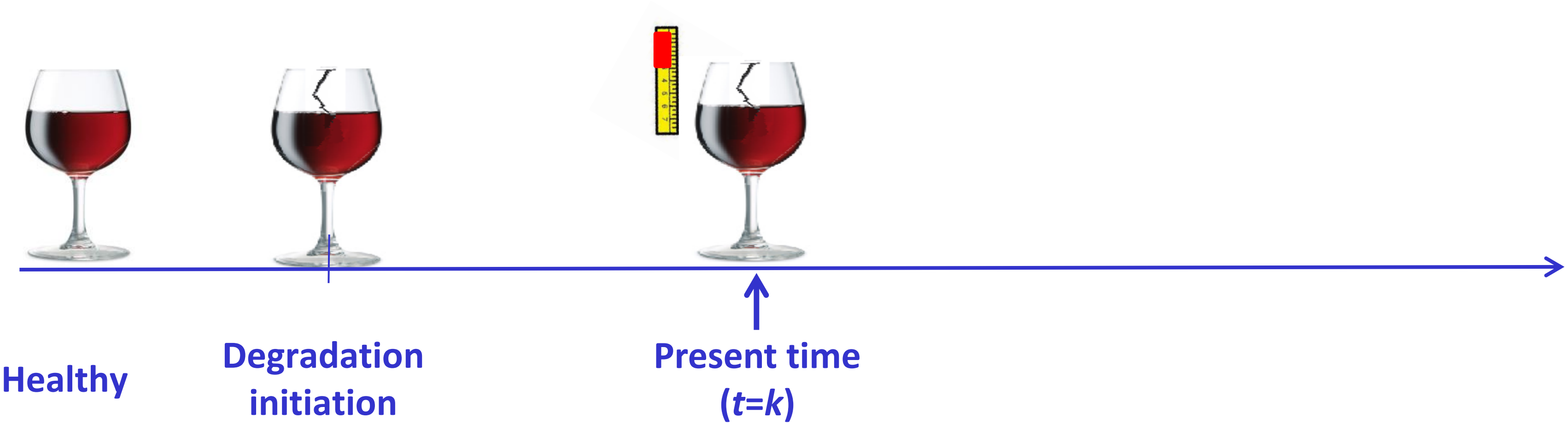
Prognostics: objectives

Evolution to… failure

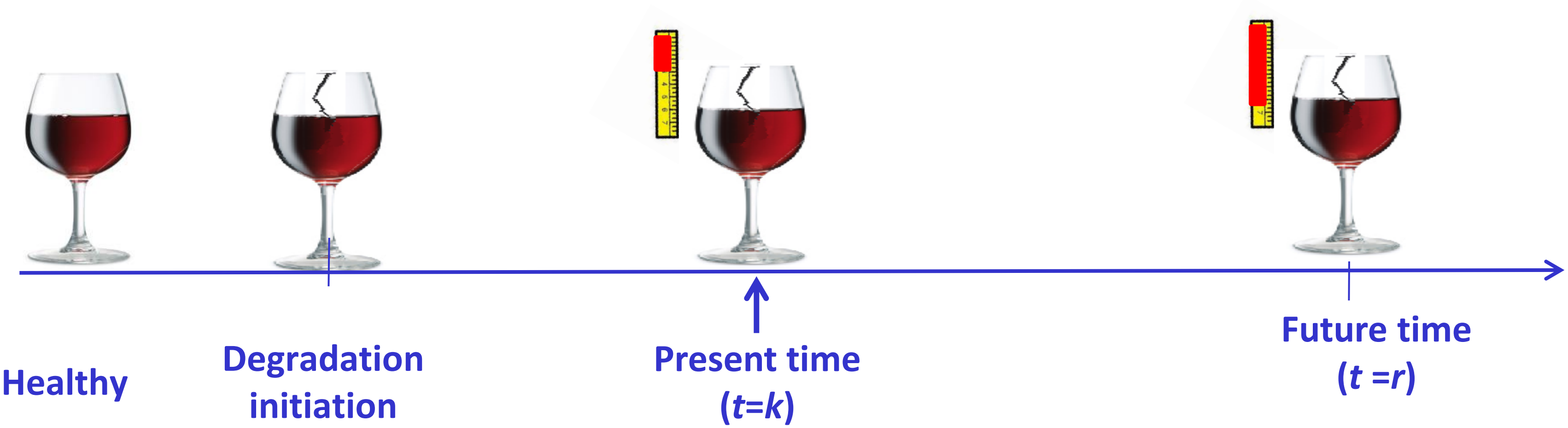


Our objectives:

1. **Estimate the component degradation at a the present time $t = k$**

Prognostics: objectives

Evolution to… failure

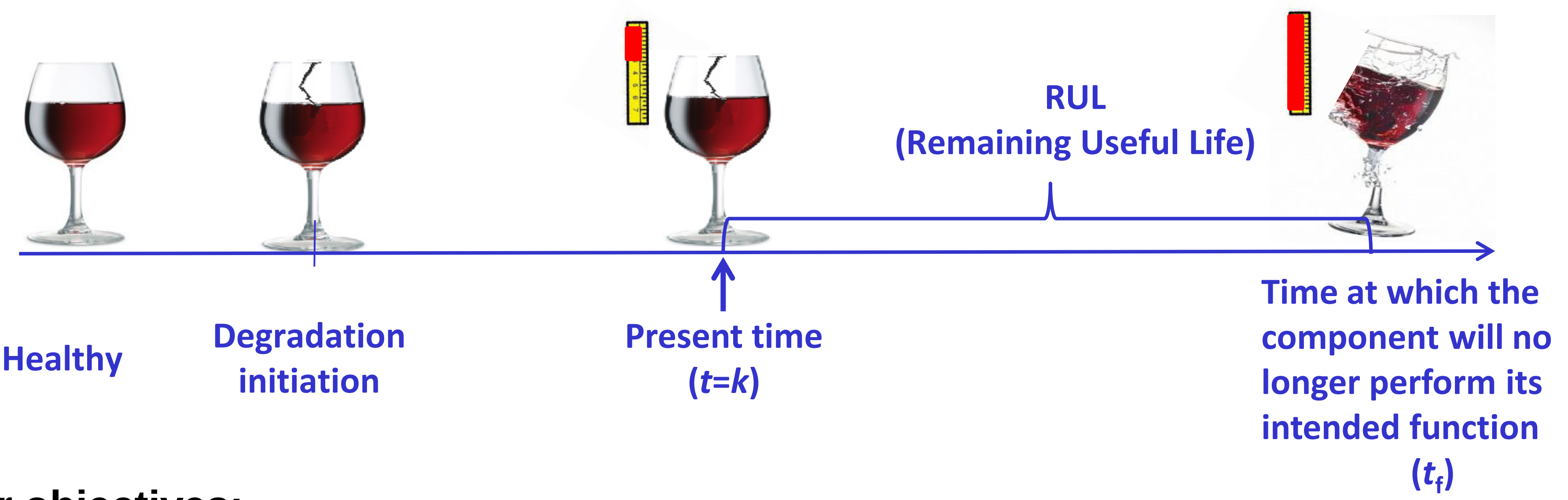


Our objectives:

1. **Estimate the component degradation at a the present time *t* =*k***
2. **Estimate the component degradation at a future time *r* > *k***

Prognostics: objectives

Evolution to… failure



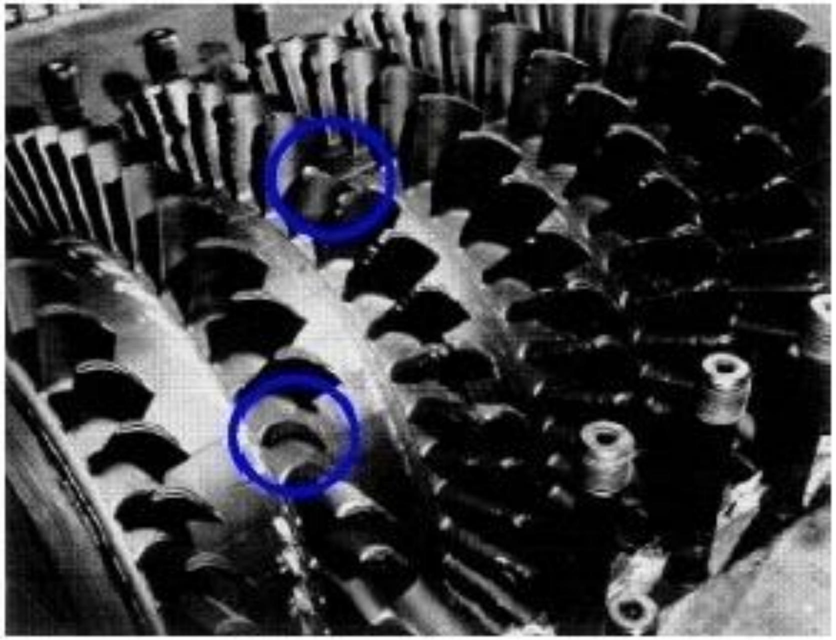
Our objectives:

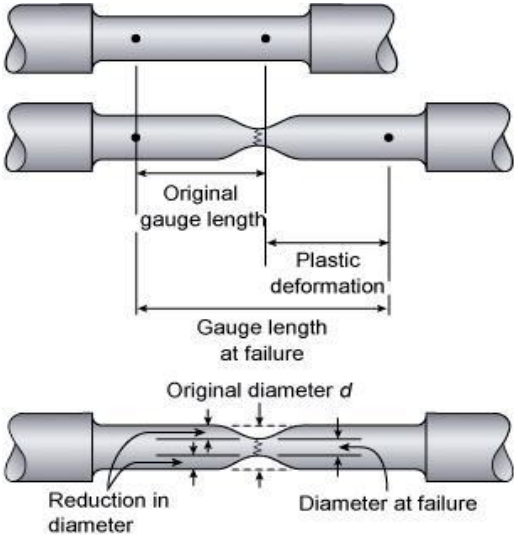
1. **Estimate the component degradation at a the present time $t = k$**
2. **Estimate the component degradation at a future time $r > k$**
3. **Estimate the component Remaining Useful Life (RUL) = $t_f - k$**

Prognostics: An example

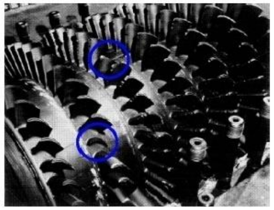
Component: turbine blade

Degradation mechanism: creeping



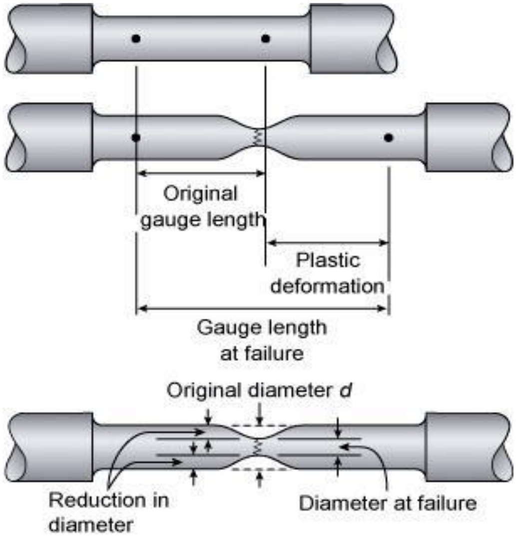


Prognostics: an example



Component: turbine blade
Degradation mechanism: creeping

Degradation indicator: blade elongation $x(t) = \frac{\text{Length}(t) - \text{initial length}}{\text{initial length}}$



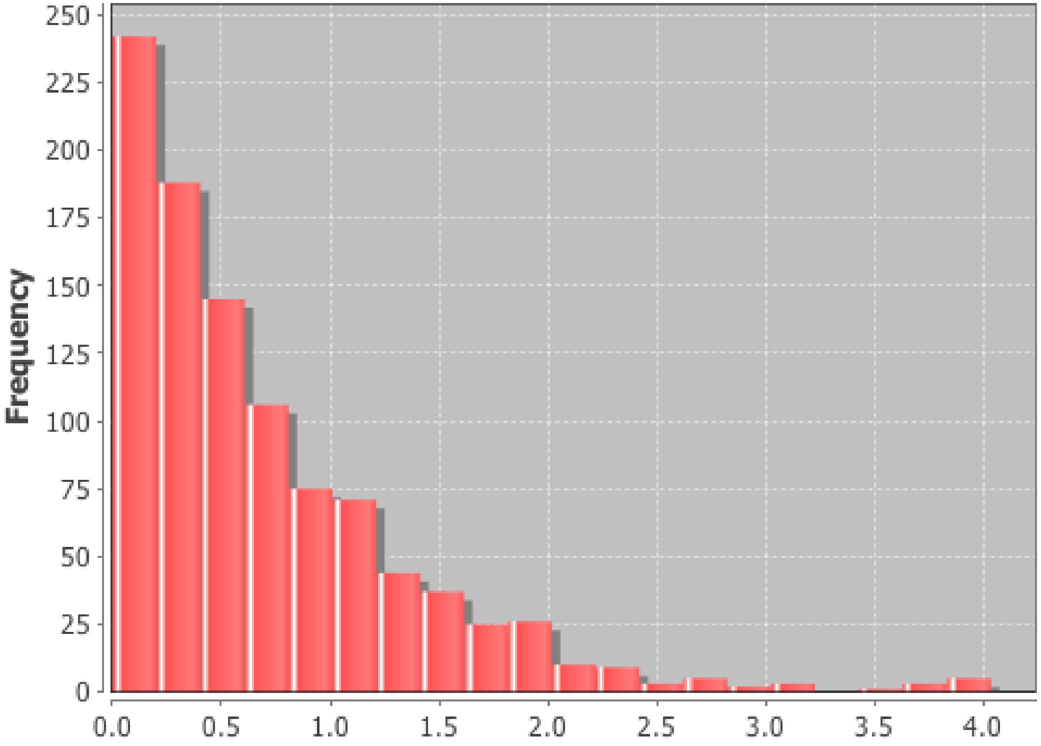
Our objectives:

1. Estimate the blade degradation at the present time $t = k$
2. Estimate the blade degradation at a future time $r > k$
3. Estimate the component Remaining Useful Life (RUL)

Sources of information for prognostics

- Life durations of a set of similar components which have already failed:

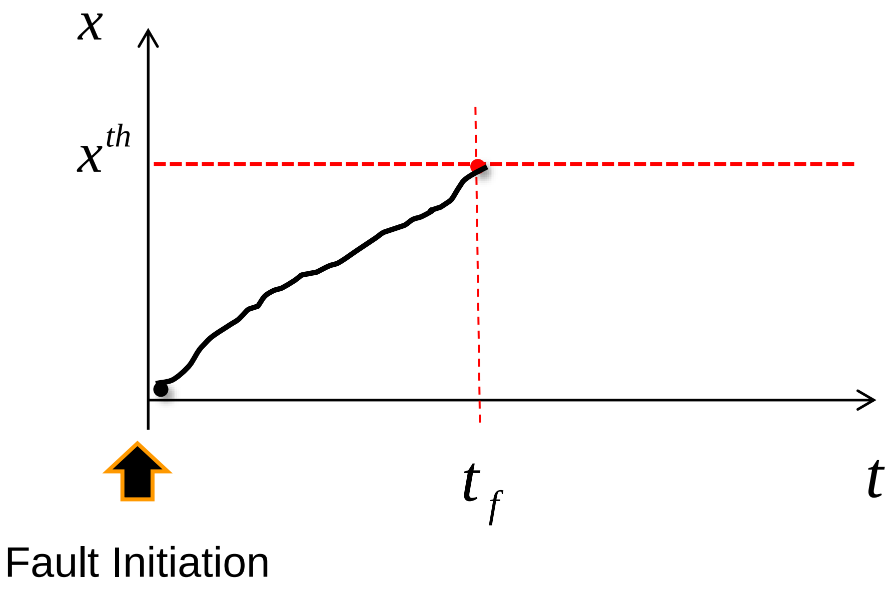
$$T_1, T_2, \dots, T_n$$



Failure time (years)

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
- Threshold of failure: x^{th}



Sources of information for prognostics

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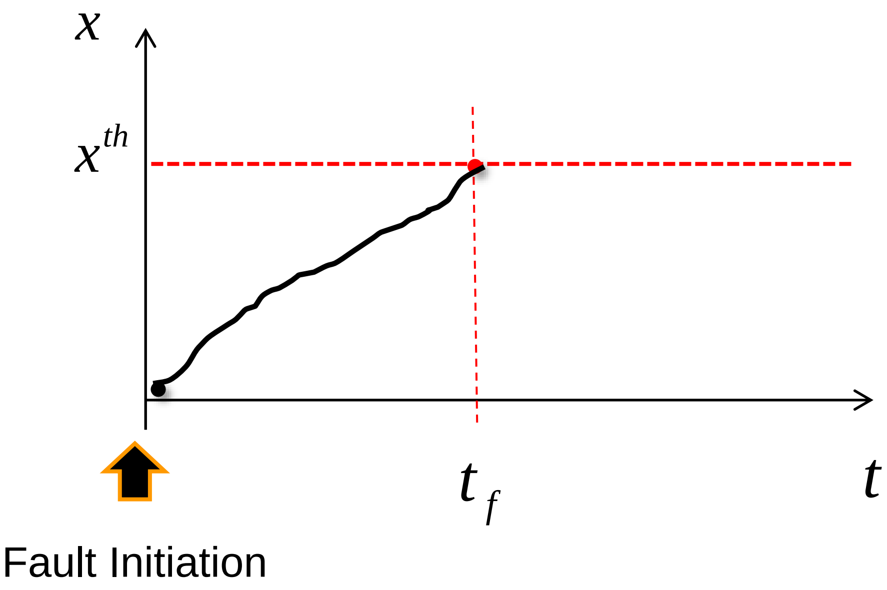
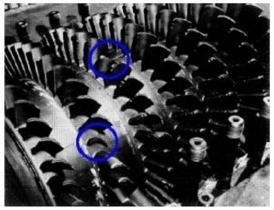
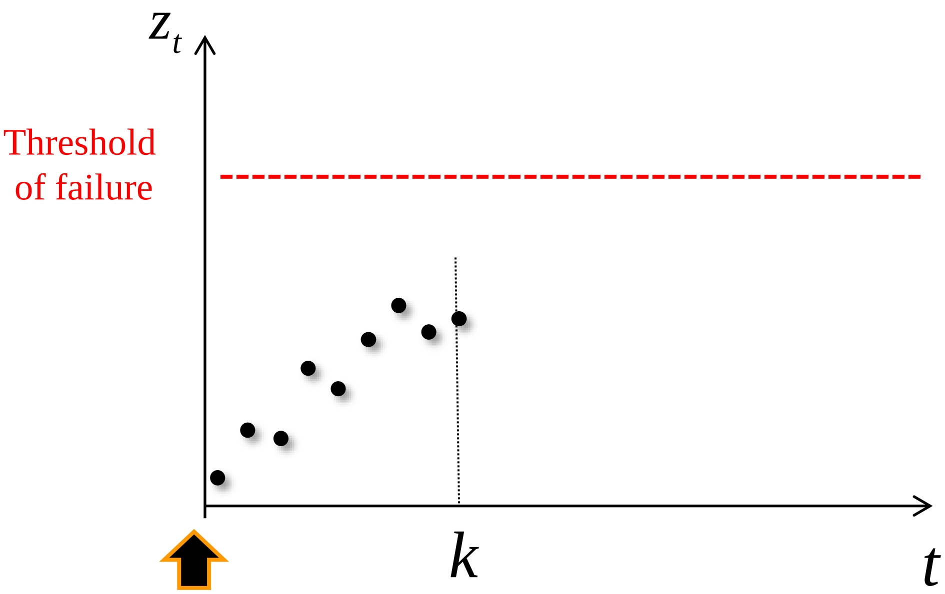




«A blade is discarded when the elongation, *x*, reaches 1.5%»

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
- Threshold of failure
- A sequence of observations collected from the degradation initiation to the present time (current degradation trajectory): $z_1, z_2 \dots, z_k$



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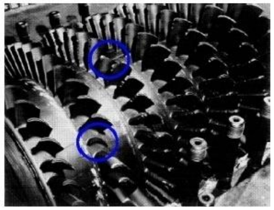
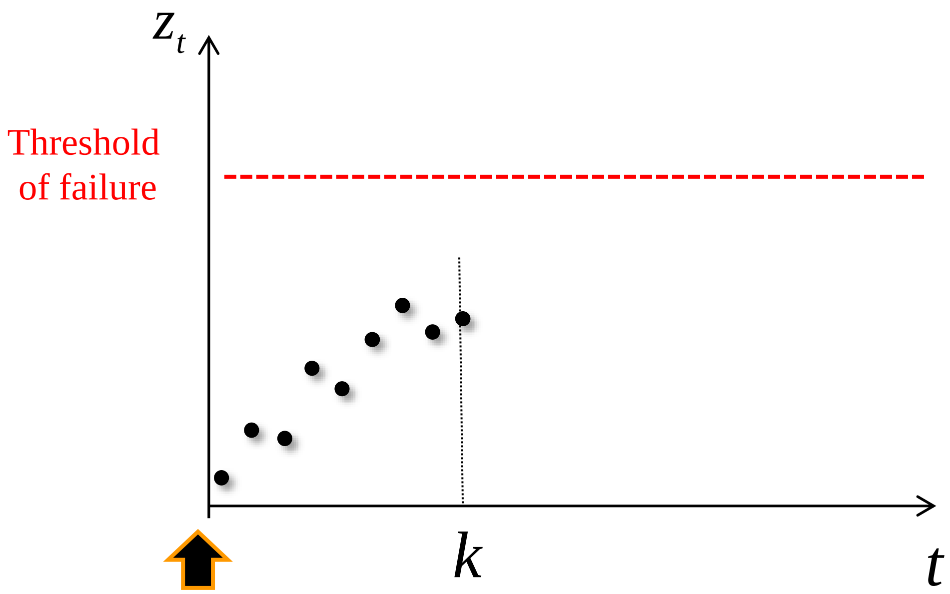
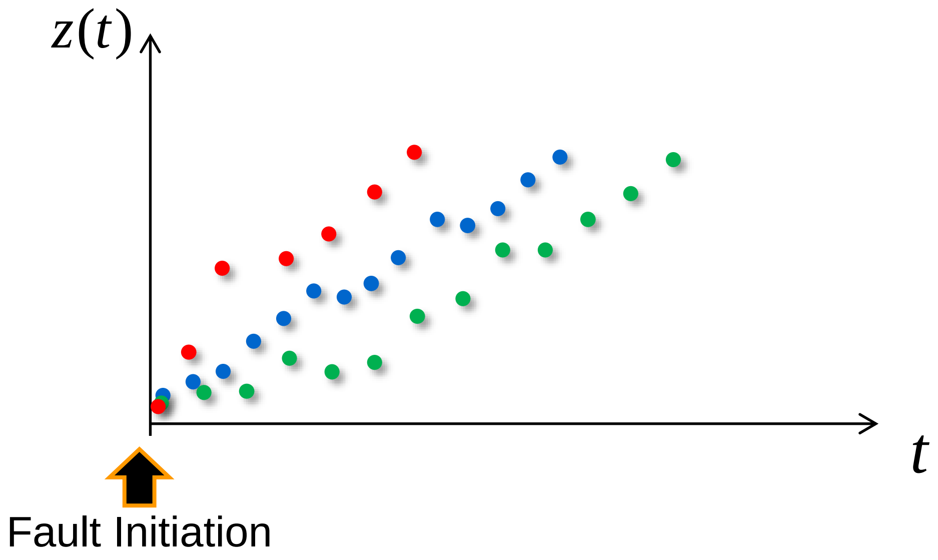


Elongation measurements = past evolution of the degradation indicator



Sources of information for prognostics

- Life durations of a set of similar components which have already failed
- Threshold of failure
- A sequence of observations collected from the degradation initiation to the present time (current degradation trajectory)
- Degradation trajectories of similar components



Sources of information for prognostics

- Life durations of a set of similar components which have already failed
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- A sequence of observations collected from the degradation initiation to the present time (current degradation trajectory)
- Degradation trajectories of similar components
- Information on external/operational conditions (past – present - future)

Past, present and future time evolution of: $u_1, u_2, \ldots, u_k, u_{k+1}, \ldots$

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
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Past, present and future time evolution of: $u_1, u_2, \ldots, u_k, u_{k+1}, \ldots$

$\boldsymbol{u_1} =$ **T = temperature**

$\boldsymbol{u_2} = \boldsymbol{\theta_r}$ **= rotational speed**

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
- Threshold of failure
- A sequence of observations collected from the degradation initiation to the present time (current degradation trajectory)
- Degradation trajectories of similar components
- Information on external/operational conditions (past – present - future)
- Measurement equation

$$z = h(x, v)$$

Random noise with
known distribution

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
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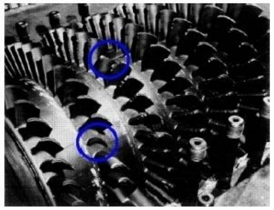
$$z = h(x, \nu)$$

Random noise with known distribution



$$z = x + \nu$$
$$\nu \propto N\left(0, \sigma^2\right)$$

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
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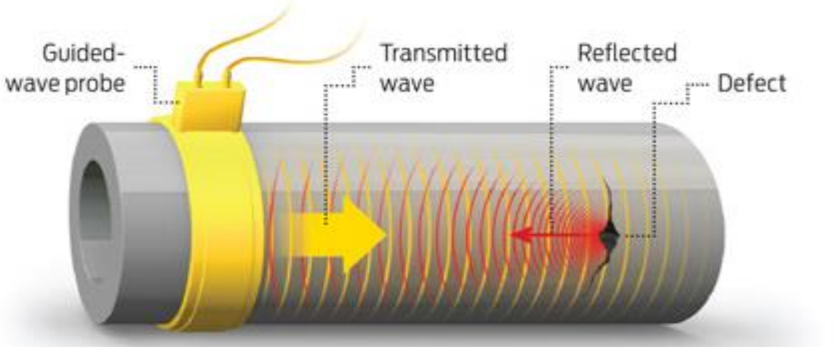
- A sequence of observations collected from the degradation initiation to the present time (current degradation trajectory)
- Degradation trajectories of similar components
- Information on external/operational conditions (past – present - future)
- Measurement equation

$$z = h(x, v)$$

$$z_k = d\left[1 - \exp\left(\beta_0 + \beta_1 \ln \frac{x_k}{d - x_k} + \upsilon_k\right)\right]^{-1}$$



Ultrasonic Monitoring (regularly used in the oil and gas industry)

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
- Threshold of failure
- A sequence of observations collected from the degradation initiation to the present time (current degradation trajectory)
- Degradation trajectories of similar components
- Information on external/operational conditions (past – present - future)
- Measurement equation
- A physics-based model of the degradation process

$$x_k = f_k\left(x_{k-1},\dots,x_1,u_{k-1},\omega_{k-1}\right)$$

Sources of information for prognostics

- Life durations of a set of similar components which have already failed
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Norton law for creep growth

$$\frac{dx}{dt} = \underbrace{A\exp\left(-\frac{Q}{RT}\right)}_{\text{Arrhenius law}}\varphi^{n}$$

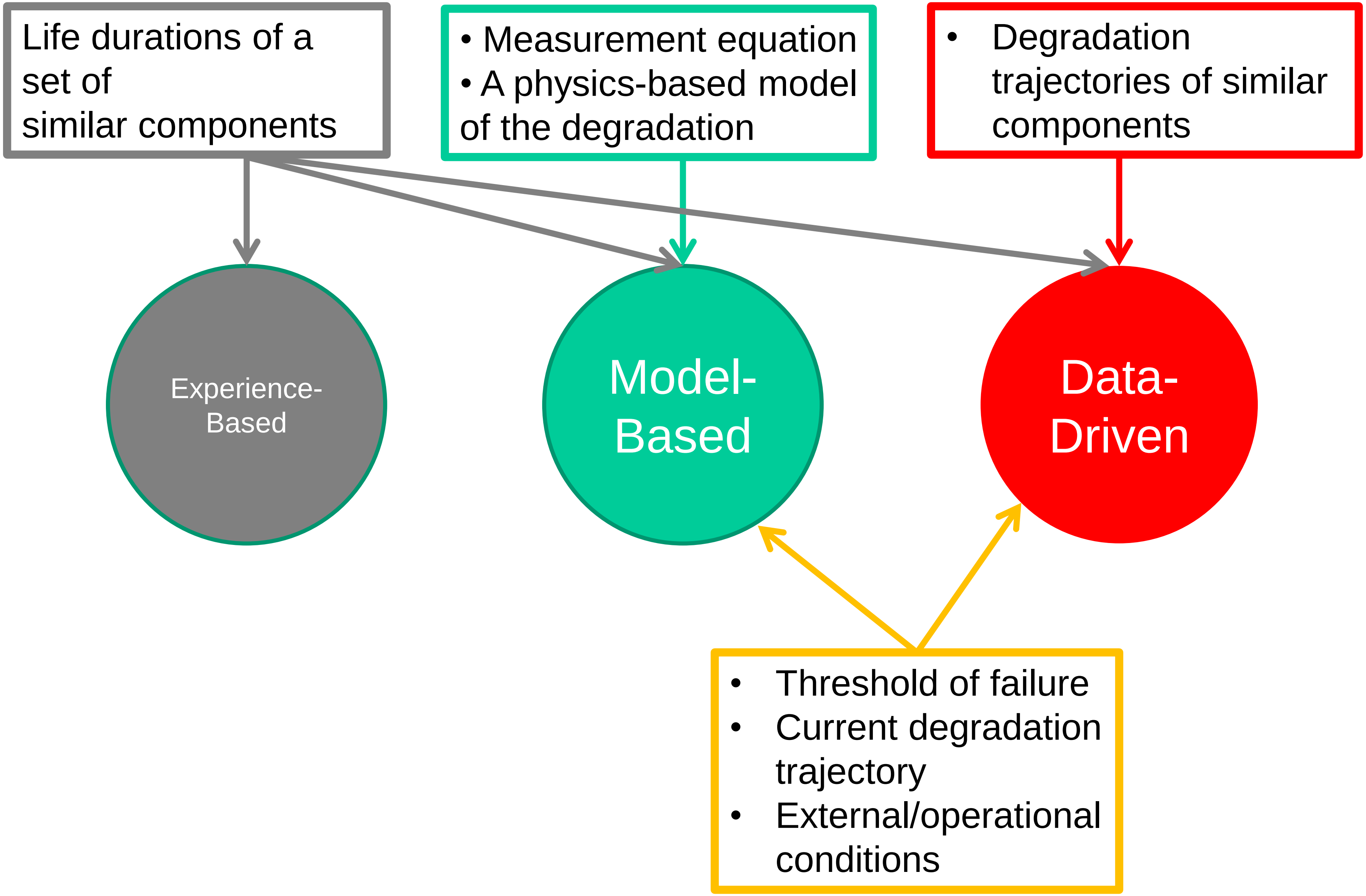
x = blade elongation

$$\left.\begin{array}{l} T = \text{temperature} \\ \varphi = K\theta_r^{\,2} = \text{applied stress} \\ \theta_r = \text{rotational speed} \end{array}\right\} \text{External/operational conditions}$$

A, Q, n = equipment inherent parameters

- Prognostics
  - What is it?
  - Prognostics in practice
  - Sources of information
  - Prognostic approaches

Prognostic approaches



Life durations of a set of similar components

- Measurement equation
- A physics-based model of the degradation

- Degradation trajectories of similar components

- Threshold of failure
- Current degradation trajectory
- External/operational conditions

Outline

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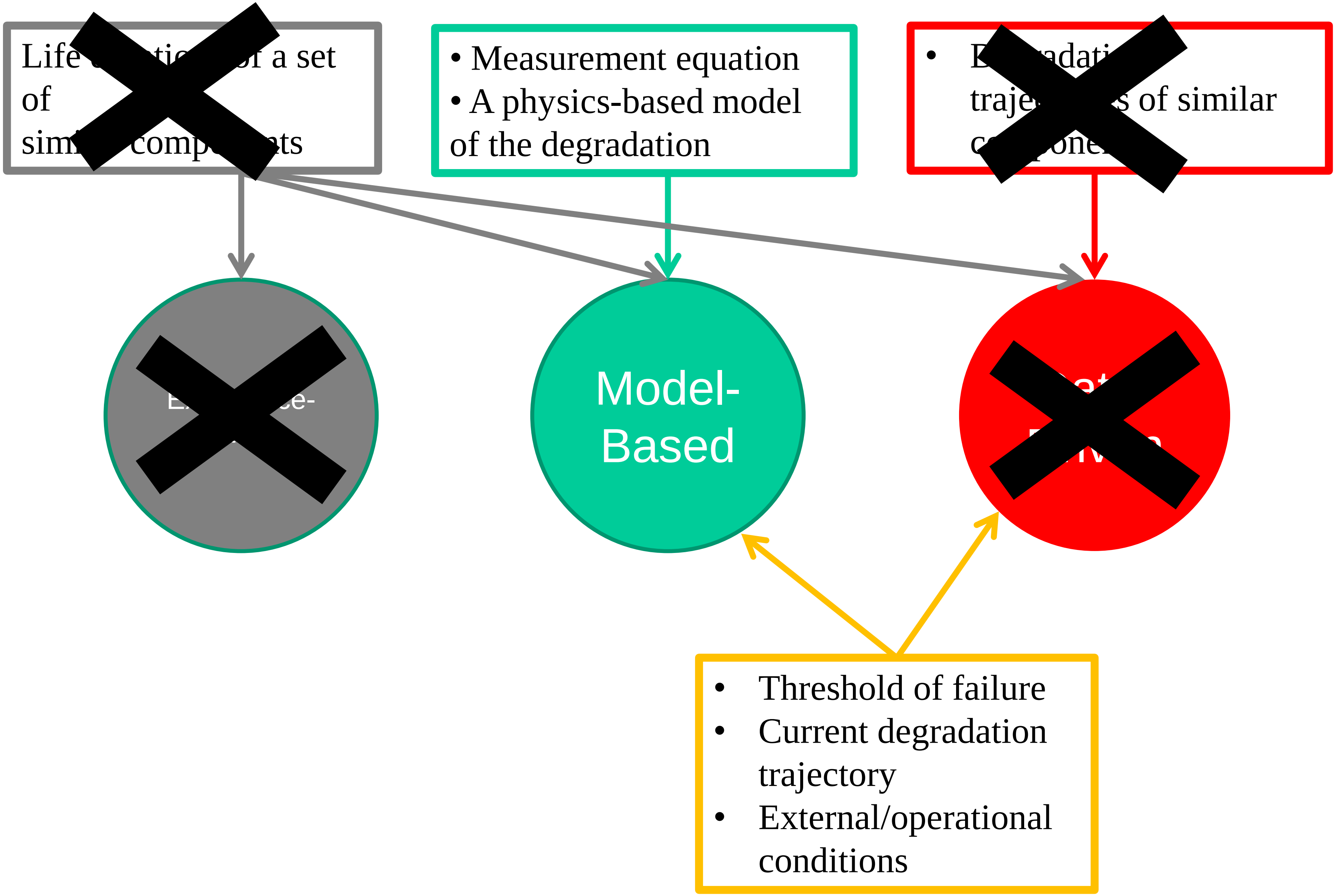
Model-Based prognostics: information available



~~Life d tio r a set of sim comp ts~~

- Measurement equation
- A physics-based model of the degradation

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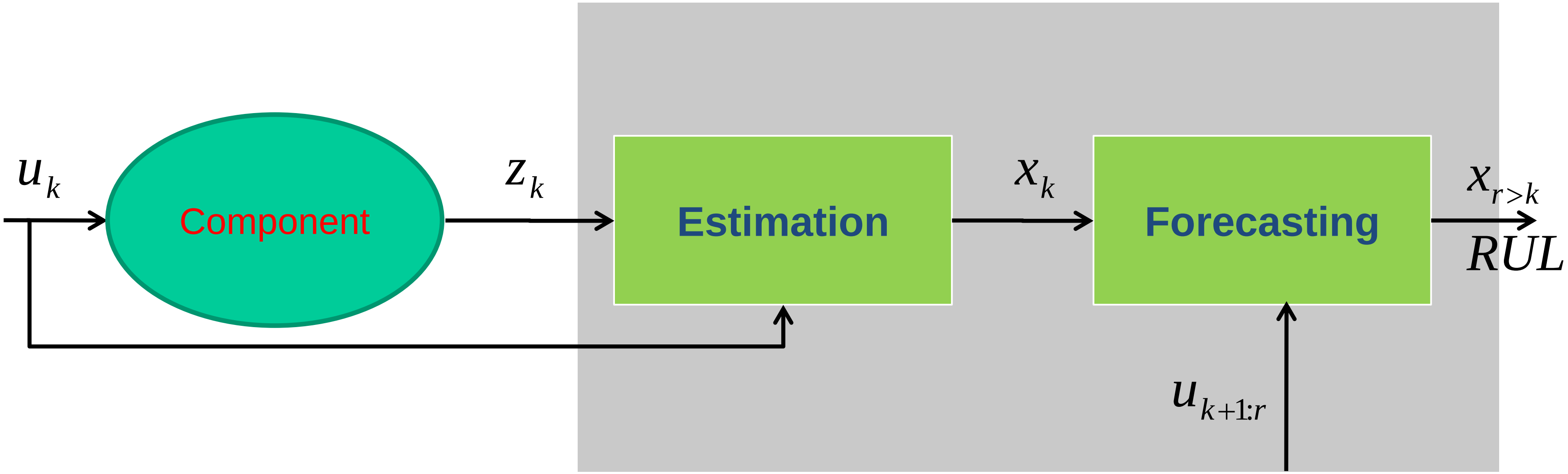
Model-Based

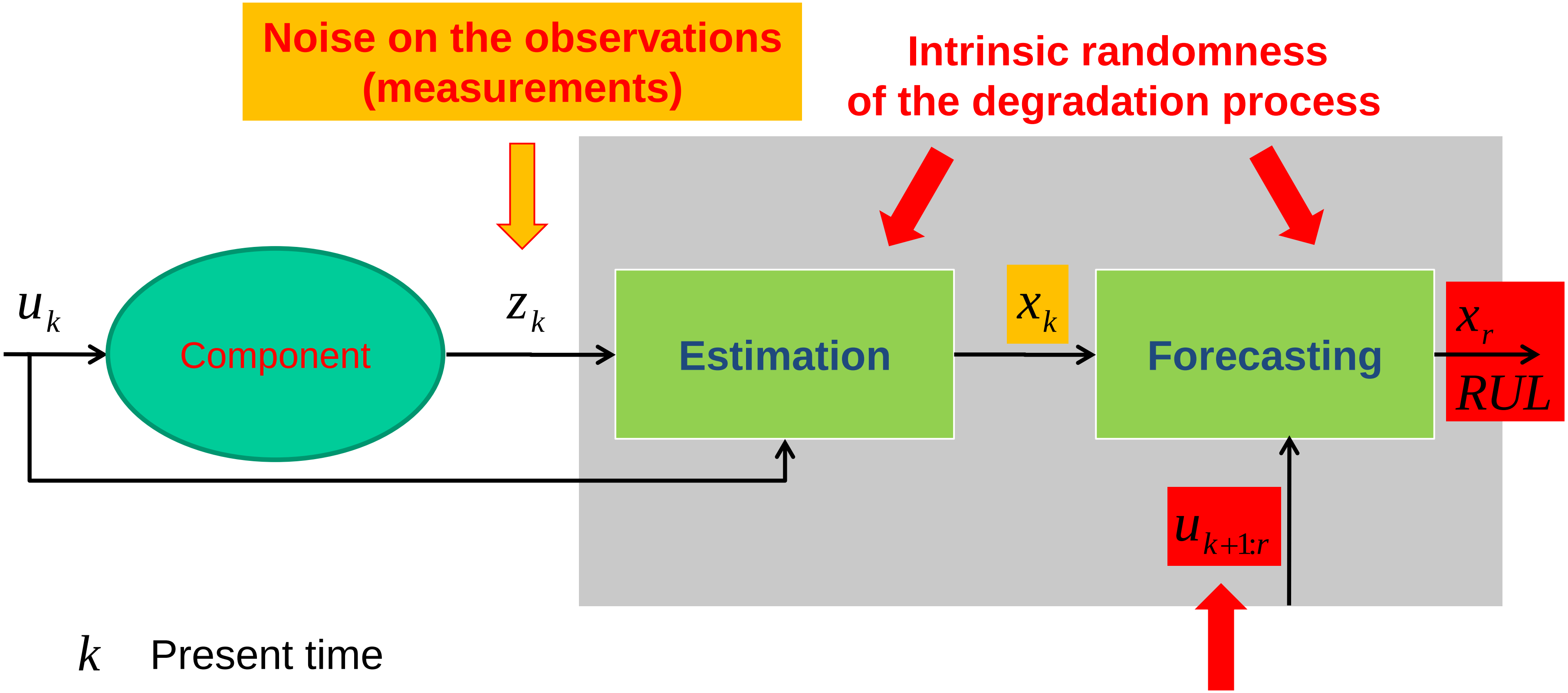
- Threshold of failure
- Current degradation trajectory
- External/operational conditions

Model-based prognostics: the methodology



k	Present time
u	External/operating conditions
z	Observations
x	Degradation state

Main sources of uncertainty

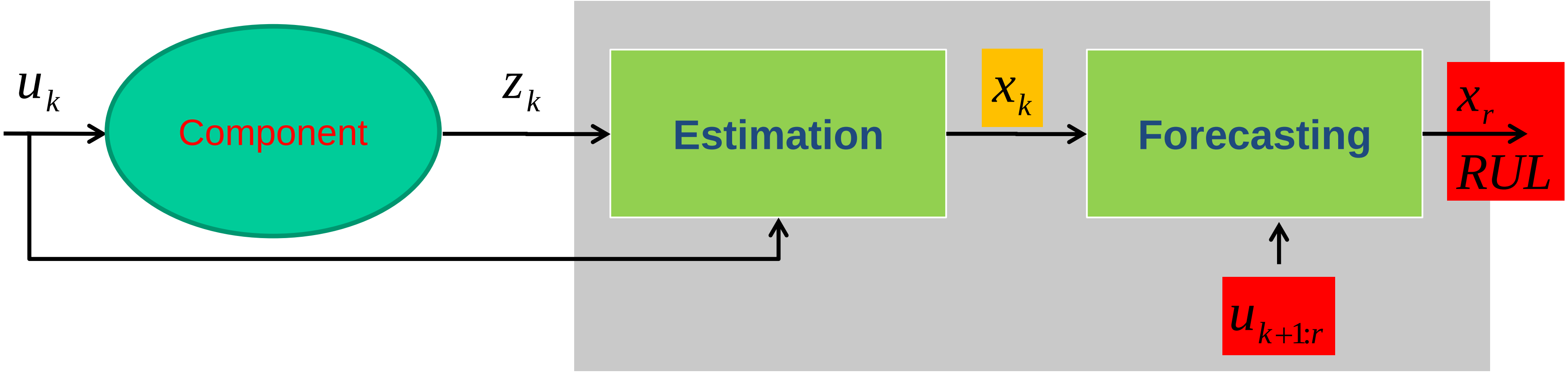

Noise on the observations (measurements)

Intrinsic randomness of the degradation process

Future external/operating conditions are never exactly known

k	Present time
u	External/operating conditions
z	Observations
x	Degradation state

Prognostics = Filtering + Forecasting



1. The filtering problem: to estimate the degradation state, x_k , at the present time
2. The forecasting problem:
   - to predict the degradation state, x_r , at a future time *r*
   - to predict the component RUL

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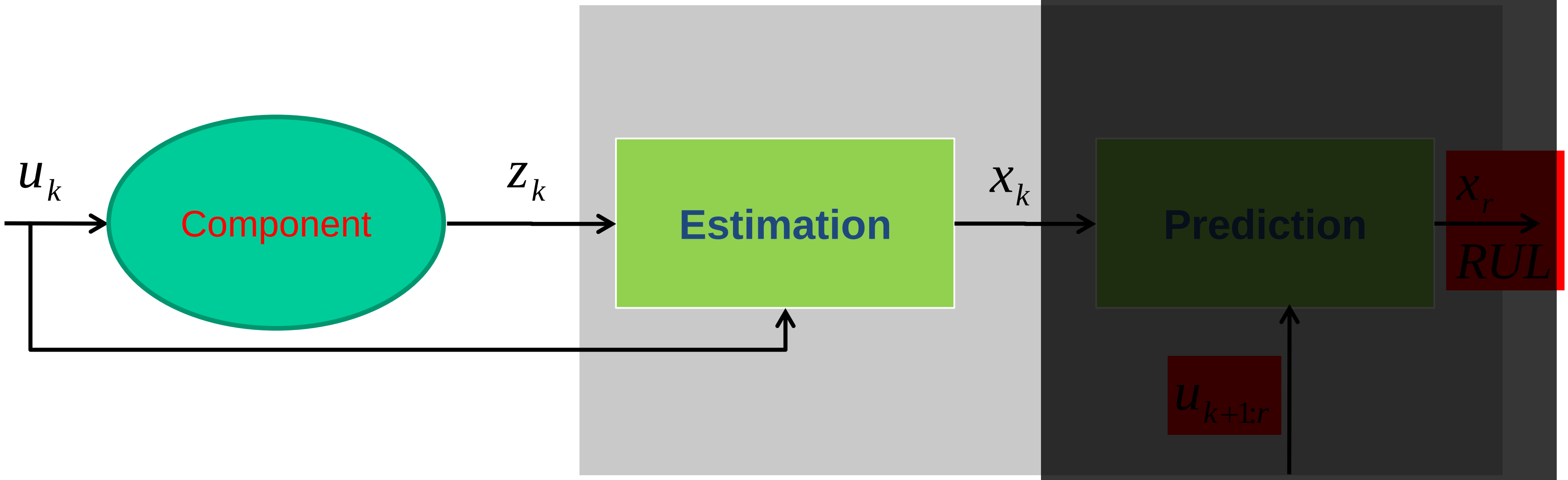
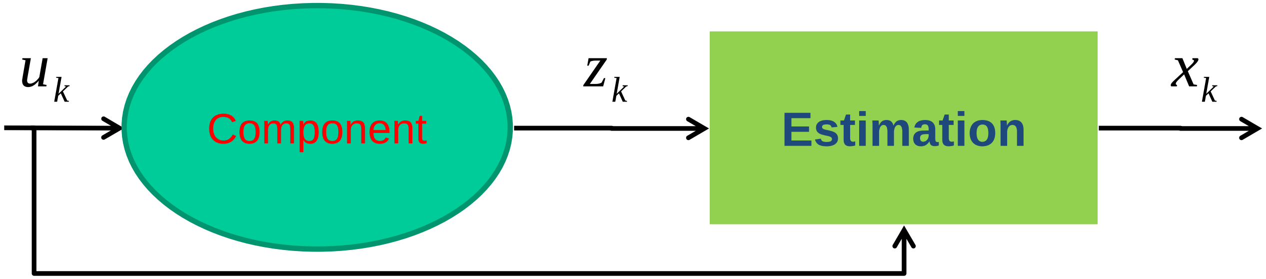
- Model-based prognostics:
    - The filtering problem
    - The forecasting problem

The Filtering Problem



k	Present time
u	External/operating conditions
z	Observations
x	Degradation state

Problem Setting



- Physical model of the degradation process

$$x_k = f_k\left(x_{k-1}, \omega_{k-1}\right)$$

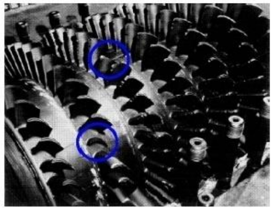
Time-discrete, hidden Markov process

  - x = **hidden** degradation state
  - ω = random **process noise**
  - *f* = physical model of the degradation process (non-linear dynamic law)
  - *k* = time step index

- Measurement equation:
  - υ = random **measurement noise**
  - *h* = non-linear measurement equation

$$z_k = h\left(x_k, v_k\right)$$

The filtering problem in practice
(Physical model of the degradation process)



- x = hidden degradation state (blade elongation)
- T_0, ϑ_0 = operating conditions
- $\omega_1, \omega_2, \omega_3$ = random process noises $\omega_i \propto N(0, \sigma_i^2)$
- *A*, *K* and *n* = constants related to the material properties

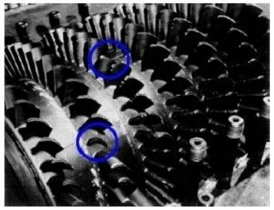
$$\frac{dx}{dt} = A \cdot \exp\left(- \frac{Q}{R \cdot (T_0 + \omega_1)} \right) \cdot \left(K \cdot (\vartheta_0 + \omega_2)^2 + \omega_3 \right)^n$$

Norton law for creep growth

Discretization of the dynamics

$$x_k = x_{k-1} + A \cdot \exp\left(- \frac{Q}{R \cdot (T_0 + \omega_1)} \right) \cdot \left(K \cdot (\theta_0 + \omega_2)^2 + \omega_3 \right)^n$$

The filtering problem in practice (Measurement Equation)

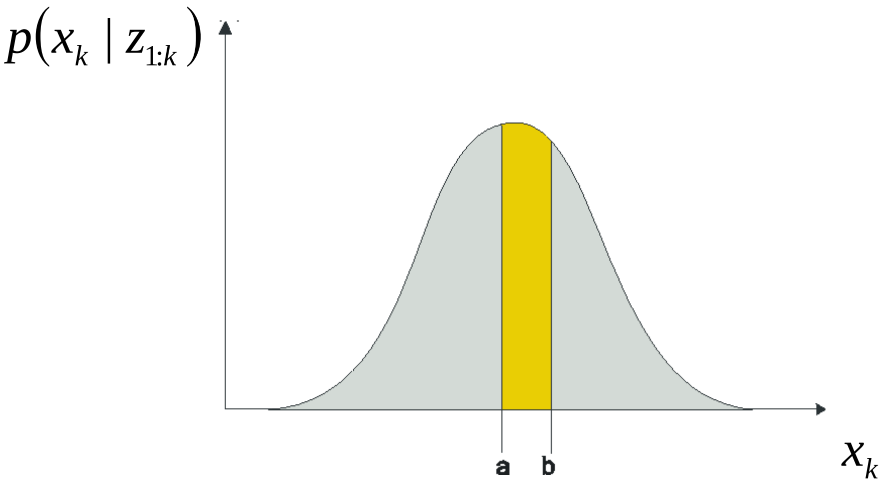


$$z_k = h(x_k, v_k) = x_k + v_k$$

- z_k = degradation observation (measure of the creep elongation)
- υ_k = gaussian measurement noise

The Bayesian framework

OBJECTIVE: $p\left(x_k \mid z_{1:k}\right)$

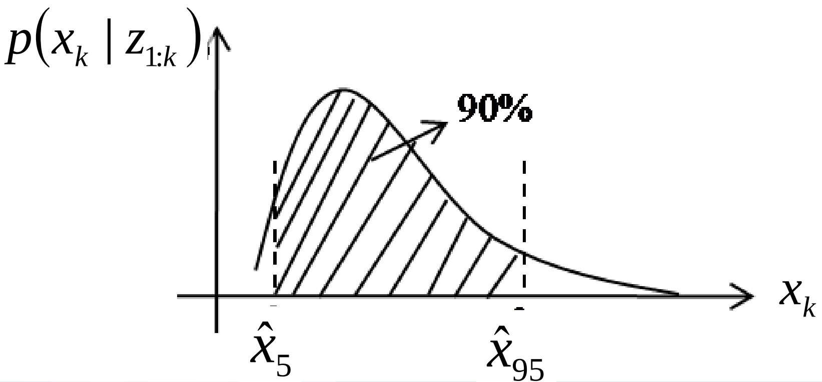


- Interpretation of the bayesian probability $p\left(x_k \mid z_{1:k}\right)$?
  - conditional on the background knowledge: the noisy measurements $z_{1:k} = z_1, z_2, \ldots, z_k$

The state estimate and its uncertainty

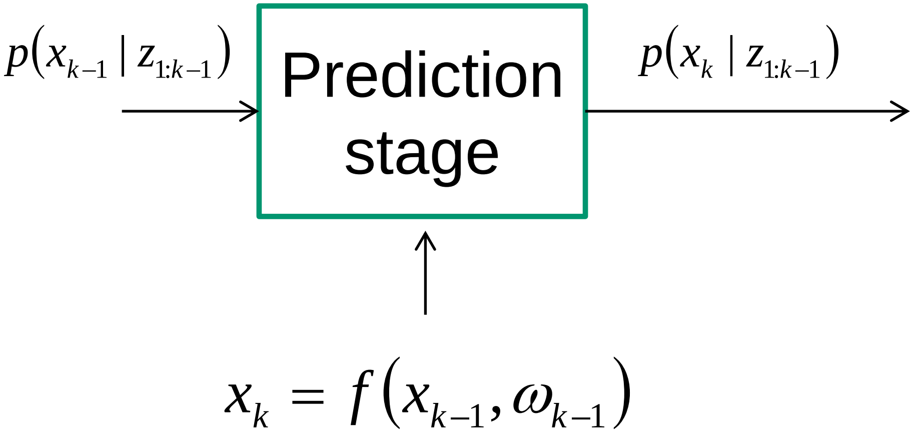
$$p\left(x_k \mid z_{1:k}\right)$$

- state **mean** (estimate) $\hat{x}_k = \int p\left(x_k \mid z_{1:k}\right) \cdot x_k \, dx_k$
- state **variance** (uncertainty) $\hat{\sigma}^2_{x_k} = \int \left(x_k - \hat{x}_k\right)^2 \cdot p\left(x_k \mid z_{1:k}\right) dx_k$
- state **percentiles** $\hat{x}_5, \hat{x}_{95}$



The sequential solution (I)

- Let us assume that we know $p\left(x_{k-1} \mid z_{1:k-1}\right)$ at time *k*-1



$p\left(x_{k-1} \mid z_{1:k-1}\right)$

Prediction stage

$p\left(x_{k} \mid z_{1:k-1}\right)$

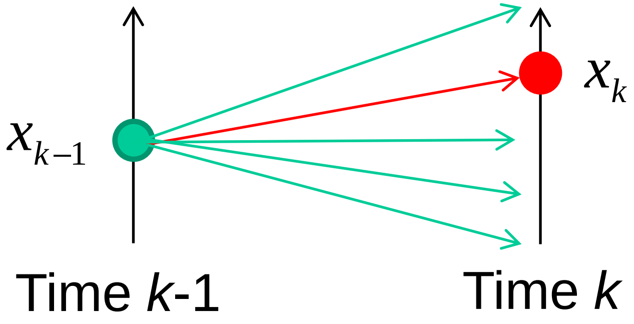
$$x_k = f\left(x_{k-1}, \omega_{k-1}\right)$$

The sequential solution (I)

- Let us assume that we know $p\left(x_{k-1} \mid z_{1:k-1}\right)$ at time *k*-1

- Prediction stage:

$p(X_{k-1} =$ 🟢 $AND\ X_k =$ 🔴 $) =$



$$p(x_{k-1}, x_k | z_{k-1}) = p(x_{k-1}|z_{1:k-1}) p(x_k|x_{k-1})$$

The sequential solution (I)

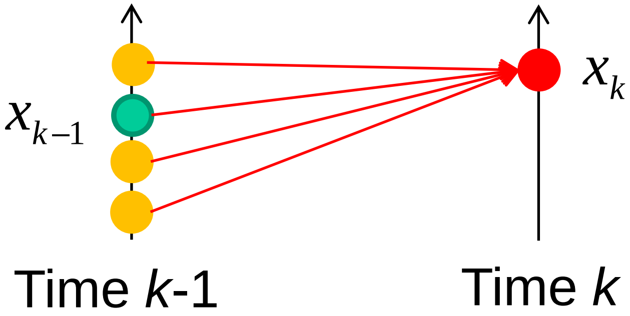
- Let us assume that we know $p(x_{k-1} \mid z_{1:k-1})$ at time *k*-1

$p(x_{k-1} \mid z_{1:k-1})$

Prediction stage

$p(x_k \mid z_{1:k-1})$

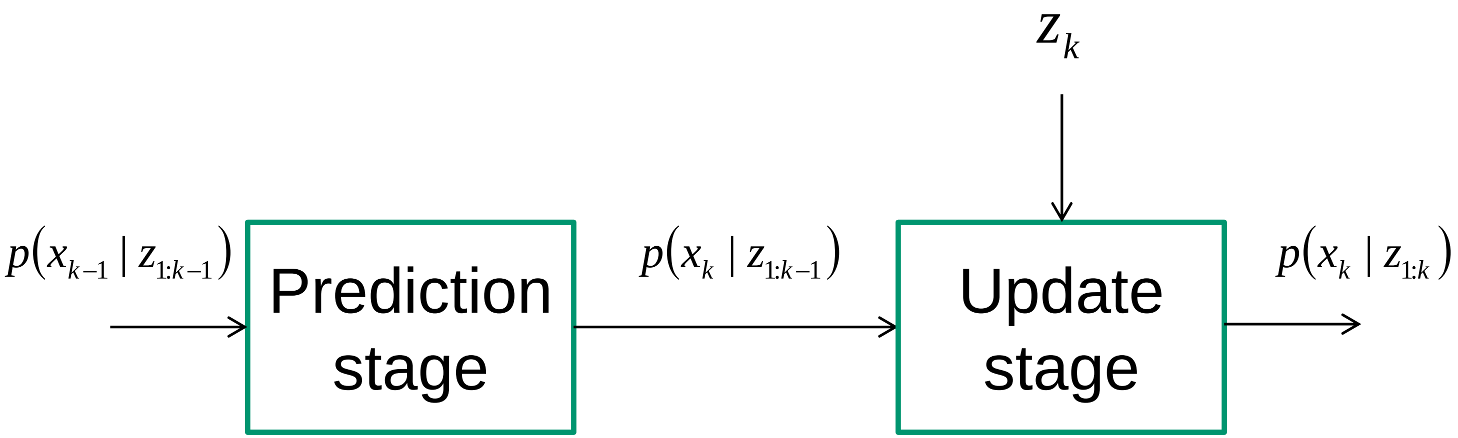
- Prediction stage: Chapman-Kolmogorov equation



$$p(\, X_k = \bullet) = \sum_{\bullet} P(X_{k-1} = \bullet \, AND \; X_k = \bullet)$$

$$p(x_k \mid z_{1:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid z_{1:k-1}) dx_{k-1}$$

The sequential solution (II)



z_k

$p(x_{k-1} \mid z_{1:k-1})$

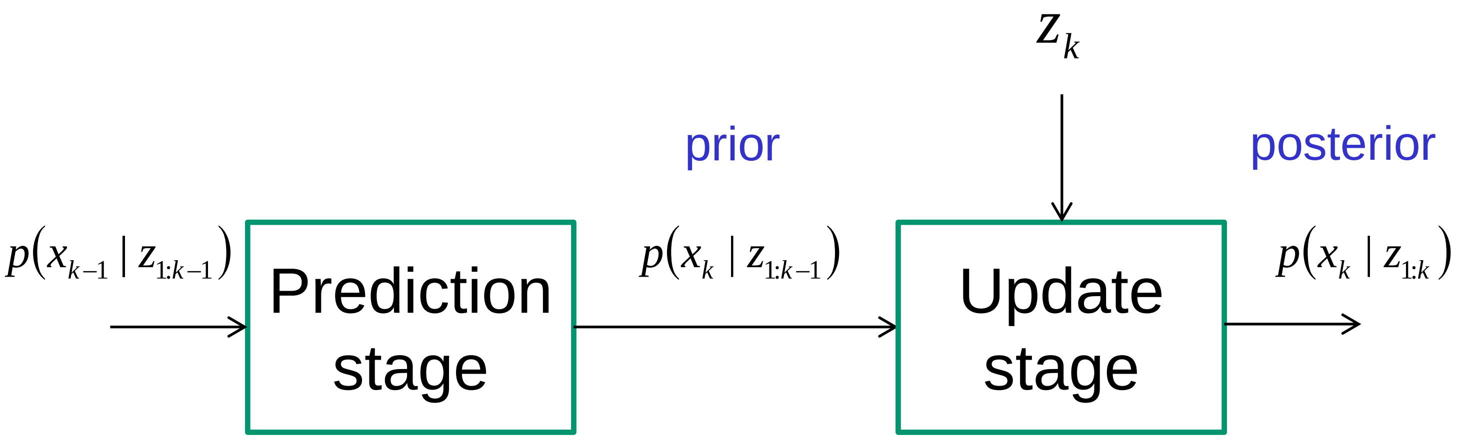
Prediction stage

$p(x_k \mid z_{1:k-1})$

Update stage

$p(x_k \mid z_{1:k})$

The sequential solution (II)



z_k

prior

posterior

$p(x_{k-1} \mid z_{1:k-1})$

Prediction stage

$p(x_k \mid z_{1:k-1})$

Update stage

$p(x_k \mid z_{1:k})$

- Update stage: Bayes Rule

$$\overset{\text{posterior}}{p(x_k \mid z_{1:k})} = \frac{\overset{\text{Likelihood}}{p(z_k \mid x_k)}}{const}\,\overset{\text{prior}}{p(x_k \mid z_{1:k-1})}$$

From the normalization

$$\int p(x_k \mid z_{1:k})dx_k = 1 \qquad \longrightarrow \qquad const = \int p(z_k \mid x_k)\cdot p(x_k \mid z_{1.k-1})dx_k$$

The sequential solution: What is difficult in practice?

- The integrals are difficult to solve analytically!

Chapman-Kolmogorov equation

$$p\left(x_k \mid z_{1:k-1}\right) = \int p\left(x_k \mid x_{k-1}\right) p\left(x_{k-1} \mid z_{1:k-1}\right) dx_{k-1}$$

Bayes Rule

$$p\left(x_k \mid z_{1:k}\right) = \frac{p\left(z_k \mid x_k\right)}{const} p\left(x_k \mid z_{1:k-1}\right)$$

$$const = \int p\left(z_k \mid x_k\right) \cdot p\left(x_k \mid z_{1:k-1}\right) dx_k$$

Available model-based filtering techniques

Kalman Filter

Exact only for linear systems and additive Gaussian noises

PARTICLE FILTERING

Numerical solution which, in the limit, tends to the exact posterior pdf:

$$p\left(x_k \mid z_{1:k}\right)$$

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  - Maintenance planning
  - Prediction of the remaining useful life of electrolyte capacitors
  - Prediction of the remaining useful life of batteries

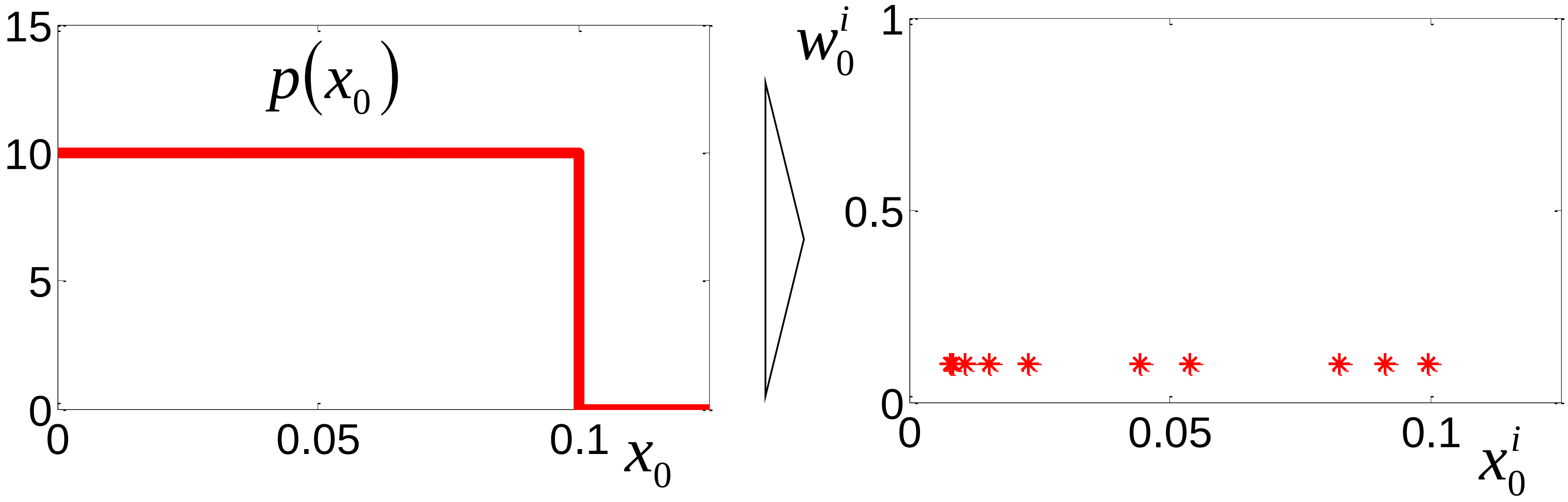
- Particle filtering for degradation state estimate
    - The intuitive representation
    - State estimate in practice
    - Detailed analytical approach to the problem
    - The pseudocode

The intuitive representation:
1. pdf approximation

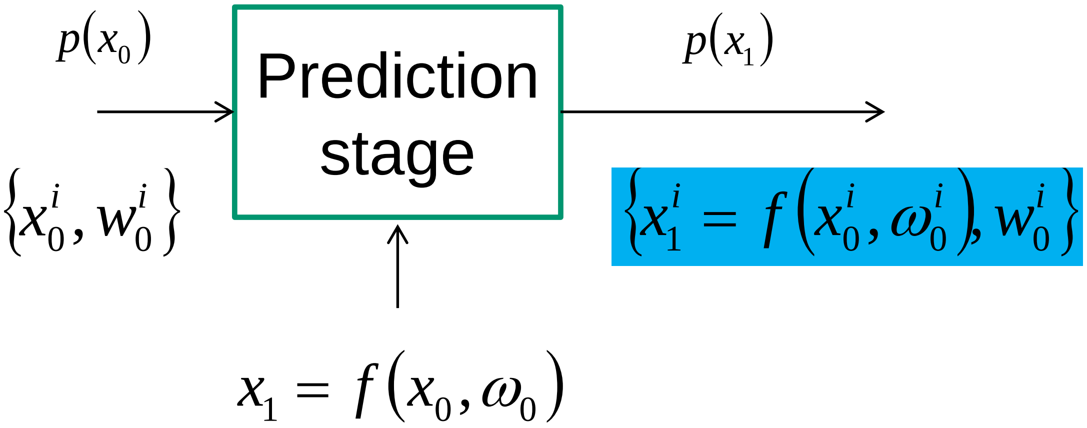
- Time 0, we approximate $p(x_0)$ in the form of a set of N_s random samples x_0^i with associated weights $w_0^i = \frac{1}{N_s}$

- $p(x_0)$ is approximated by a population of particles: $\{x_0^i, w_0^i\}, i = 1, \ldots, N_s$

  with: $\sum_{i=1}^{N_s} w_0^i = 1$



The intuitive representation: prediction stage: Monte Carlo Simulation



$p(x_0)$

$\{x_0^i, w_0^i\}$

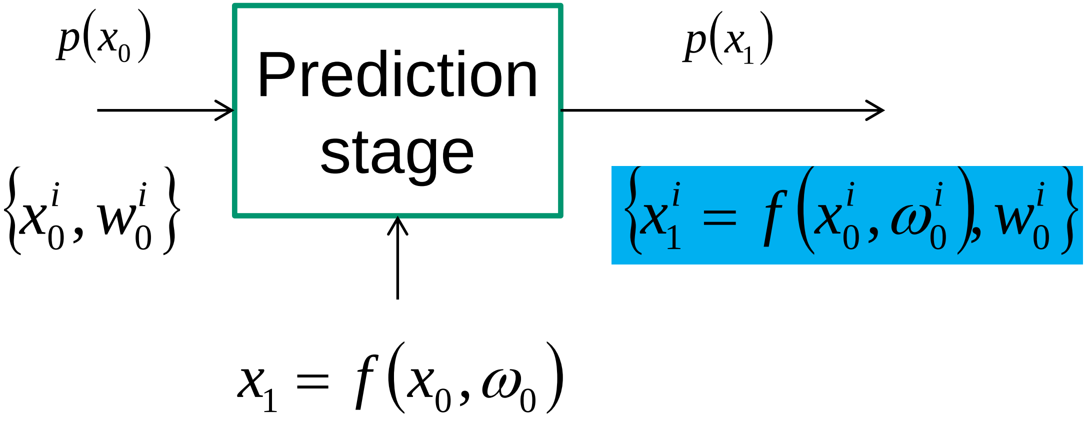
Prediction stage

$p(x_1)$

$\{x_1^i = f(x_0^i, \omega_0^i), w_0^i\}$

$$x_1 = f(x_0, \omega_0)$$

The intuitive representation: prediction stage: Monte Carlo Simulation



$p(x_0)$

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Prediction stage

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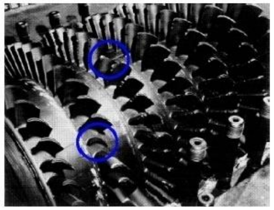


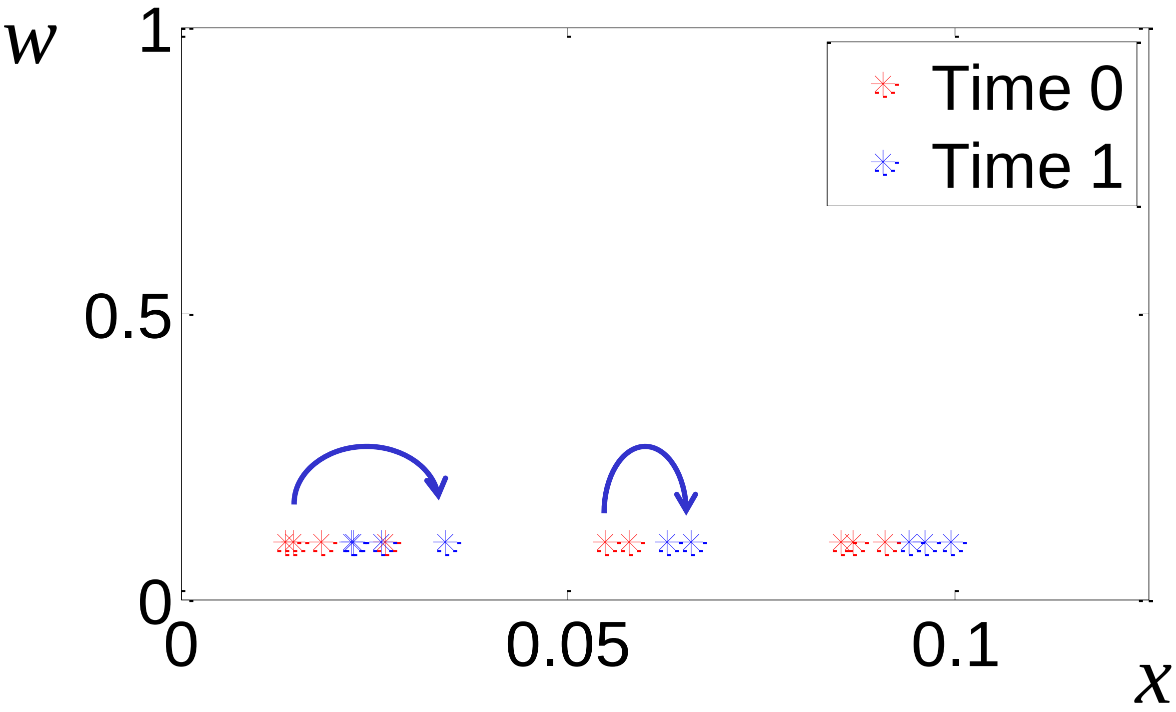
Prediction stage for particle *i*

1. Sample a value of $\omega_1^i, \omega_2^i, \omega_3^i$
2. Apply:

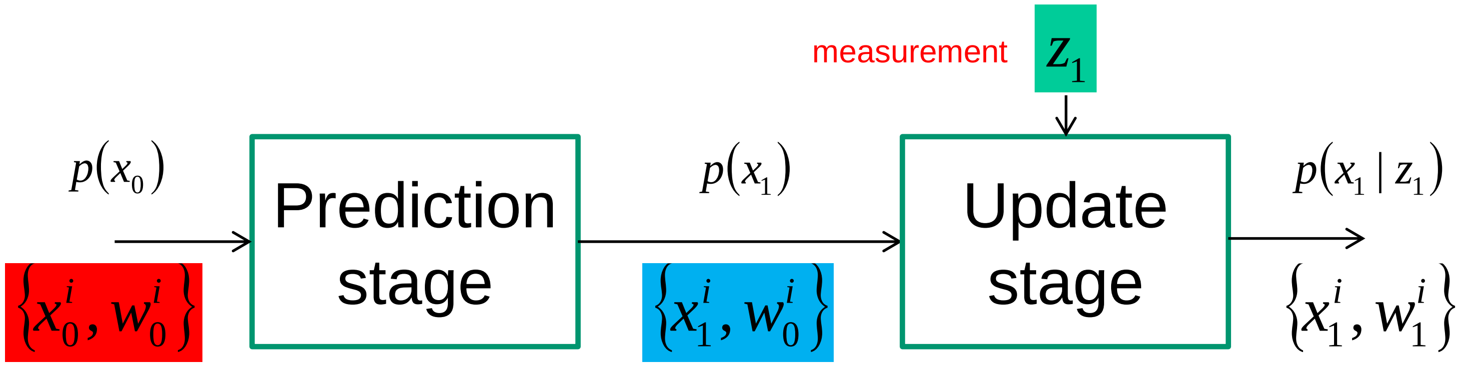
$$x_1^i = x_0^i + A\exp\left(-\frac{Q}{R(T_0+\omega_1^i)}\right)\left(K(\theta_0+\omega_2^i)^2+\omega_3^i\right)^n$$

The intuitive representation: prediction stage: Monte Carlo Simulation

$$x_1^i = x_0^i + A\exp\left(-\frac{Q}{R\left(T_0+\omega_1^i\right)}\right)\left(K\left(\theta_0+\omega_2^i\right)^2+\omega_3^i\right)^n$$



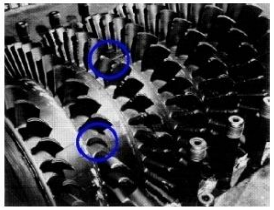
The intuitive representation: update stage: weight modification

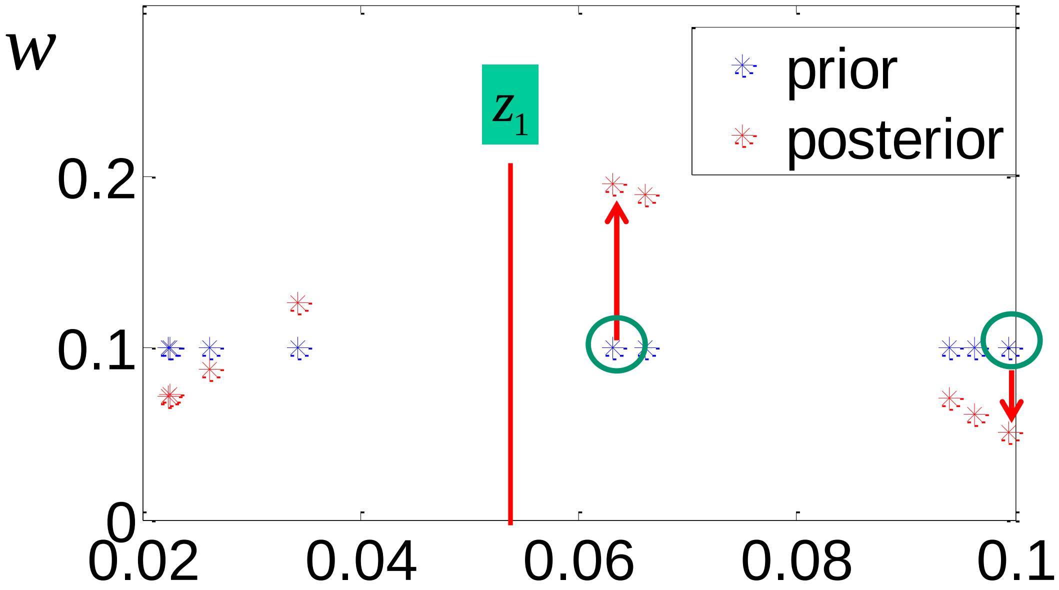


measurement z_1

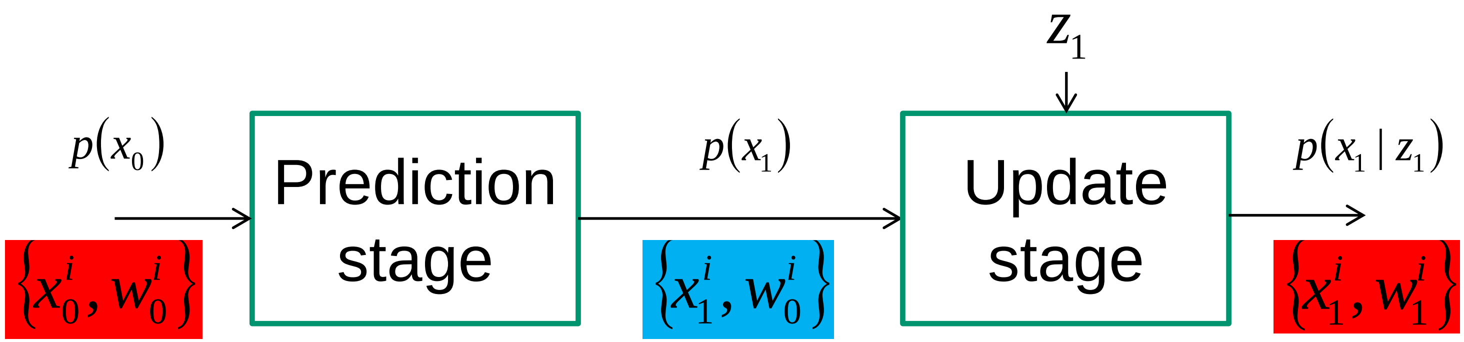
$p(x_0)$ $\{x_0^i, w_0^i\}$ → Prediction stage → $p(x_1)$ $\{x_1^i, w_0^i\}$ → Update stage → $p(x_1 | z_1)$ $\{x_1^i, w_1^i\}$

- Time 1: measure $z_1 = 0.058$ becomes available → particle weights' update





The intuitive representation: update stage: weight modification



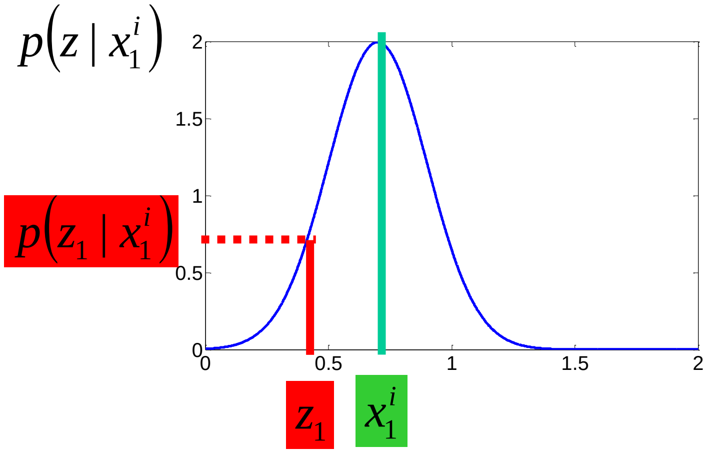
- Time 1: measure z_1 becomes available

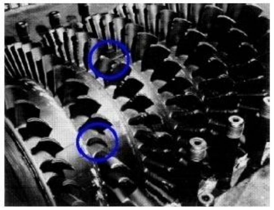
- Compute likelihood of the particles: $p(z_1 | x_1^i)$

$$z = x_1^i + \nu$$

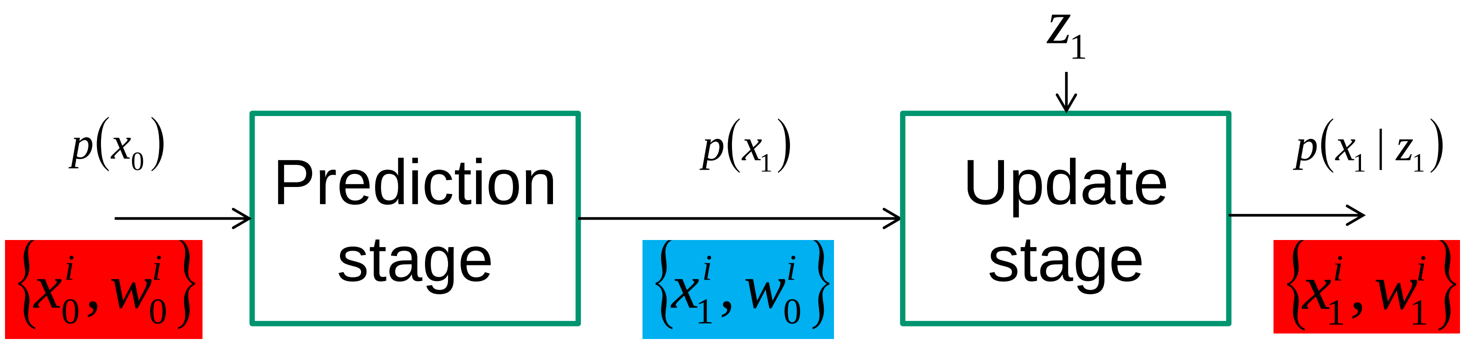
$$\nu \propto N(0, \sigma^2)$$

$$z \propto N(x_1^i, \sigma^2)$$





The intuitive representation:
update stage: weight modification



$p(x_0)$ $\{x_0^i, w_0^i\}$

Prediction stage

$p(x_1)$ $\{x_1^i, w_0^i\}$

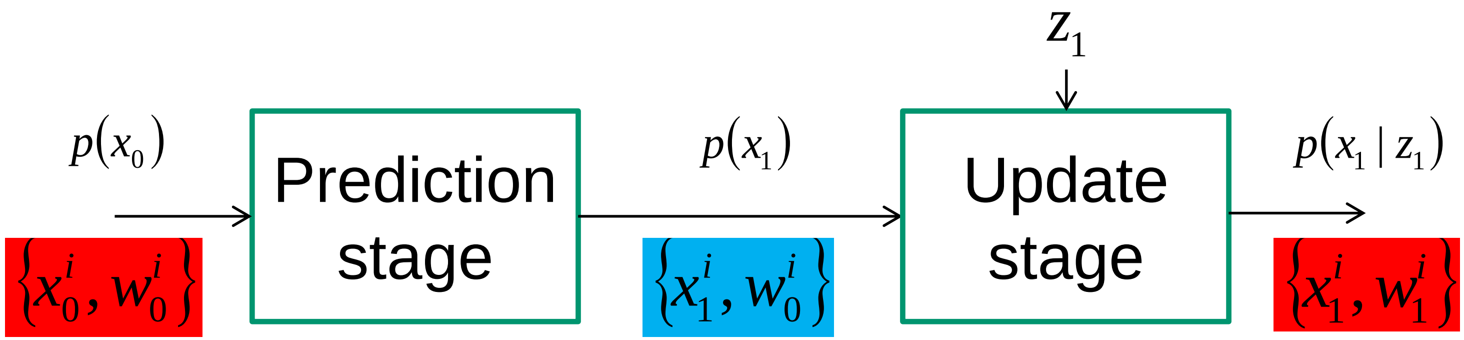
z_1

Update stage

$p(x_1 | z_1)$ $\{x_1^i, w_1^i\}$

- Time 1: measure z_1 becomes available
- Compute likelihood of the particles: $p(z_1 | x_1^i)$
- $\tilde{w}_1^i = w_0^i \cdot p(z_1 | x_1^i)$

The intuitive representation: update stage: weight modification



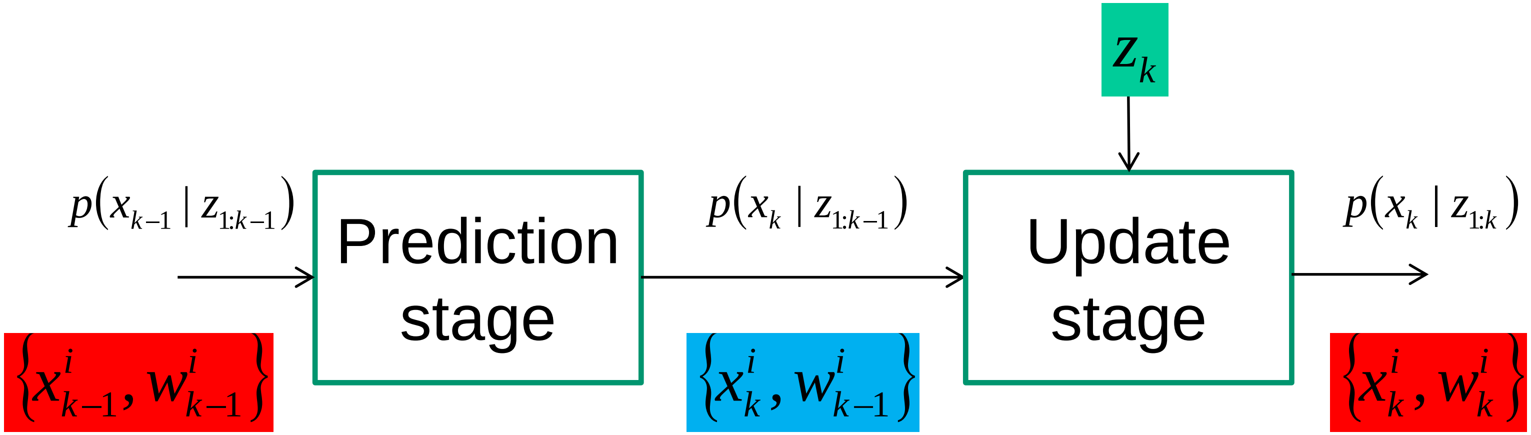
- Time 1: measure z_1 becomes available

- Compute likelihood of the particles: $p\left(z_1 \mid x_1^i\right)$

- $\tilde{w}_1^i = w_0^i \cdot p\left(z_1 \mid x_1^i\right)$ ▶ $w_1^i = \dfrac{\tilde{w}_1^i}{\sum_{i=1}^{N} \tilde{w}_1^i}$

The intuitive representation

- Repeat prediction and update stage each time a new measure becomes available



z_k

$p(x_{k-1} \mid z_{1:k-1})$

$\{x_{k-1}^i, w_{k-1}^i\}$

Prediction stage

$p(x_k \mid z_{1:k-1})$

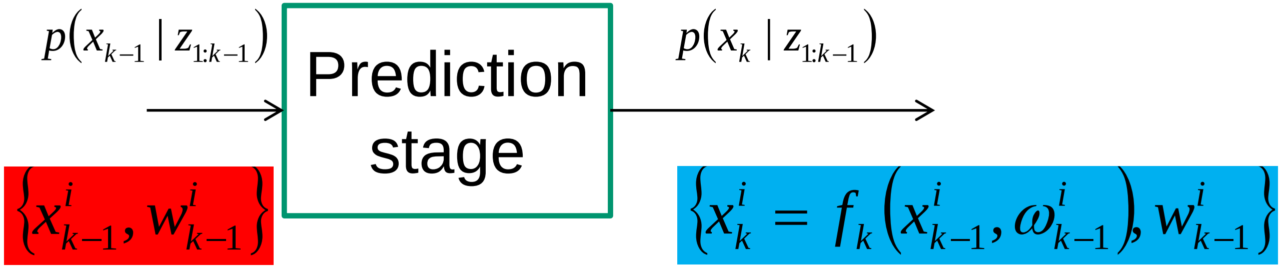
$\{x_k^i, w_{k-1}^i\}$

Update stage

$p(x_k \mid z_{1:k})$

$\{x_k^i, w_k^i\}$

The intuitive representation:
prediction stage: Monte Carlo Simulation



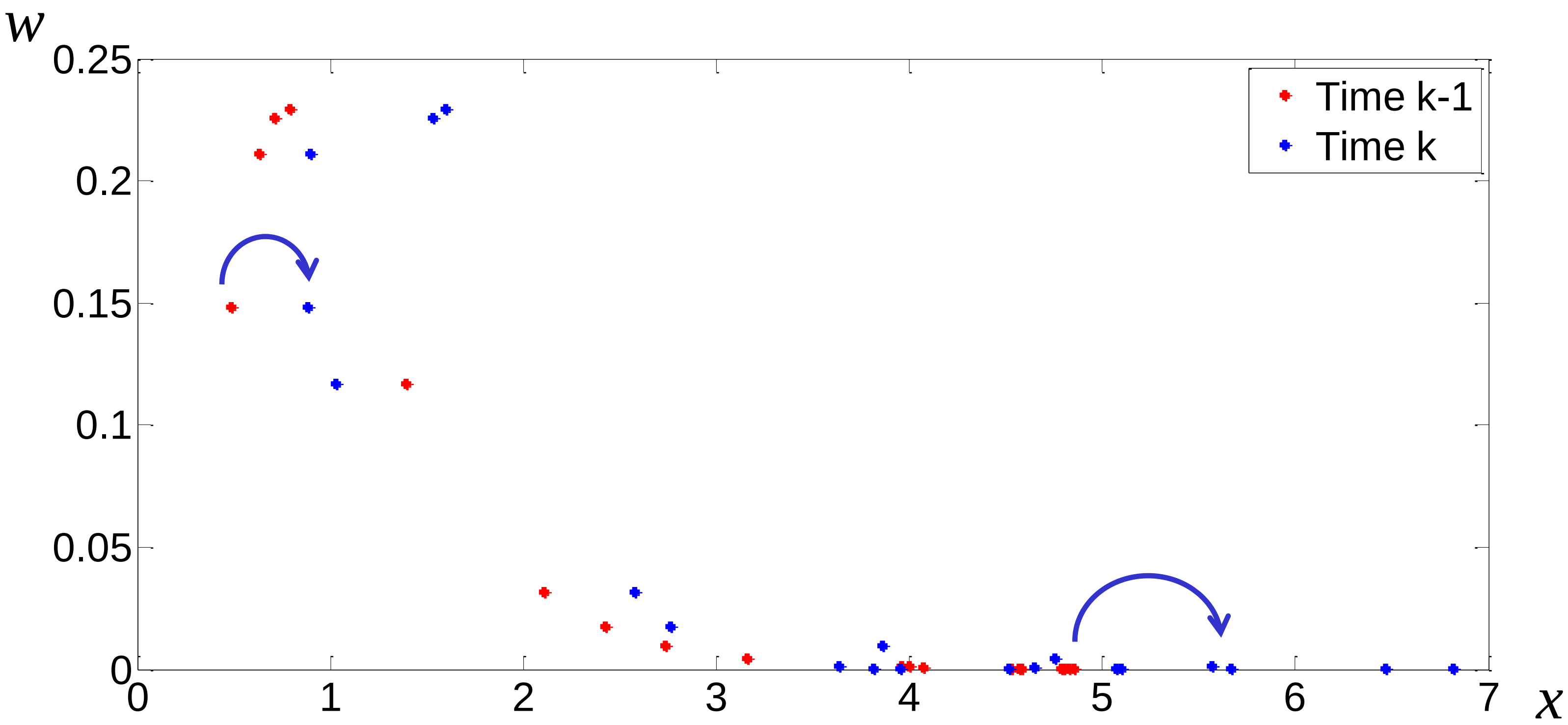
$p\left(x_{k-1} \mid z_{1:k-1}\right)$

Prediction stage

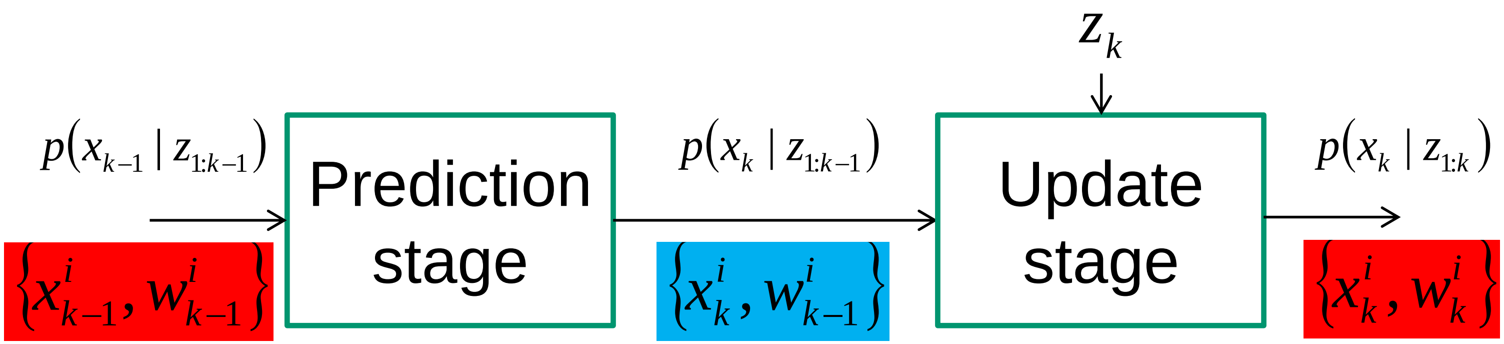
$p\left(x_k \mid z_{1:k-1}\right)$

$\left\{x_{k-1}^{i}, w_{k-1}^{i}\right\}$

$\left\{x_k^i = f_k\left(x_{k-1}^i, \omega_{k-1}^i\right), w_{k-1}^i\right\}$



The intuitive representation: update stage: weight modification



z_k

$p(x_{k-1} \mid z_{1:k-1})$

$\{x_{k-1}^i, w_{k-1}^i\}$

Prediction stage

$p(x_k \mid z_{1:k-1})$

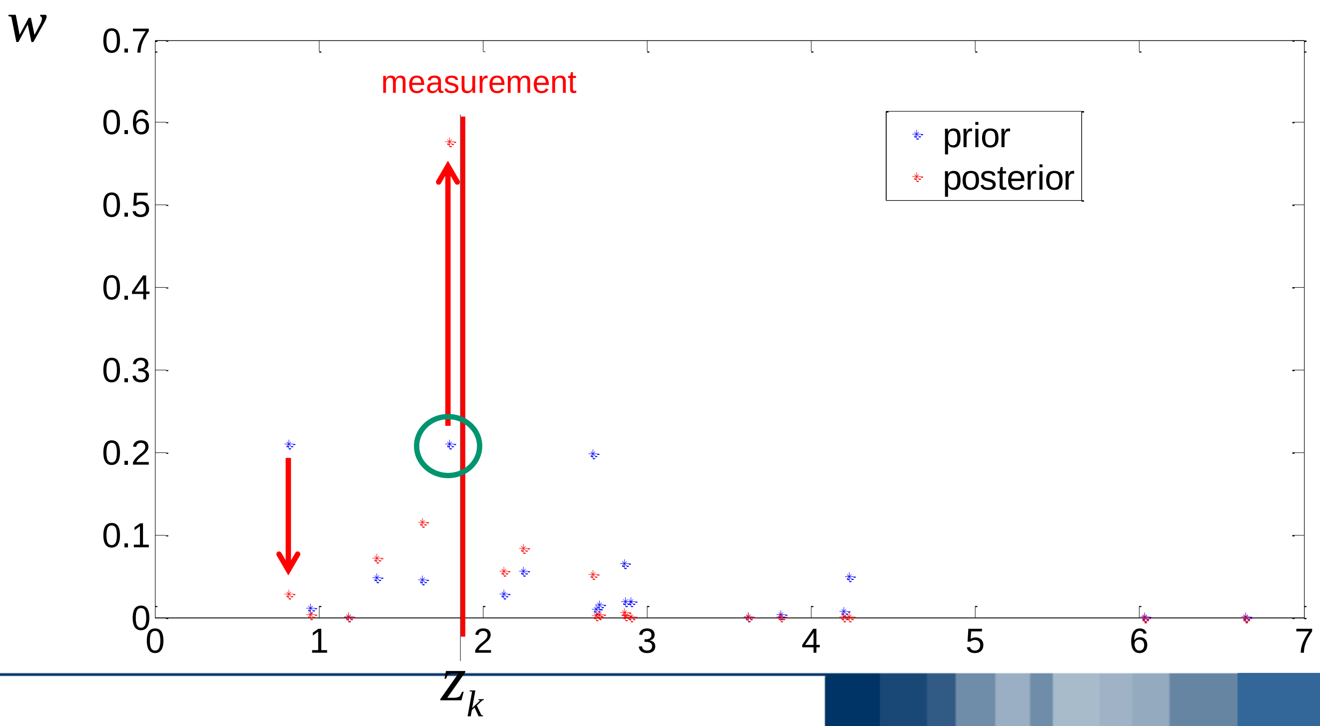
$\{x_k^i, w_{k-1}^i\}$

Update stage

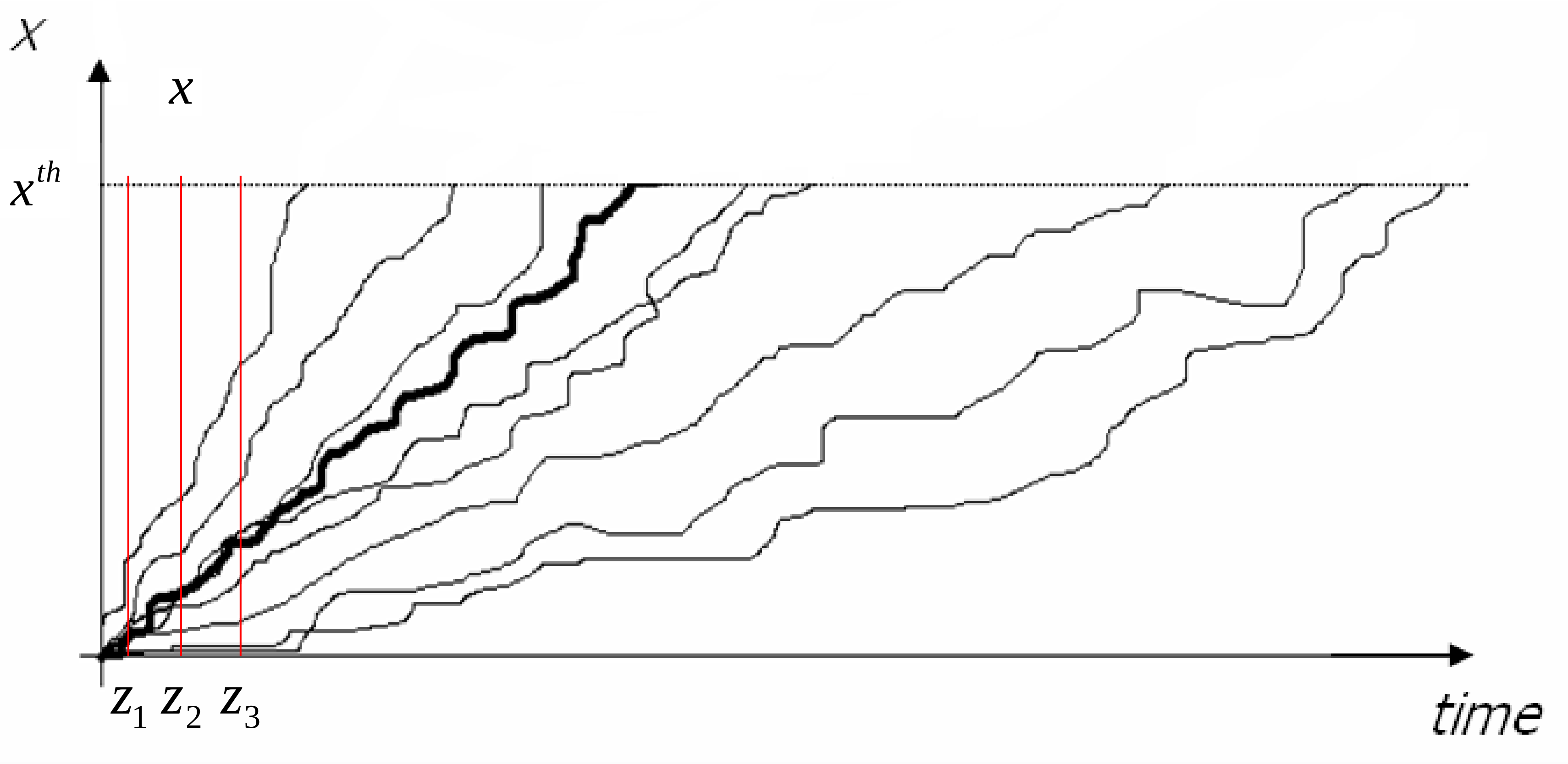
$p(x_k \mid z_{1:k})$

$\{x_k^i, w_k^i\}$

- Time *k*: measure z_k becomes available → particle weight modification



Example of Particle Trajectories



Estimates of interest

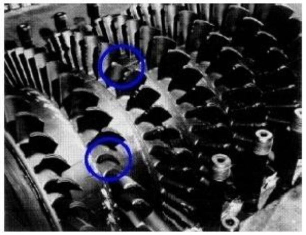
$$\left\{x_k^i, w_k^i\right\} \Longleftrightarrow p\left(x_k \mid z_{1:k}\right)$$

▼

- degradation state **mean** (estimate) $$\hat{x}_k = \sum_{i=1}^{N_s} w_k^i x_k^i$$
- degradation state **variance** (uncertainty) $$\hat{\sigma}_k^2 = \sum_{i=1}^{N_s} w_k^i \left(x_k^i - \hat{x}_k\right)^2$$

- Particle filtering for degradation state estimate
    - The intuitive representation
    - State estimate in practice
    - Detailed analytical approach to the problem
    - The pseudocode

Degradation state estimate in practice



$$x_k = x_{k-1} + A\exp\left(-\frac{Q}{R(T_0+\omega_1)}\right)\left(K(\theta_0+\omega_2)^2\right)^n$$

n=6

A= 7.5e^{-3} %/(MPa^{n}*day)

Q: Activation energy = 290000 J/mol

R: Ideal gas constant = 8.31 J/(mol*K)

K=0. 0011 MPa

T_0 = 1100 K

θ_0 = 3000 rpm

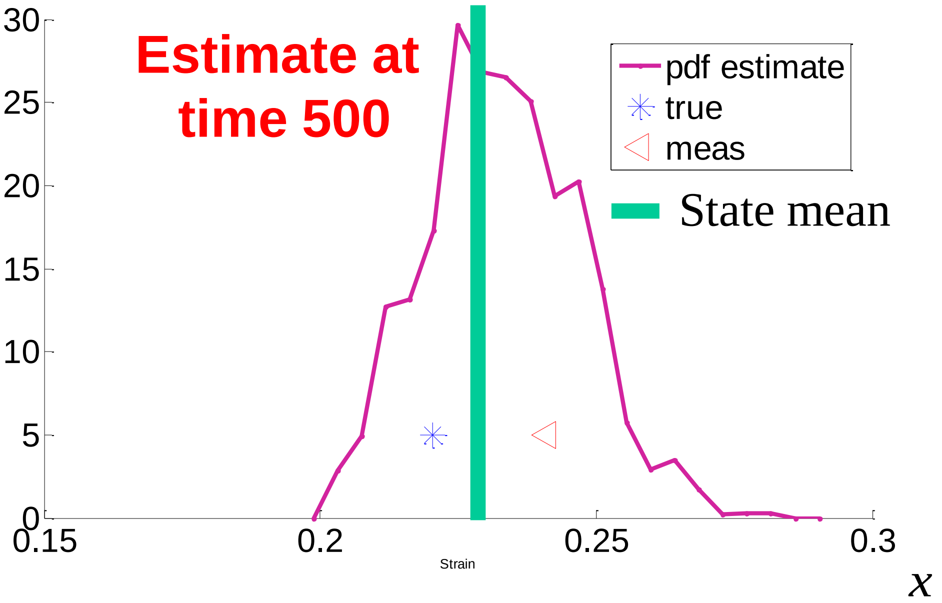
ω_1~ N(0; 11) K

ω_2~ N(0; 30) rpm

Initial Condition: Time *t*=0 → $x_0 = 0$

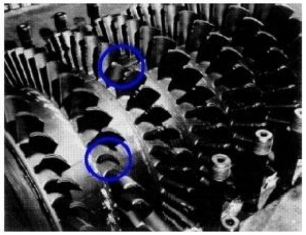
Number of Particles: N_p =1000

Time	**Elongation Measure**
500	0.2411%



Estimate at time 500

Degradation state estimate in practice

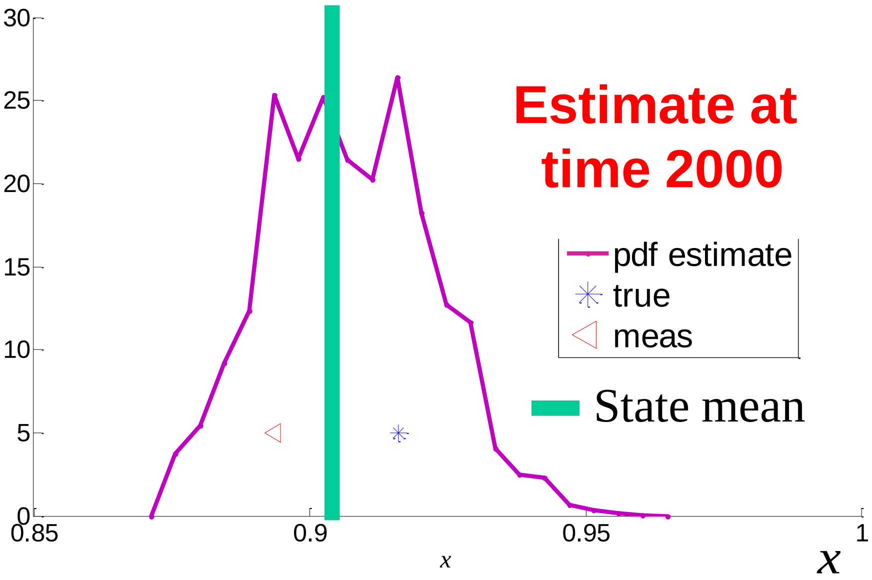


$$x_k = x_{k-1} + A\exp\left(-\frac{Q}{R\left(T_0 + \omega_1\right)} \right)\left(K\left(\theta_0 + \omega_2\right)^2 \right)^n$$

n=6
A= 7.5e^{-3} %/(MPa^{n}*day)
Q: Activation energy = 290000 J/mol
R: Ideal gas constant = 8.31 J/(mol*K)
K=0. 0011 MPa
T_0 = 1100 K
θ_0 = 3000 rpm
ω_1~ N(0; 11) K
ω_2~ N(0; 30) rpm

Initial Condition: Time t=0 → $x_0 = 0$
Number of Particles: N_p =1000

Time	Elongation Measure
500	0.2411%
1000	0,4600%
1500	0,7129%
2000	0,8938%



Estimate at time 2000

- Particle filtering for degradation state estimate
  - The intuitive representation
  - State estimate in practice
  - Detailed analytical approach to the problem
  - The pseudocode

Basic Idea: Importance sampling

OBJECTIVE: $p\left(x_{0:k} \mid z_{1:k}\right)$

MAIN IDEA: IMPORTANCE SAMPLING

$$p\left(x_{0:k} \mid z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{0:k} - x_{0:k}^i\right)$$

Importance sampling

- Let $p(x) \propto \pi(x)$ be a probability density function (pdf) difficult to sample from, with $\pi(x)$ easy to evaluate
- Let $q(x)$ be a proposal pdf easy to sample from: $\left\{x^{i}\right\}_{i=1:N_S}$

Importance density

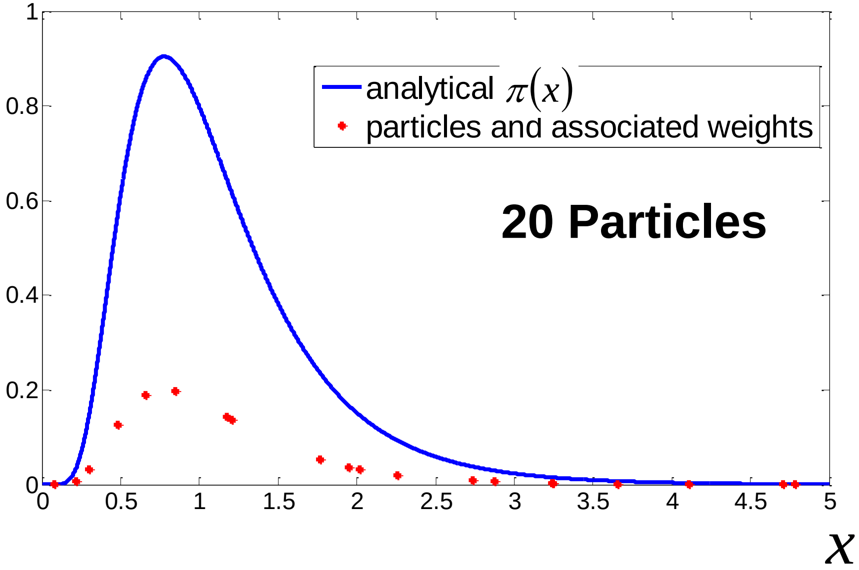
$$p(x) \approx \sum_{i=1}^{N_s} w^i \delta\left(x - x^i\right)$$

where:

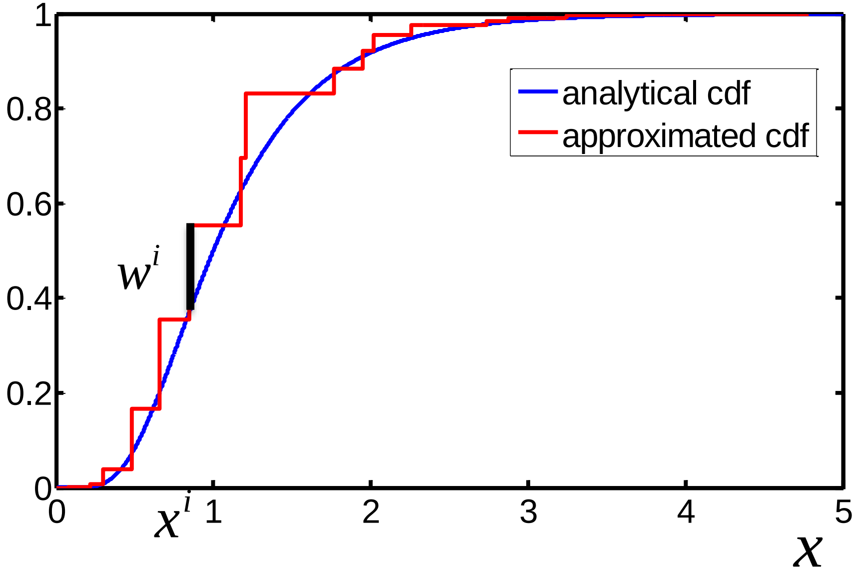
$$\tilde{w}^i = \frac{\pi\left(x^i\right)}{q\left(x^i\right)}$$

$$w^i = \frac{\tilde{w}^i}{\sum_{i=1,N_S} \tilde{w}^i}$$

Example: approximation of the pdf distribution



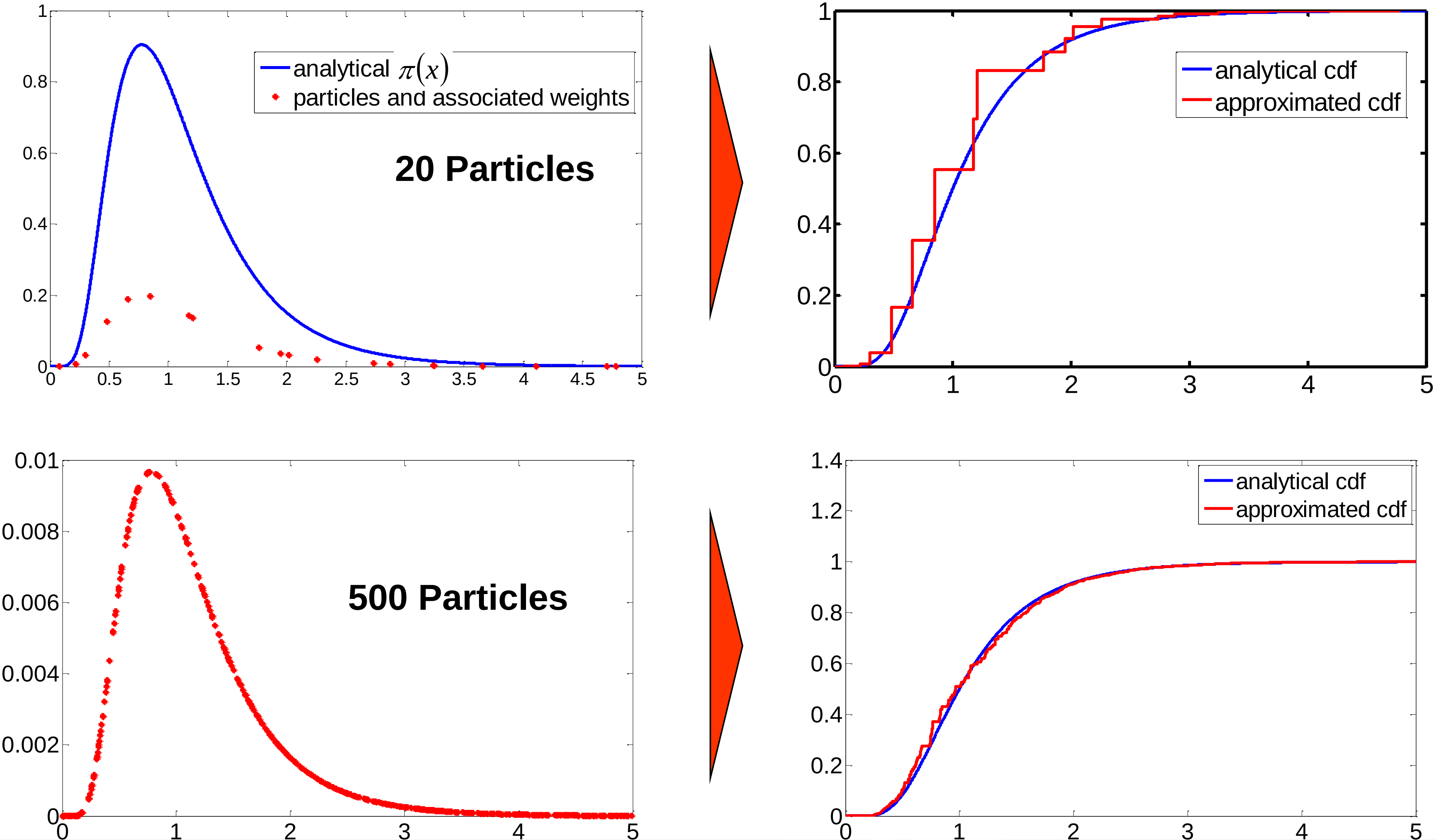
$$\int_{-\infty}^{x} p(\xi)d\xi$$



- Particles sampled from: $q(x)=U[0,5]$
- Corresponding weight obtained from:

$$\tilde{w}^i = \frac{\pi\left(x^i\right)}{q\left(x^i\right)} = \frac{\pi\left(x^i\right)}{1/5}$$

Example: approximation of the pdf distribution



Particle Filter: Estimate of the posterior

$$p\left(x_{0:k} \mid z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{0:k} - x_{0:k}^i\right)$$

Arbitrarily chosen

↓

In practice:

- Sample N_s particles from $q\left(x_{0:k} \mid z_{1:k}\right)$
- Compute weights from:

$$\boxed{w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)}}$$

Sequential Importance Sampling

Arbitrarily chosen

↓

$$q\left(x_{0:k} \mid z_{1:k}\right) = q\left(x_k \mid x_{0:k-1}, z_{1:k}\right) q\left(x_{0:k-1} \mid z_{1:k-1}\right)$$

↑

Known from previous time step

Sample at time *k*-1: $x_0^i, x_1^i, \dots, x_{k-1}^i$

▼

Sample at time *k*: $x_0^i, x_1^i, \dots, x_{k-1}^i, x_k^i$

↓

from $q\left(x_k \mid x_{0:k-1}, z_{1:k}\right)$

Particle Filter: Estimate of the posterior

$$p\left(x_{0:k} \mid z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{0:k} - x_{0:k}^i\right)$$

In practice:

- Sample N_s particles from $q\left(x_{0:k} \mid z_{1:k}\right) = q\left(x_k \mid x_{0:k-1}, z_{1:k}\right) q\left(x_{0:k-1} \mid z_{1:k-1}\right)$
- Compute weights from:

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)} = \frac{\boxed{p\left(x_{0:k}^i \mid z_{1:k}\right)}^{\;\;\textbf{?}}}{q\left(x_k^i \mid x_{0:k-1}^i, z_{1:k}\right) q\left(x_{0:k-1}^i \mid z_{1:k-1}\right)}$$

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$p\left(x_{0:k} \mid z_{1:k}\right)=\frac{p\left(x_{0:k} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const}$$

- Bayes Rule

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

(conditional probability formula)

$$P(A,B) = P(A \mid B)P(B)$$

$$p\left(x_k, x_{0:k-1} \mid z_{1:k-1}\right) = p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)$$

$$p\left(x_{0:k} \mid z_{1:k}\right) = \frac{p\left(x_{0:k} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const}$$

$$= \frac{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const}$$

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$p\left(x_{0:k} \mid z_{1:k}\right) = \frac{p\left(x_{0:k} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const}$$

$$= \frac{\boxed{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)} p\left(x_{0:k-1} \mid z_{1:k-1}\right) \boxed{p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}}{const}$$

$$= \frac{\boxed{p\left(x_k \mid x_{0:k-1}\right)} p\left(x_{0:k-1} \mid z_{1:k-1}\right) \boxed{p\left(z_k \mid x_{0:k}\right)}}{const}$$

↓ (observational independence)

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$
\begin{aligned}
p\left(x_{0:k} \mid z_{1:k}\right) &= \frac{p\left(x_{0:k} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const} \\
&= \frac{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right) p\left(x_{0:k-1} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const} \\
&= \frac{p\left(x_k \mid x_{0:k-1}\right) p\left(x_{0:k-1} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}\right)}{const} \\
&= p\left(x_{0:k-1} \mid z_{1:k-1}\right) \frac{p\left(z_k \mid x_{0:k}\right) p\left(x_k \mid x_{0:k-1}\right)}{const}
\end{aligned}
$$

↓ Rearrangement

Recursive formula for $p\left(x^{i}_{0:k} \mid z_{1:k}\right)$

$$
\begin{aligned}
p\left(x_{0:k} \mid z_{1:k}\right) &= \frac{p\left(x_{0:k} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const} \\
&= \frac{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{const} \\
&= \frac{p\left(x_k \mid x_{0:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}\right)}{const} \\
&= p\left(x_{0:k-1} \mid z_{1:k-1}\right)\frac{\boxed{p\left(z_k \mid x_{0:k}\right)}\boxed{p\left(x_k \mid x_{0:k-1}\right)}}{const} \\
&= p\left(x_{0:k-1} \mid z_{1:k-1}\right)\frac{\boxed{p\left(z_k \mid x_k\right)}\boxed{p\left(x_k \mid x_{k-1}\right)}}{const}
\end{aligned}
$$

↓ (Markov model)

▼

$$
p\left(x_{0:k} \mid z_{1:k}\right) \propto p\left(z_k \mid x_k\right)p\left(x_k \mid x_{k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)
$$

Weight updating equation – Sequential Importance Sampling (SIS)

- Where were we?
  - SLIDE 62:

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)}$$

$$q\left(x_{0:k} \mid z_{1:k}\right) = q\left(x_k \mid x_{0:k-1}, z_{1:k}\right) q\left(x_{0:k-1} \mid z_{1:k-1}\right)$$

  - SLIDE 67:

$$p\left(x_{0:k} \mid z_{1:k}\right) \propto p\left(z_k \mid x_k\right) p\left(x_k \mid x_{k-1}\right) p\left(x_{0:k-1} \mid z_{1:k-1}\right)$$

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)} \propto \frac{p\left(z_k \mid x_k^i\right) p\left(x_k^i \mid x_{k-1}^i\right) p\left(x_{0:k-1}^i \mid z_{1:k-1}\right)}{q\left(x_k^i \mid x_{0:k-1}^i, z_{1:k}\right) q\left(x_{0:k-1}^i \mid z_{1:k-1}\right)} = \frac{p\left(z_k \mid x_k^i\right) p\left(x_k^i \mid x_{k-1}^i\right)}{q\left(x_k^i \mid x_{0:k-1}^i, z_{1:k}\right)} w_{k-1}^i$$

A possible choice for $q\left(x_k \mid x_{0:k-1}, z_{1:k}\right)$

MOST POPULAR CHOICE

$$\boxed{q\left(x_k \mid x_{0:k-1}, z_{1:k}\right) = p\left(x_k \mid x_{k-1}\right)}$$

Easy! We know the Physical model of the degradation process

$$\Downarrow$$

$$\tilde{w}_k^i = w_{k-1}^i \frac{p\left(z_k \mid x_k^i\right)\cancel{p\left(x_k^i \mid x_{k-1}^i\right)}}{\cancel{q\left(x_k^i \mid x_{0:k-1}^i, z_k\right)}} = \enclose{circle}{w_{k-1}^i}\, p\left(z_k \mid x_k^i\right)$$

Easy! We know the measurement equation

Advantage:

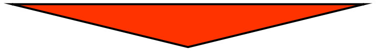
- ➢ easy to implement (both sampling and evaluation of weights)

Drawbacks:

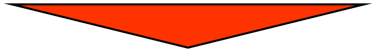
- ➢ state-space explored without knowledge of observations
- ➢ degeneracy phenomenon

SIS: degeneracy problem

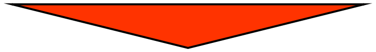
Variance of the weights can only increase over time: $w_k^i = w_{k-1}^i p\left(z_k \mid x_k^i\right)$



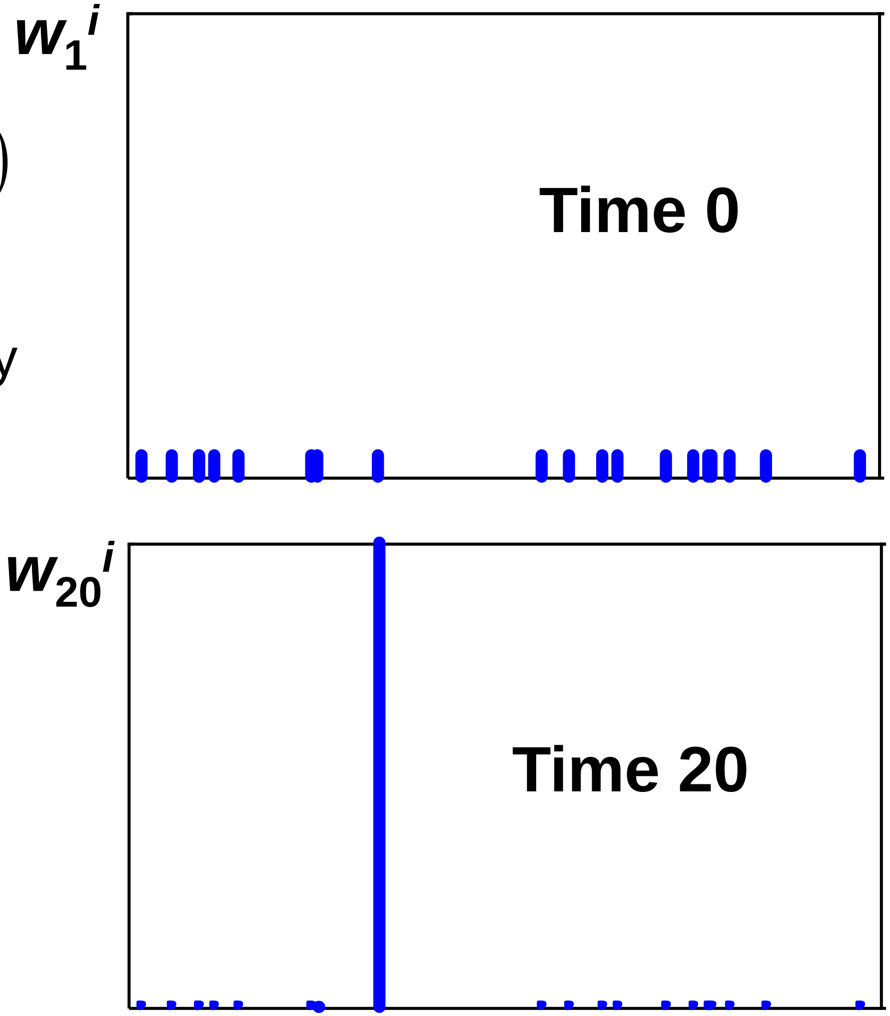
Weight distribution becomes progressively more skewed



Large effort in updating particles whose contribution to final estimate is almost 0



Resampling Algorithm

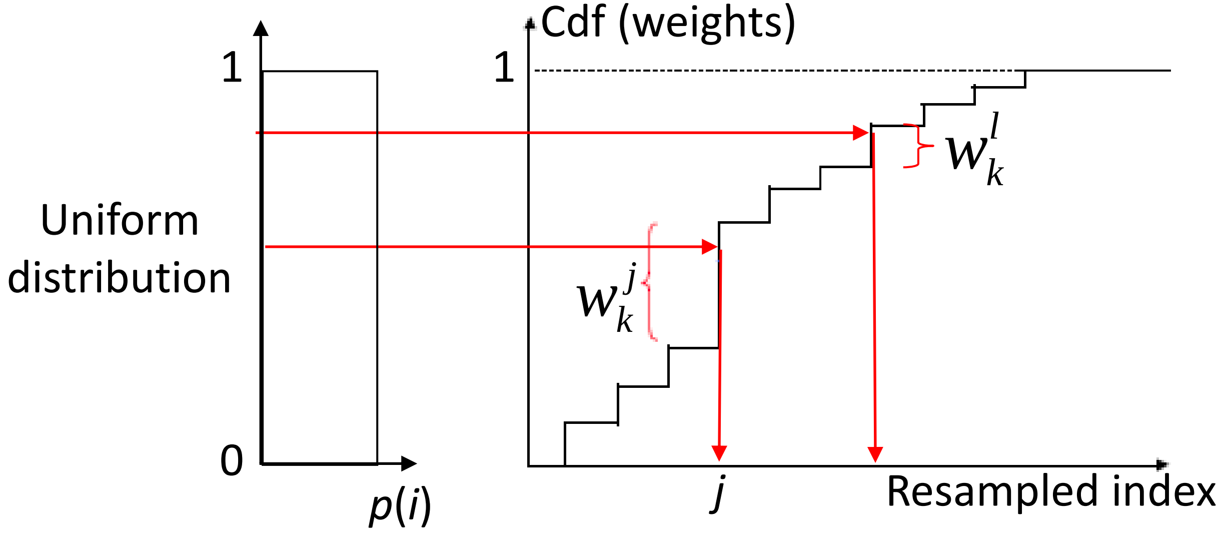


Bootstrap resampling procedure

- Reduce number of samples with low weights and increase number of samples with large weights
- Set of unequally weighted samples → set of equally weighted particles

$$\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s} \rightarrow \left\{x_k^{j*}, 1/N_s\right\}_{j=1}^{N_s}$$

BOOTSTRAP RESAMPLING WITH REPLACEMENT



$$p\left(x_k^{j*} = x_k^i\right) = w_k^i$$

- Particle filtering for degradation state estimate
    - The intuitive representation
    - State estimate in practice
    - Detailed analytical approach to the problem
    - The pseudocode

Sampling Importance Resampling (SIR) PF

$$\left[\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s}\right] = \mathrm{SIR\text{-}PF}\left[\left\{x_{k-1}^i, w_{k-1}^i\right\}_{i=1}^{N_s}, z_k\right]$$

```
For $i$ = 1: $N_s$
    - Sample: $x_k^i$ using $x_{k-1}^i$ and $x_k = f_k\left(x_{k-1}, \omega_{k-1}\right)$
    - Assign the particles a weight: $\tilde{w}_k^i = w_{k-1}^i p\left(z_k \mid x_k^i\right)$
End For
For $i$ = 1: $N_s$
    - Normalize the weights: $w_k^i = \tilde{w}_k^i \Big/ \sum_{i=1}^{N_s} \tilde{w}_k^i$
End For
…
```

Sampling Importance Resampling (SIR) PF

- …
  - $\left[\left\{x_k^{j*}, w_k^{j*} = 1/N_s\right\}_{i=1}^{N_s}\right]$ = RESAMPLE $\left[\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s}\right]$
    - Bootstrap sample the system states (with replacement)
    - Update the weights: $w_k^{j*} = 1/N_s$

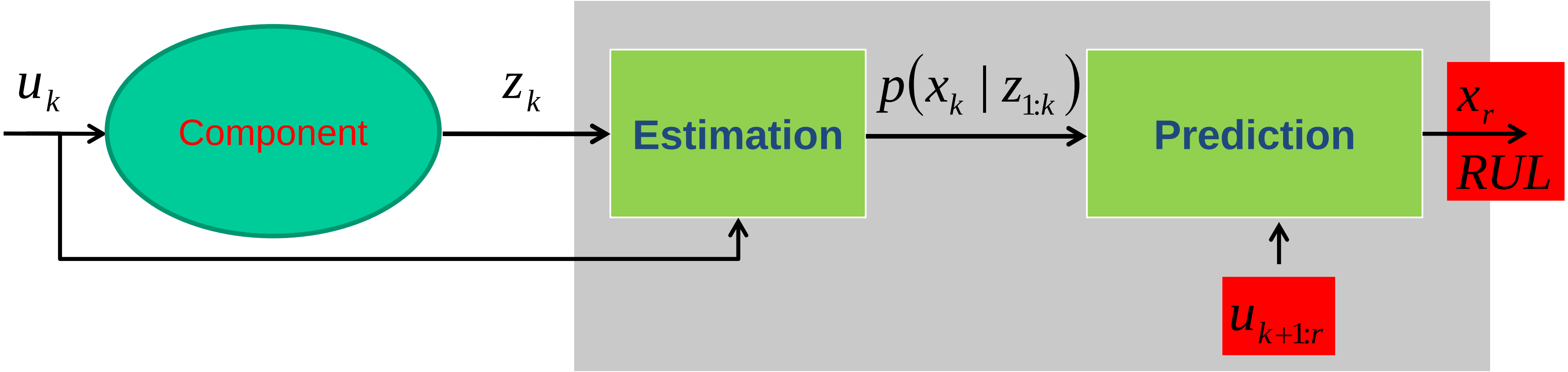
  - Compute estimates of interest:
    - Posterior mean: $\hat{x}_k = \sum_{i=1}^{N_s} w_k^i x_k^i$
    - Posterior variance: $\hat{\sigma}_k^2 = \sum_{i=1}^{N_s} w_k^i \left(x_k^i - \hat{x}_k\right)^2$

End SIR-PF

Outline

- Model-based prognostics
- Particle filtering for degradation state estimate
- Particle filtering for RUL estimate
- Applications
  - Maintenance planning
  - Prediction of the remaining useful life of electrolyte capacitors
  - Prediction of the remaining useful life of batteries

The forecasting problem



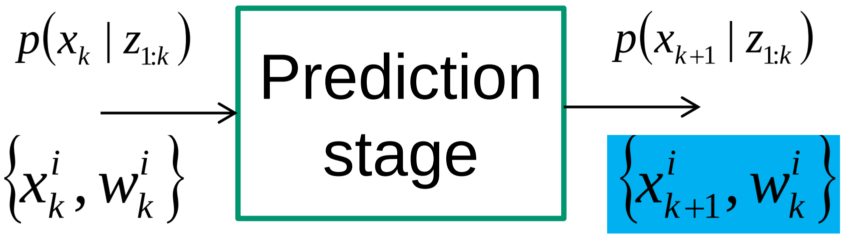
Information available:

- Estimate of the pdf of the state at the current time (from PF): $p(x_k \mid z_{1:k})$ in the form of $\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s}$
- future (random) distribution of the operational/external conditions: $p_r(u_r, \omega_r)$
- physical model of the degradation process $x_k = f_k(x_{k-1}, \omega_{k-1})$

- Estimate $p(x_r \mid z_{1:k})$
- Estimate RUL

RUL estimate: Method

- Prediction of the degradation state one time step ahead:



$p(x_k \mid z_{1:k})$

$\{x_k^i, w_k^i\}$

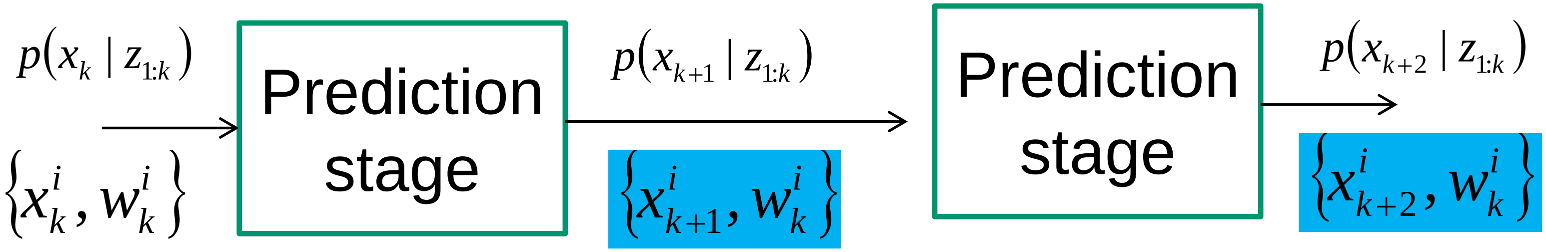
Prediction stage

$p(x_{k+1} \mid z_{1:k})$

$\{x_{k+1}^i, w_k^i\}$

$$x_{k+1}^i = f\left(x_k^i, \omega_k^i\right)$$

- Prediction of the degradation state 2 time steps ahead



$p(x_k \mid z_{1:k})$

$\{x_k^i, w_k^i\}$

Prediction stage

$p(x_{k+1} \mid z_{1:k})$

$\{x_{k+1}^i, w_k^i\}$

Prediction stage

$p(x_{k+2} \mid z_{1:k})$

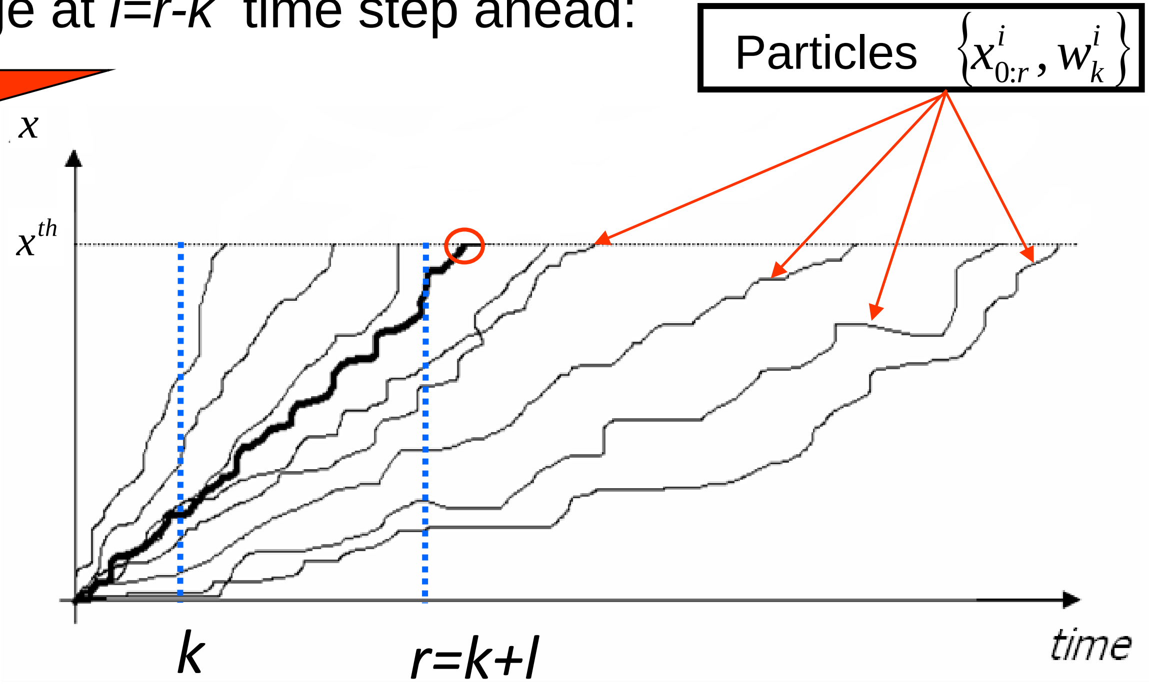
$\{x_{k+2}^i, w_k^i\}$

$$x_{k+1}^i = f\left(x_k^i, \omega_k^i\right)$$

$$x_{k+2}^i = f\left(x_{k+1}^i, \omega_{k+1}^i\right)$$

RUL estimate: Method

- Prediction stage at *l=r-k* time step ahead:

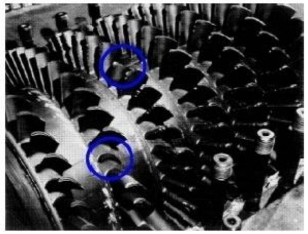


Particles $\left\{x_{0:r}^i, w_k^i\right\}$

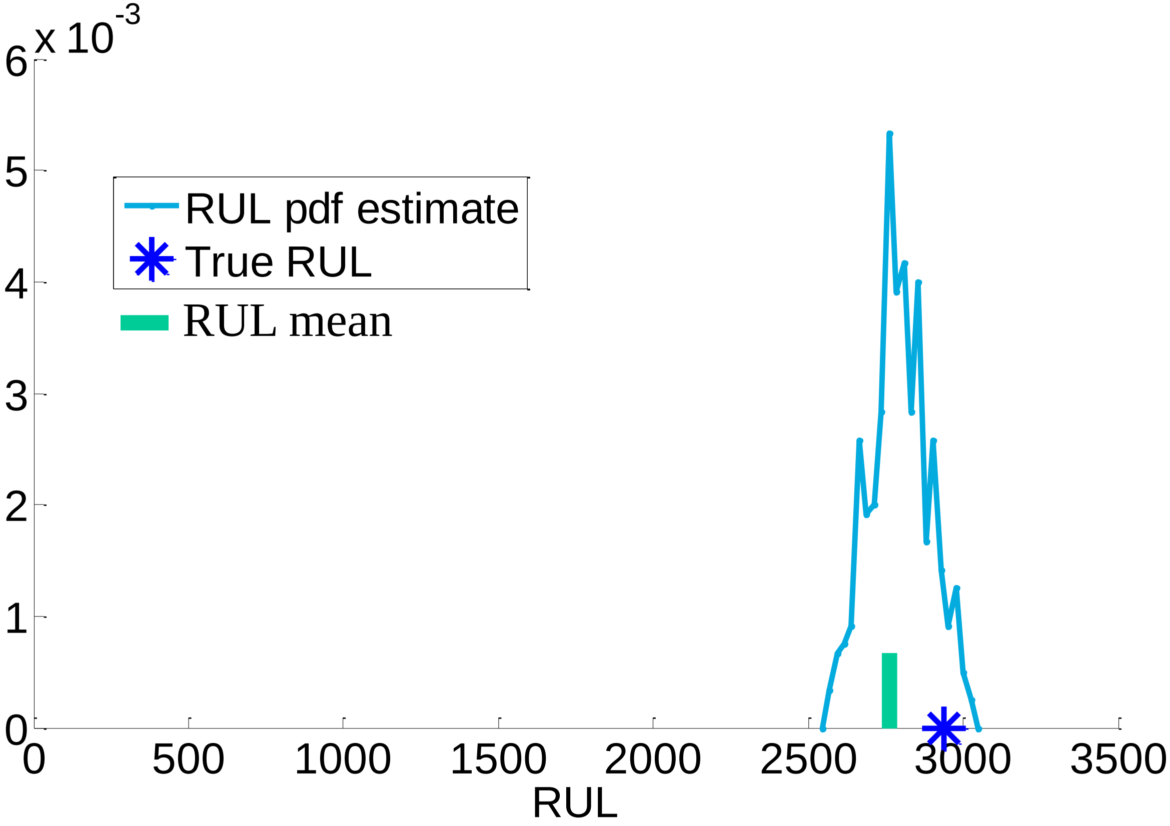
- RUL estimate

$$\hat{rul}_k = \sum_{i=1}^{N_s} w_k^i rul_k^i$$

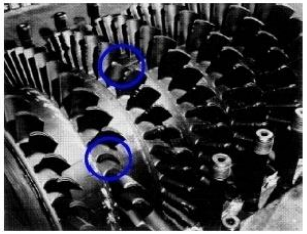
RUL estimate in practice



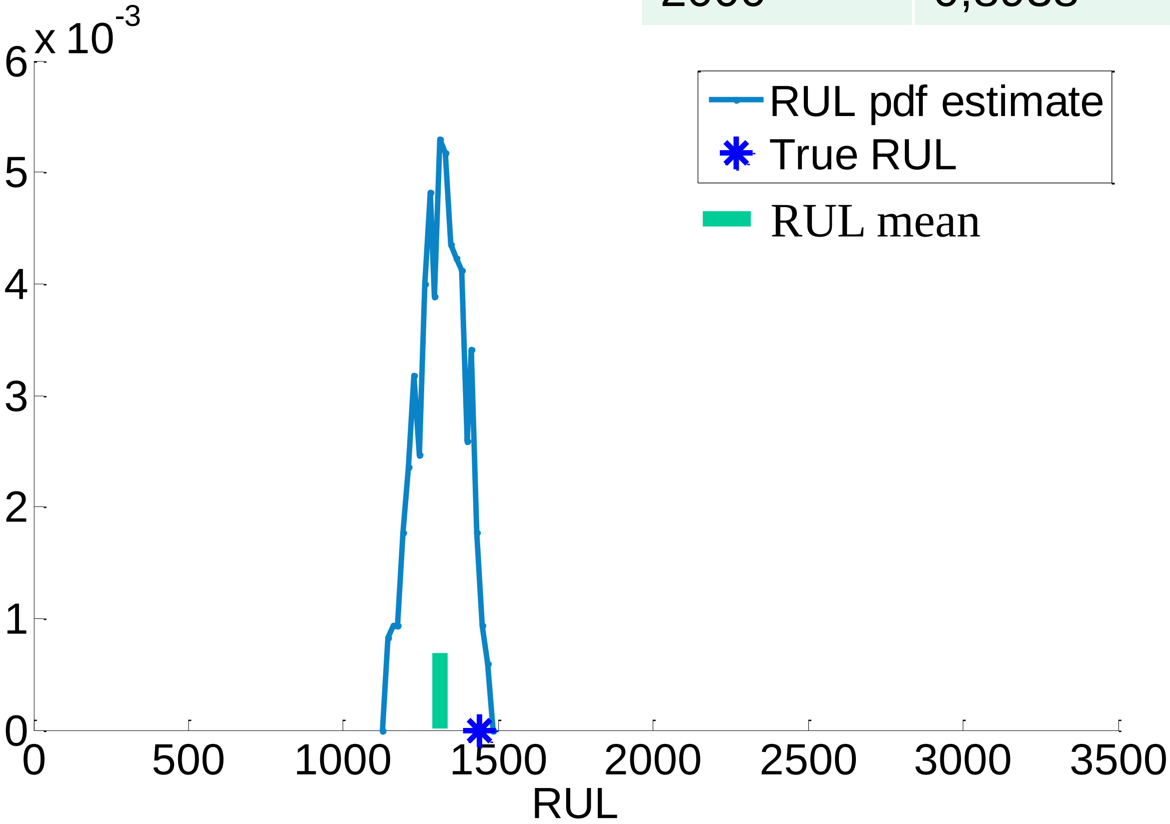
Time	Elongation Measure
500	0.2411%



RUL estimate in practice

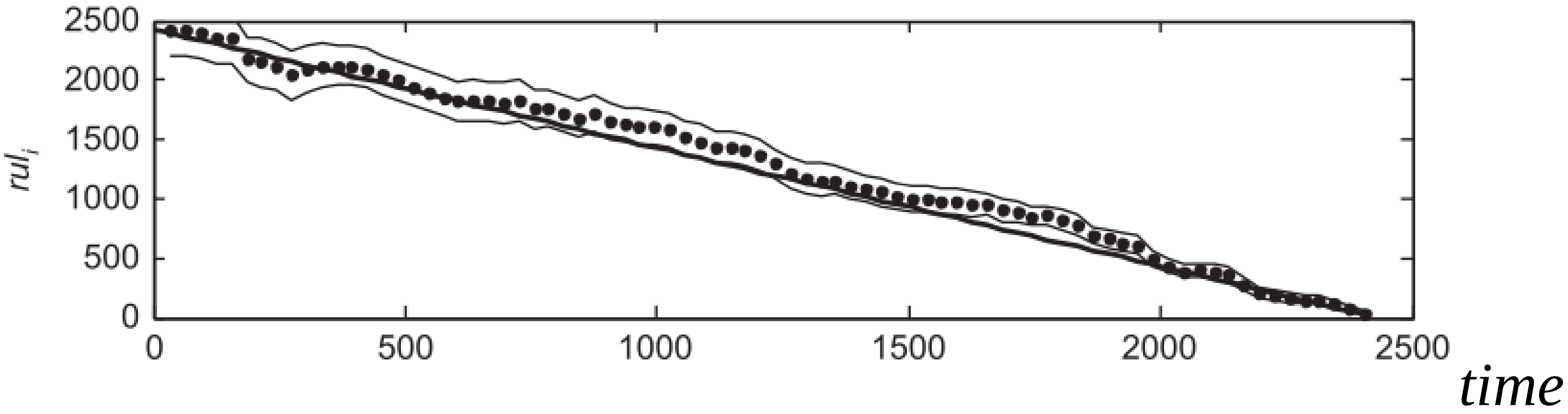


Time	Elongation Measure
500	0.2411%
1000	0,4600%
1500	0,7129
2000	0,8938



RUL estimate in practice: performance

- Another test case: one creep elongation measure every month



- Test over N_{tst} = 250 different creep growth trajectories
  - Mean Relative Absolute Error:

$$rMAE = \frac{1}{N_{tst}} \sum_{i=1}^{N_{tst}} \left| \frac{rul_i - r\hat{u}l_i}{rul_i} \right| = 0.150 \pm 0.009$$

  - Coverage:

$$Cov = \frac{1}{N_{tst}} \sum_{i=1}^{N_{tst}} c_i ; \quad c_i = \begin{cases} 1 & if \quad rul_i \in C_i^{68\%} \\ 0 & if \quad rul_i \notin C_i^{68\%} \end{cases} \qquad Cov = 0.663 \pm 0.018$$

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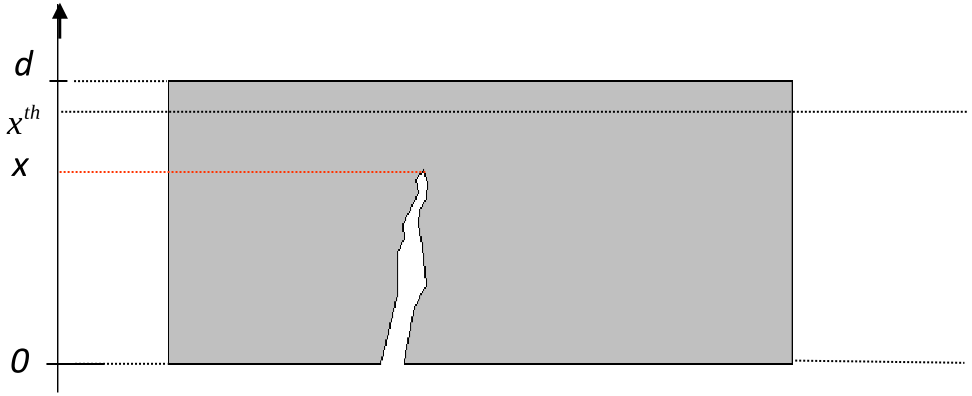
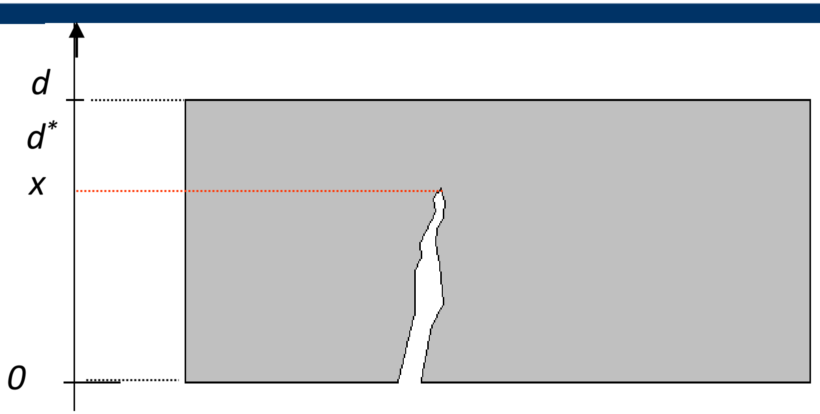
- Applications
  - Maintenance planning
  - Prediction of the remaining useful life of batteries

The degrading component

Component: structure
Degradation mechanism: crack propagation
Degradation Indicator: crack depth, x (not directly measurable)
Threshold of failure: x^{th}



Physical model of the degradation process



Paris-Erdogan model

$$\frac{dx}{dN} = e^{\omega} C\left(\beta\sqrt{x}\right)^{n} \longrightarrow$$

Discretization of the dynamics

$$\longrightarrow x_k = x_{k-1} + e^{\omega_{k-1}} C\left(\beta\sqrt{x_{k-1}}\right)^{n} \Delta N$$

- x = **hidden** degradation state (crack depth)
- ω = independent Gaussian **process noise**
- N = load cycle → time k
- C, β and n = constants related to the material properties

Measurement equation

$$z_k = d\left[1-\exp\left(\beta_0+\beta_1\ln\frac{x_k}{d-x_k}+\upsilon_k\right)\right]^{-1}$$

Logit model: non-destructive ultrasonic inspections

- z_k = degradation observation (vibration measurements)
- υ_k = independent non additive **measurement noise**
- β_0, β_1 = constants related to the material properties

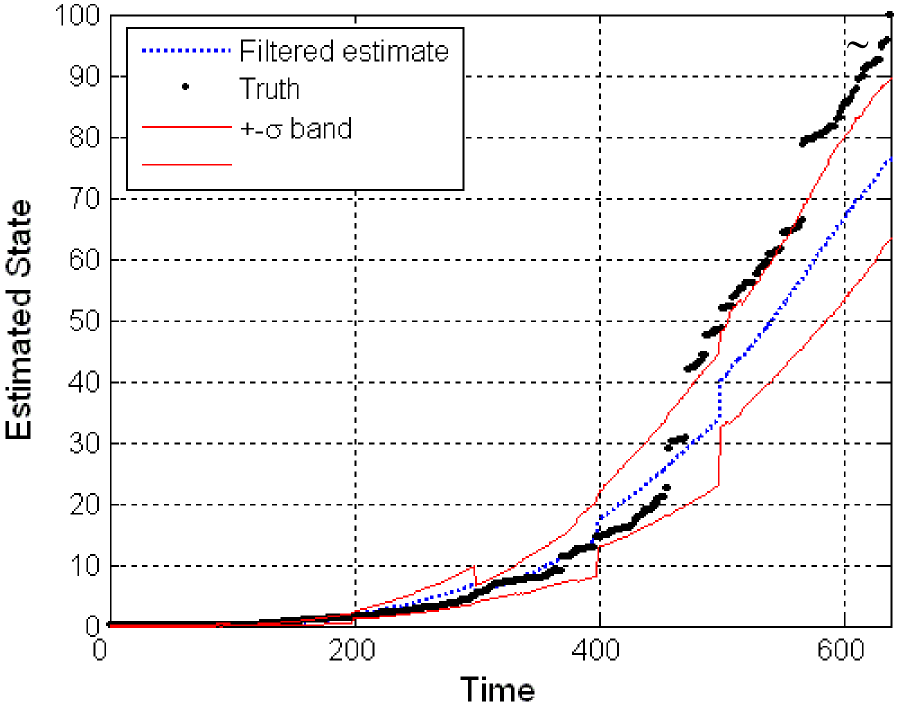
Objectives

- Degradation state (crack depth) estimate at the present time
- RUL prediction
- Maintenance planning

Crack growth evolution

- 5 measurements at: k_1 = 100; k_2 = 200; k_3 = 300; k_4 = 400; k_5 = 500
- 5000 particles

$$\boxed{p\left(x_k \middle| z_{0:k}\right)}$$



F. Cadini, E. Zio, D. Avram "*Monte Carlo-based filtering for fatigue crack growth estimation*", Probabilistic Engineering Mechanics, **24**, n. 3, pp. 367-373, 2009

Crack growth evolution

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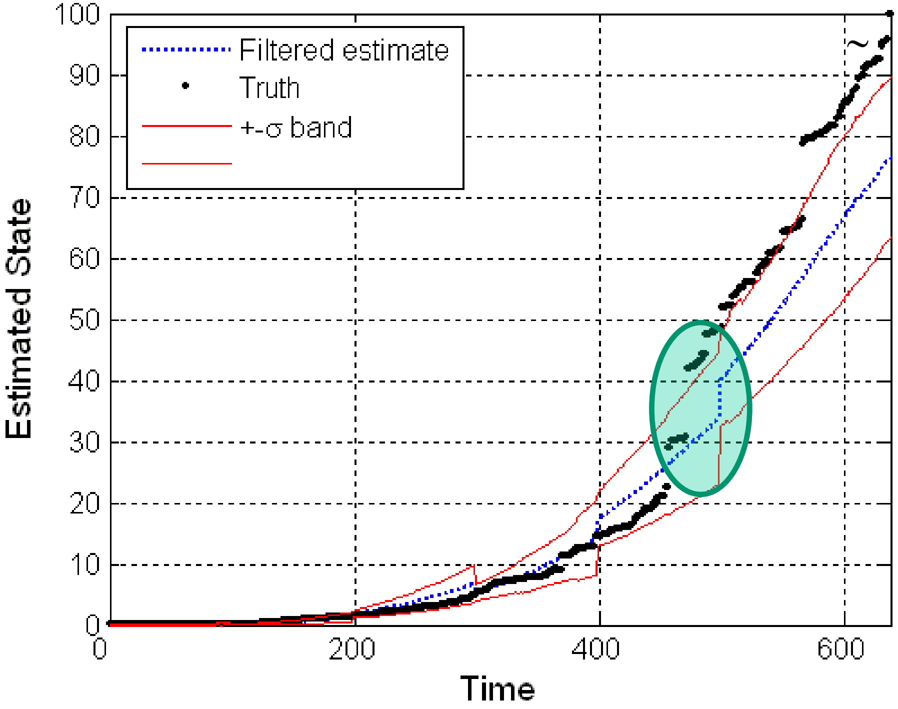
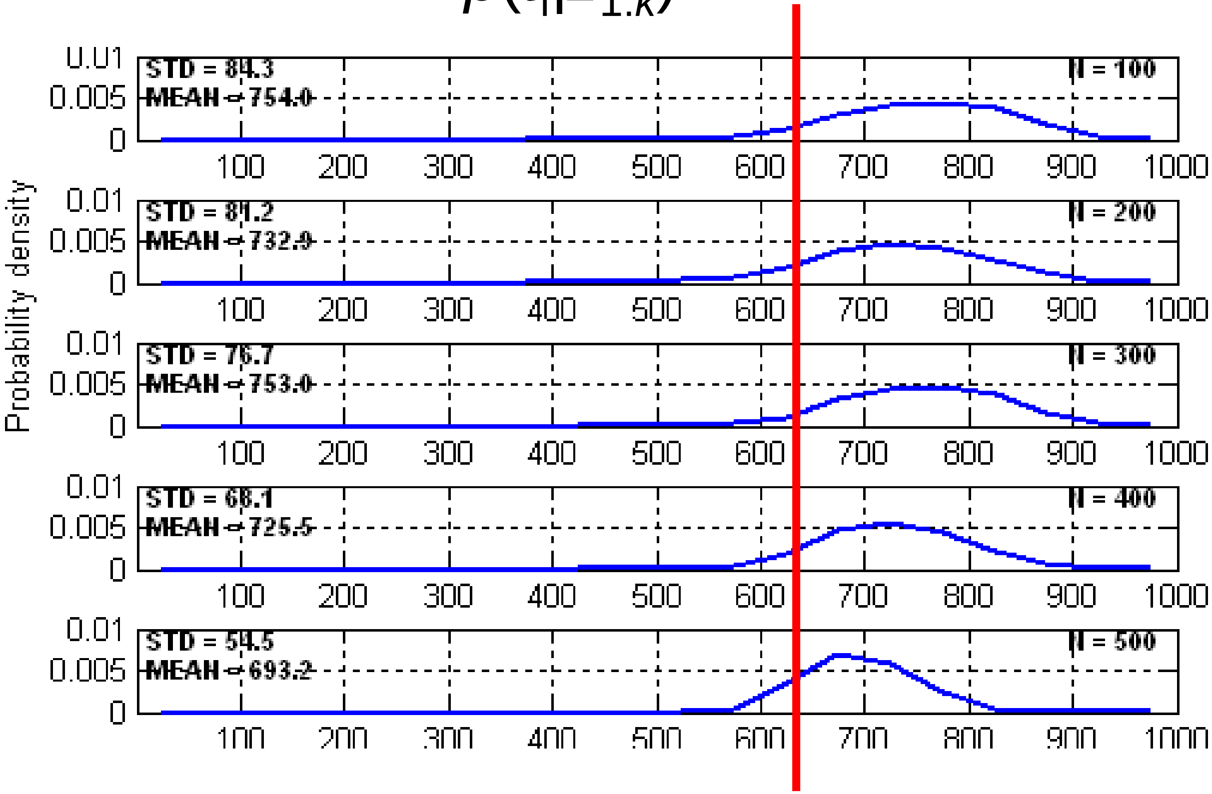


F. Cadini, E. Zio, D. Avram "*Monte Carlo-based filtering for fatigue crack growth estimation*", Probabilistic Engineering Mechanics, **24**, n. 3, pp. 367-373, 2009

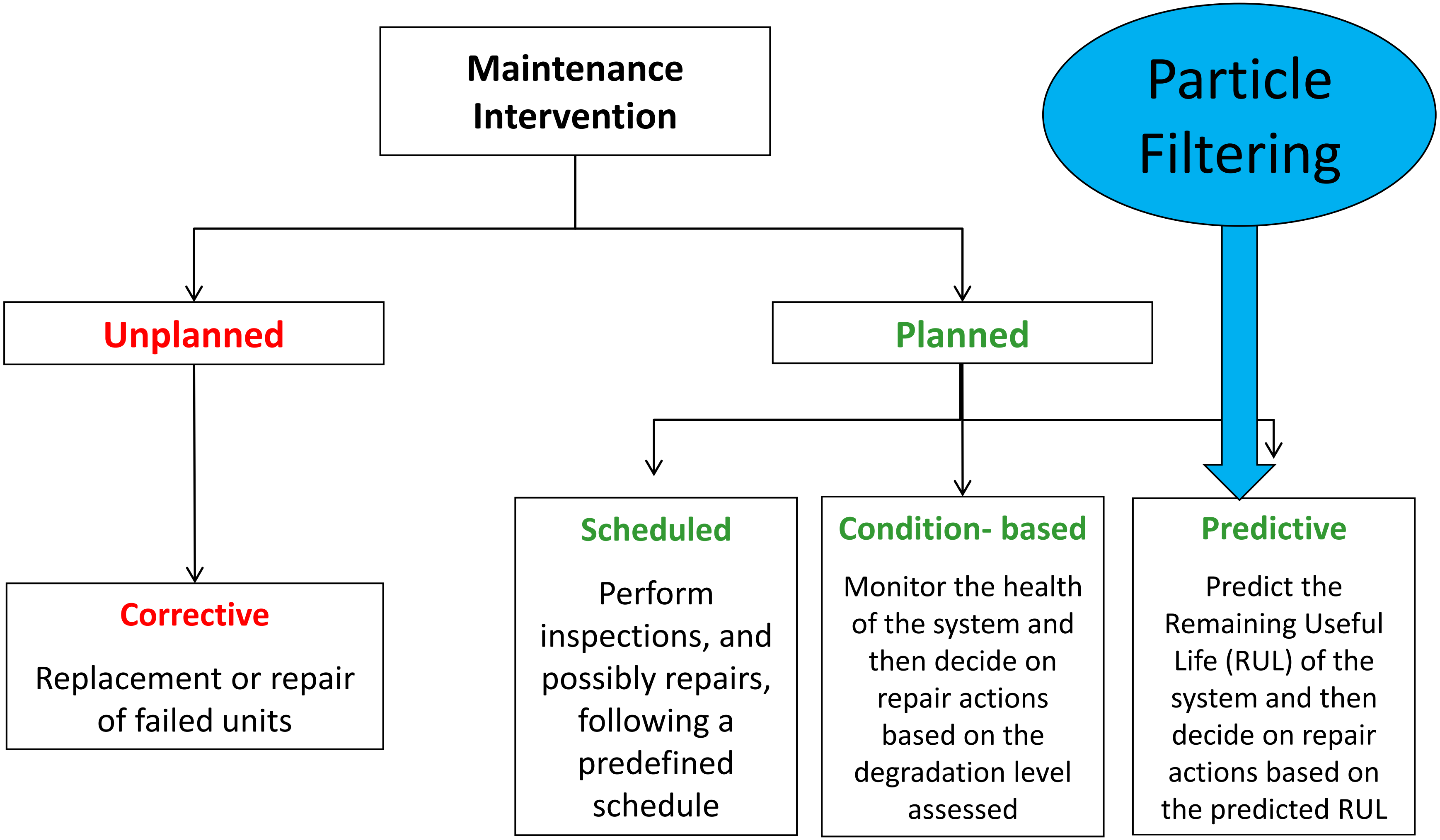
RUL estimate

- 5 measurements at: k_1 = 100; k_2 = 200; k_3 = 300; k_4 = 400; k_5 = 500
- 5000 particles
- *True failure time is 631*

$$p(t_f|z_{1:k})$$



Maintenance: ultimate goal of PHM



Maintenance Intervention

Particle Filtering

Unplanned

Planned

Corrective

Replacement or repair of failed units

Scheduled

Perform inspections, and possibly repairs, following a predefined schedule

Condition- based

Monitor the health of the system and then decide on repair actions based on the degradation level assessed

Predictive

Predict the Remaining Useful Life (RUL) of the system and then decide on repair actions based on the predicted RUL

Predictive maintenance

- A cost model of literature[*] is considered for the quantification of the costs driving the maintenance strategy
- Hypotheses:
  - **Inspection procedure**: periodic inspections are performed at given scheduled times. Results of the inspection are $z_{1:k}$
  - **Maintenance actions**: either replacement upon failure (cost c_f) or preventive replacement (cost c_p)
- Decision-making policy: at any time a decision can be made on whether to replace the component or to further extend its life, albeit assuming the risk of a possible failure

[*] A.H. Christer, W. Wang, J.M. Sharp, A state space condition monitoring model for furnace erosion prediction and replacement, European Journal of Operational Research, Vol. 101, 1997, pp. 1-14

Predictive maintenance planning

- Present time: k
- Replacement time $=k+l$
- Expected cost per unit time, $C(k,l)$ (evaluated at the present time k, assuming that the component will be replaced at time $k+l$)

$$C(k,l)= f(c_p, c_f, P(RUL<l))$$

- $\boxed{P(RUL<l)}$ ➤ Particle filter!!

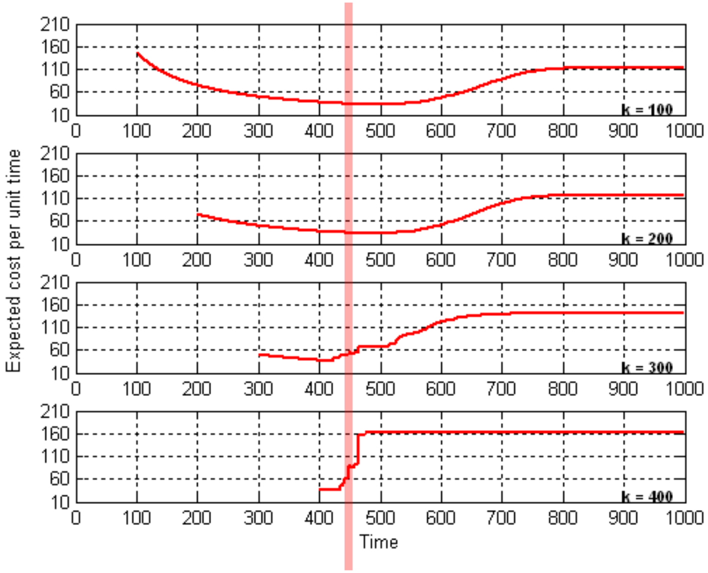
- Among all future time steps l, the best time to replacement l_{min} is the one which minimizes:

$$C(k,l)= f(c_p, c_f, P(RUL<l))$$

Predictive maintenance: results

- Measurements at time steps: $k_1 = 100$, $k_2 = 200$, $k_3 = 300$, $k_4 = 400$
- Number of particles: 5000
- TRUE FAILURE TIME = 452

Expected cost per unit time



k_1=100

k_2=200

k_3=300

k_4=400

Time step (k)	K_{min}
100	505
200	516
300	423
400	434

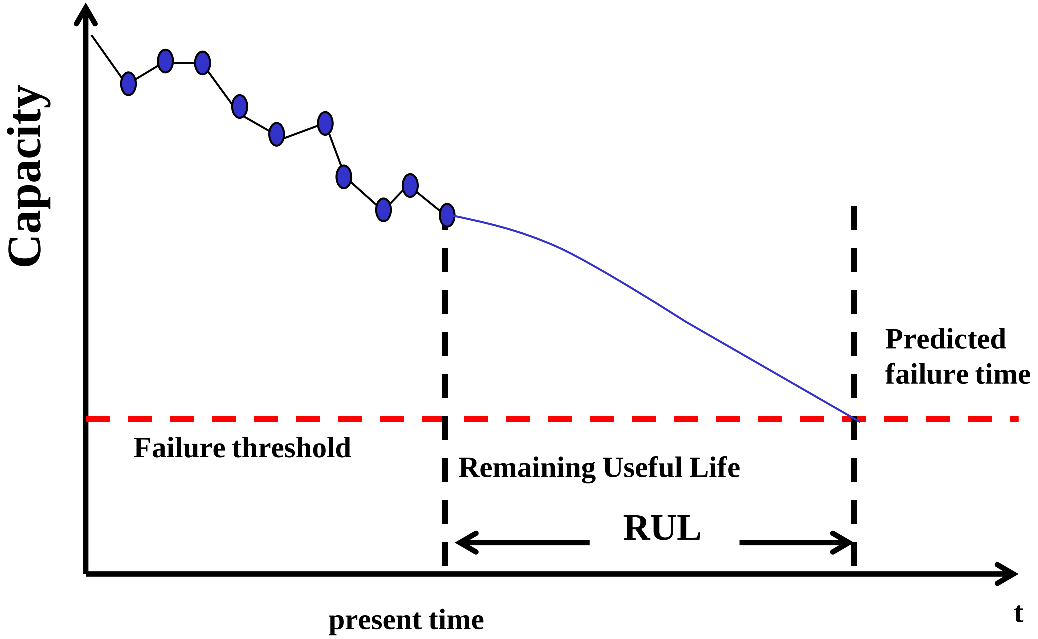
F. Cadini, E. Zio "*Model-based Monte Carlo state estimation for condition-based component replacement*", Reliability Engineering and System Safety, doi:10.1016/j.ress.2008.08.003, **94**, n. 3, pp. 752-758, 2009

- Applications
  - Maintenance planning
  - Prediction of the remaining useful life of batteries

Reference Case Study

Component: **Li-on Battery**

Objective: RUL prediction



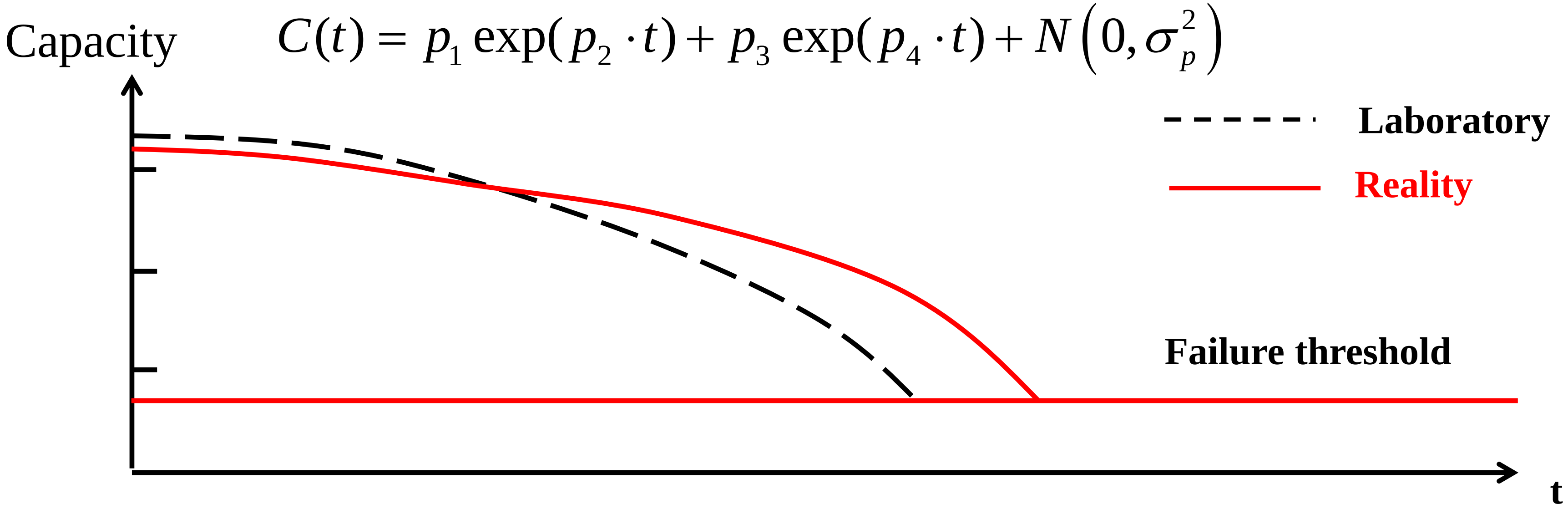
Relevance of Li-ion Battery

Issues:

- ➢ Most common used energy supply device
- ➢ Related with many safety critical functions
- ➢ Potential risk: overheat, expand, fire, explosion

Reference Case Study

$$C(t) = p_1 \exp(p_2 \cdot t) + p_3 \exp(p_4 \cdot t) + N\left(0, \sigma_p^2\right)$$



- **Laboratory environment**

p_1= 0.88, p_2=-0.001
p_3= -0.04, p_4=0.36

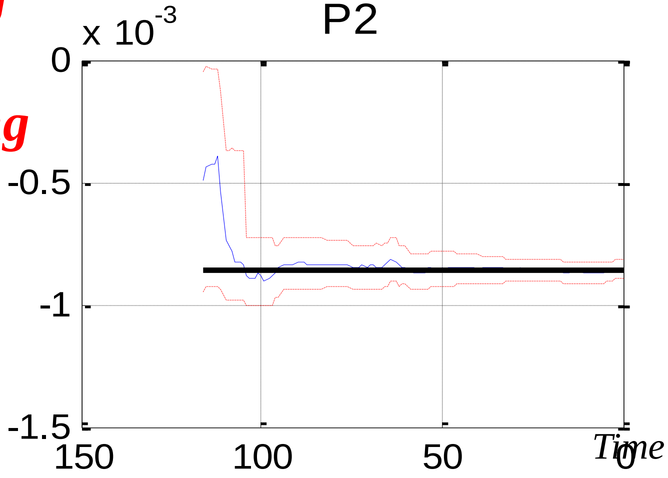
- **Reality:**

p_1=??
p_2=??
p_3=??
p_4=??

Unknown but fixed

Prognostics: challenges (3)

Particle Filtering and Kernel Smoothing



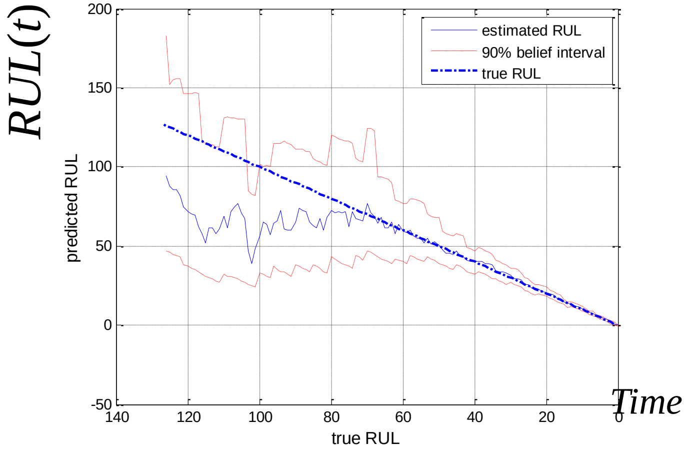
Information available:

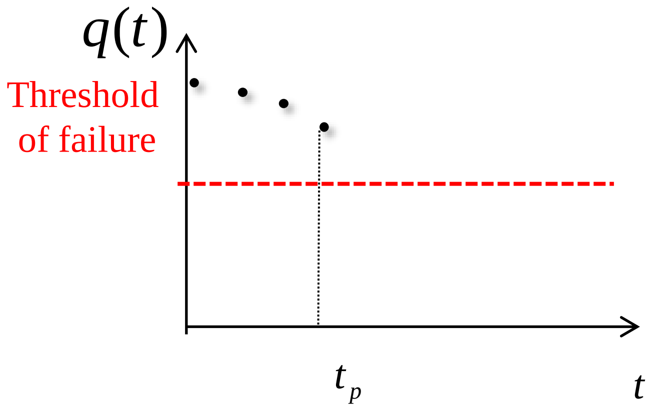
- A model of the degradation process;

$$q(t) = p_1 \exp(p_2 \cdot t) + p_3 \exp(p_4 \cdot t) + N\left(0, \sigma_p^2\right)$$

unknown parameters *p_1,p_2,p_3,p_4*!!

- Current degradation trajectory





References

- P. Baraldi, F. Cadini, F. Mangili, E. Zio, “*Model-based and data-driven prognostics under different available information*”, Probabilistic Engineering Mechanics, Vol. 32, pp. 66-79, 2013.
- M. Rigamonti, P. Baraldi, E. Zio, D. Astigarraga, A. Galarza, “*Particle Filter-Based Prognostics for an Electrolytic Capacitor Working in Variable Operating Conditions*”, (2016) IEEE Transactions on Power Electronics, 31 (2), pp. 1567-1575.
- Y. Hu, P. Baraldi, F. Di Maio, E. Zio, “A particle filtering and kernel smoothing-based approach for new design component prognostics”, Reliability Engineering & System Safety, 134, pp. 19-31, 2015
- F. Cadini, E. Zio, D. Avram “*Monte Carlo-based filtering for fatigue crack growth estimation*”, Probabilistic Engineering Mechanics, **24**, n. 3, pp. 367-373, 2009
- P. Baraldi, F. Mangili, E. Zio, “*Investigation of uncertainty treatment capability of model-based and data-driven prognostic methods using simulated data*” Reliability Engineering and System Safety, Vol. 112, pp. 94-108, 2013
- F. Cadini, E. Zio “*Model-based Monte Carlo state estimation for condition-based component replacement*”, Reliability Engineering and System Safety, doi:10.1016/j.ress.2008.08.003, **94**, n. 3, pp. 752-758, 2009
- A. Doucet, S. Godsill, C. Andrieu, “*On Sequential Simulation-Based Methods for Bayesian Filtering*”, Statistics and Computing, 2000
- A. Doucet, J.F.G. de Freitas and N.J. Gordon, *An Introduction to Sequential Monte Carlo Methods, in Sequential Monte Carlo in Practice*, A. Doucet, J.F.G. de Freitas and N.J. Gordon, Eds., New York: Springer-Verlag, 2001.
- A. Doucet, S. Godsill and C. Andreu, *On Sequential Monte Carlo Sampling Methods for Bayesian Filtering*, Statistics and Computing (2000), Vol 10, pp. 197-208.
- M.S. Arulampalam, S. Maskell, N. Gordon and T. Clapp, *A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking, IEEE Trans. On Signal Processing*, Vol. 50, No. 2, 2002, pp. 174-188