



$T=TTT$ (Total Time on Test)

r =number of failures

| | I, fixed t_0 | II, fixed r |
|---|---|--|
| <p>one-sided (lower)</p> <p>$P(MTTF > \vartheta_1) = \alpha$</p>  | $\vartheta_1 = \frac{2T}{\chi_{\alpha}^2 \underbrace{(2r+2)}_{\substack{\text{\#of degrees of freedom} \\ \downarrow \\ \text{percentile}}}}$ | $\vartheta_1 = \frac{2T}{\chi_{\alpha}^2(2r)}$ |
| <p>two-sided (lower and upper)</p> <p>$P(\vartheta_1 < MTTF < \vartheta_2) = \alpha$</p>  | $(\vartheta_1, \vartheta_2) = \left(\frac{2T}{\chi_{\frac{1+\alpha}{2}}^2(2r+2)}, \frac{2T}{\chi_{\frac{1-\alpha}{2}}^2(2r)} \right)$ | $(\vartheta_1, \vartheta_2) = \left(\frac{2T}{\chi_{\frac{1+\alpha}{2}}^2(2r)}, \frac{2T}{\chi_{\frac{1-\alpha}{2}}^2(2r)} \right)$ |

α Percentile values of the $\chi^2(f)$ distribution

| $\alpha \backslash f$ | 0.005 | 0.025 | 0.050 | 0.900 | 0.950 | 0.975 | 0.990 | 0.995 | 0.999 |
|-----------------------|--------|---------|---------|-------|-------|-------|-------|-------|-------|
| 1 | 0.0439 | 0.03982 | 0.02393 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.8 |
| 2 | 0.0100 | 0.0506 | 0.103 | 4.61 | 5.99 | 7.38 | 9.21 | 10.6 | 13.8 |
| 3 | 0.0717 | 0.216 | 0.352 | 6.25 | 7.81 | 9.35 | 11.3 | 12.8 | 16.3 |
| 4 | 0.207 | 0.484 | 0.711 | 7.78 | 9.49 | 11.1 | 13.3 | 14.9 | 18.5 |
| 5 | 0.412 | 0.831 | 1.15 | 9.24 | 11.1 | 12.8 | 15.1 | 16.7 | 20.5 |
| 6 | 0.676 | 1.24 | 1.64 | 10.6 | 12.6 | 14.4 | 16.8 | 18.5 | 22.5 |
| 7 | 0.989 | 1.69 | 2.17 | 12.0 | 14.1 | 16.0 | 18.5 | 20.3 | 24.3 |
| 8 | 1.34 | 2.18 | 2.73 | 13.4 | 15.5 | 17.5 | 20.1 | 22.0 | 26.1 |
| 9 | 1.73 | 2.70 | 3.33 | 14.7 | 16.9 | 19.0 | 21.7 | 23.6 | 27.9 |
| 10 | 2.16 | 3.25 | 3.94 | 16.0 | 18.3 | 20.5 | 23.2 | 25.2 | 29.6 |
| 11 | 2.60 | 3.82 | 4.57 | 17.3 | 19.7 | 21.9 | 24.7 | 26.8 | 31.3 |
| 12 | 3.07 | 4.40 | 5.23 | 18.5 | 21.0 | 23.3 | 26.2 | 28.3 | 32.9 |
| 13 | 3.57 | 5.01 | 5.89 | 19.8 | 22.4 | 24.7 | 27.7 | 29.8 | 34.5 |
| 14 | 4.07 | 5.63 | 6.57 | 21.1 | 23.7 | 26.1 | 29.1 | 31.3 | 36.1 |
| 15 | 4.60 | 6.26 | 7.26 | 22.3 | 25.0 | 27.5 | 30.6 | 32.8 | 37.7 |
| 16 | 5.14 | 6.91 | 7.96 | 23.5 | 26.3 | 28.8 | 32.0 | 34.3 | 39.3 |
| 17 | 5.70 | 7.56 | 8.67 | 24.8 | 27.6 | 30.2 | 33.4 | 35.7 | 40.8 |
| 18 | 6.26 | 8.23 | 9.39 | 26.0 | 28.9 | 31.5 | 34.8 | 37.2 | 42.3 |
| 19 | 6.84 | 8.91 | 10.1 | 27.2 | 30.1 | 32.9 | 36.2 | 38.6 | 43.8 |
| 20 | 7.43 | 9.59 | 10.9 | 28.4 | 31.4 | 34.2 | 37.6 | 40.0 | 45.3 |
| 21 | 8.03 | 10.3 | 11.6 | 29.6 | 32.7 | 35.5 | 38.9 | 41.4 | 46.8 |
| 22 | 8.64 | 11.0 | 12.3 | 30.8 | 33.9 | 36.8 | 40.3 | 42.8 | 48.3 |
| 23 | 9.26 | 11.7 | 13.1 | 32.0 | 35.2 | 38.1 | 41.6 | 44.2 | 49.7 |
| 24 | 9.89 | 12.4 | 13.8 | 33.2 | 36.4 | 39.4 | 43.0 | 45.6 | 51.2 |
| 25 | 10.5 | 13.1 | 14.6 | 34.4 | 37.7 | 40.6 | 44.3 | 46.9 | 52.6 |
| 26 | 11.2 | 13.8 | 15.4 | 35.6 | 38.9 | 41.9 | 45.6 | 48.3 | 54.1 |
| 27 | 11.8 | 14.6 | 16.2 | 36.7 | 40.1 | 43.2 | 47.0 | 49.6 | 55.5 |
| 28 | 12.5 | 15.3 | 16.9 | 37.9 | 41.3 | 44.5 | 48.3 | 51.0 | 56.9 |
| 29 | 13.1 | 16.0 | 17.7 | 39.1 | 42.6 | 45.7 | 49.6 | 52.3 | 58.3 |
| 30 | 13.8 | 16.8 | 18.5 | 40.3 | 43.8 | 47.0 | 50.9 | 53.7 | 59.7 |
| 35 | 17.2 | 20.6 | 22.5 | 46.1 | 49.8 | 53.2 | 57.3 | 60.3 | 66.6 |
| 40 | 20.7 | 24.4 | 26.5 | 51.8 | 55.8 | 59.3 | 63.7 | 66.8 | 73.4 |
| 45 | 24.3 | 28.4 | 30.6 | 57.5 | 61.7 | 65.4 | 70.0 | 73.2 | 80.1 |
| 50 | 28.0 | 32.4 | 34.8 | 63.2 | 67.5 | 71.4 | 76.2 | 79.5 | 86.7 |

| Basic random variable | Parameter | Prior and posterior distributions of parameter | Mean and Variance of Parameter | Posterior Statistics |
|--|---------------|---|--|--|
| Binomial | | Beta | | |
| $p_X(x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$ | θ | $f_{\Theta}(\theta) = \frac{\Gamma(q+r)}{\Gamma(q)\Gamma(r)} \theta^{q-1} (1-\theta)^{r-1}$ | $E(\Theta) = \frac{q}{q+r}$ $\text{Var}(\Theta) = \frac{qr}{(q+r)^2(q+r+1)}$ | $q'' = q' + x$ $r'' = r' + n - x$ |
| Exponential | | Gamma | | |
| $f_X(x) = \lambda e^{-\lambda x}$ | λ | $f_{\Lambda}(\lambda) = \frac{\nu(\nu\lambda)^{k-1} e^{-\nu\lambda}}{\Gamma(k)}$ | $E(\lambda) = \frac{k}{\nu}$ $\text{Var}(\lambda) = \frac{k}{\nu^2}$ | $\nu'' = \nu' + \sum_i x_i$ $k'' = k' + n$ |
| Normal | | Normal | | |
| $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ (with known σ) | μ | $f_M(\mu) = \frac{1}{\sqrt{2\pi}\sigma_\mu} \exp\left[-\frac{1}{2}\left(\frac{\mu-\mu_\mu}{\sigma_\mu}\right)^2\right]$ | $E(\mu) = \mu_\mu$ $\text{Var}(\mu) = \sigma_\mu^2$ | $\mu_\mu'' = \frac{\mu_\mu'(\sigma^2/n) + \bar{x}\sigma_\mu'^2}{\sigma^2/n + (\sigma_\mu')^2}$ $\sigma_\mu'' = \sqrt{\frac{(\sigma_\mu')^2(\sigma^2/n)}{(\sigma_\mu')^2 + \sigma^2/n}}$ |
| Normal | | Gamma-Normal | | |
| $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ | μ, σ | $f(\mu, \sigma) = \left\{ \frac{1}{\sqrt{2\pi}\sigma/n} \exp\left[-\frac{1}{2}\left(\frac{\mu-\bar{x}}{\sigma/\sqrt{n}}\right)^2\right] \cdot \left\{ \frac{[(n-1)/2]^{(n+1)/2}}{\Gamma[(n+1)/2]} \left(\frac{s^2}{\sigma^2}\right)^{(n-1)/2} \cdot \exp\left(-\frac{n-1}{2} \frac{s^2}{\sigma^2}\right) \right\} \right\}$ | $E(\mu) = \bar{x}$ $\text{Var}(\mu) = s^2 \left[\frac{n-1}{n(n-3)} \right]$ $E(\sigma) = s \sqrt{\frac{n-1}{2} \frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}}$ $\text{Var}(\sigma) = s^2 \left(\frac{n-1}{n-3} \right) - E^2(\sigma)$ | $n'' = n' + n$ $n''\bar{x}'' = n'\bar{x}' + n\bar{x}$ $(n''-1)s''^2 + n''\bar{x}''^2 = [(n'-1)s'^2 + n'\bar{x}'^2] + [(n-1)s^2 + n\bar{x}^2]$ |
| Poisson | | Gamma | | |
| $p_X(x) = \frac{(\mu t)^x}{x!} e^{-\mu t}$ | μ | $f_M(\mu) = \frac{\nu(\nu\mu)^{k-1} e^{-\nu\mu}}{\Gamma(k)}$ | $E(\mu) = \frac{k}{\nu}$ $\text{Var}(\mu) = \frac{k}{\nu^2}$ | $\nu'' = \nu' + t$ $k'' = k' + x$ |
| Lognormal | | Normal | | |
| $f_X(x) = \frac{1}{\sqrt{2\pi}\xi x} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\xi}\right)^2\right]$ (with known ξ) | λ | $f_{\Lambda}(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\lambda-\mu}{\sigma}\right)^2\right]$ | $E(\lambda) = \mu$ $\text{Var}(\lambda) = \sigma^2$ | $\mu'' = \frac{\mu'(\xi^2/n) + \sigma^2 \ln \bar{x}}{\xi^2/n + \sigma^2}$ $\sigma'' = \sqrt{\frac{\sigma^2(\xi^2/n)}{\sigma^2 + \xi^2/n}}$ |