FAULT DETECTION
Part 1 & 2

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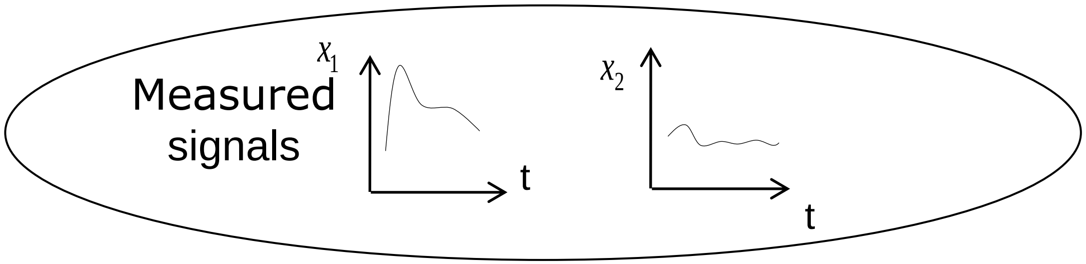
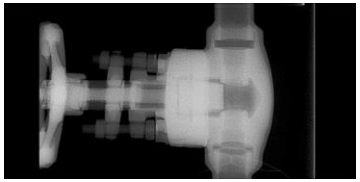
In This Lecture

- Part 1: Model of the Equipment Behavior in Normal Condition
  - 1A) Auto Associative Kernel Regression (AAKR)
  - 1B) Principal Component Analysis (PCA)
- Part 2: Statistical Test
  - 2A) Thresholds-Based
  - 2B) Q-Statistics
  - 2C) Sequential Probability Ratio Test (SPRT)

Overview

Equipment (System, Structure or Component)





$$\frac{dx}{dt} = A\exp\left(-\frac{Q}{RT}\right)\varphi^n$$

Detect

- Anomalous operation
- Normal operation

Diagnose

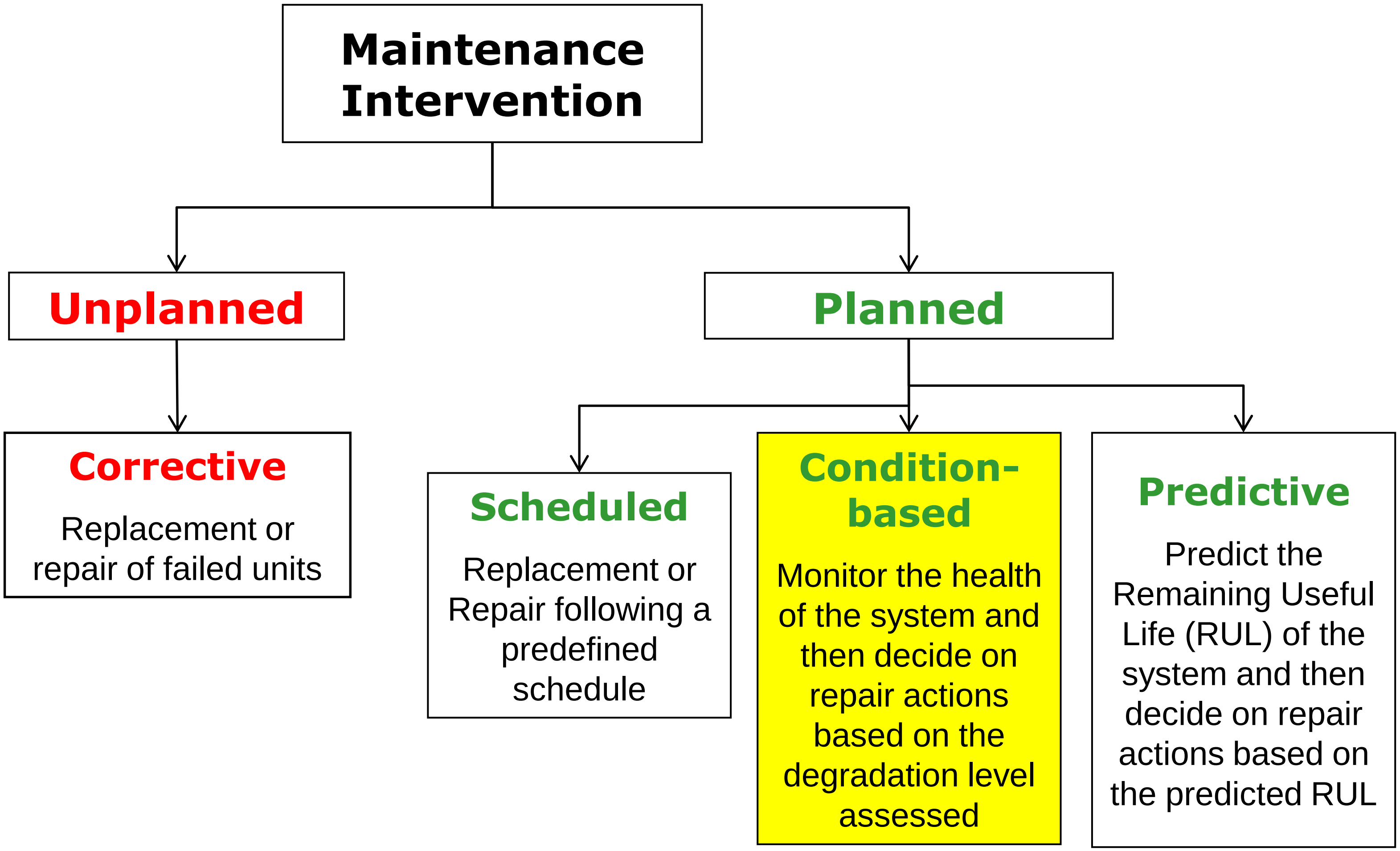
c_1 c_2 c_3

Malfunctioning type (classes)

Predict

Remaining Useful Life (RUL)

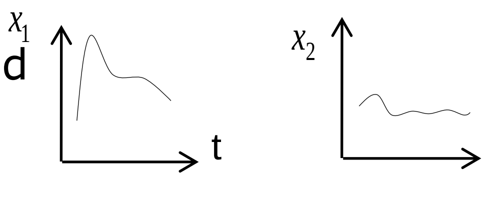
Context: Maintenance Interventions & PHM

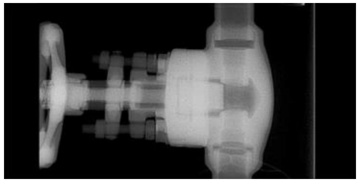


In This Lecture: Fault Detection

Equipment (System, Structure or Component)

Measured signals





$$\frac{dx}{dt} = A\exp\left(-\frac{Q}{RT}\right)\varphi^n$$

Detect

Anomalous operation

Normal operation

Diagnose

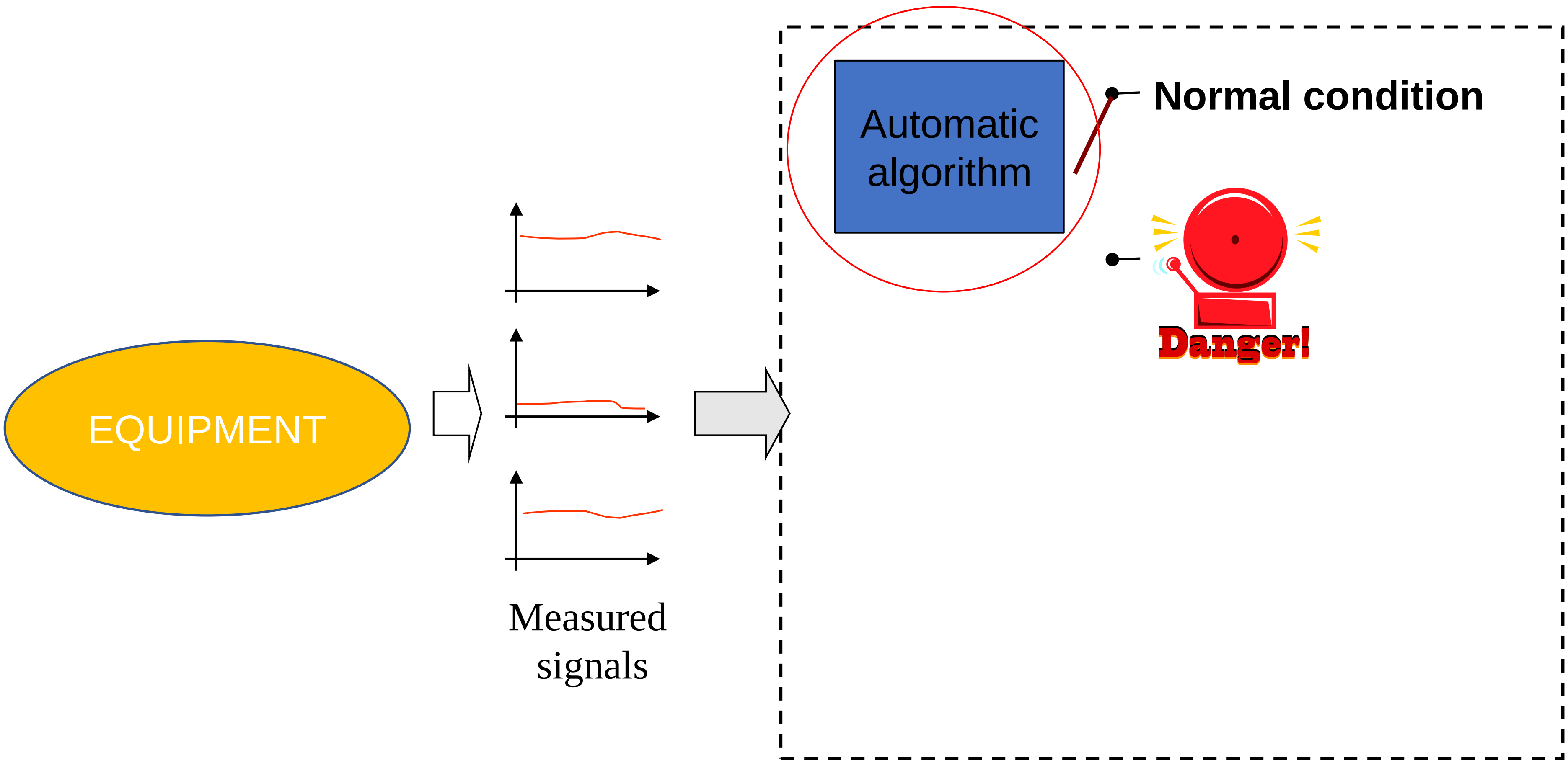
c_1 c_2 c_3

Malfunctioning type (classes)

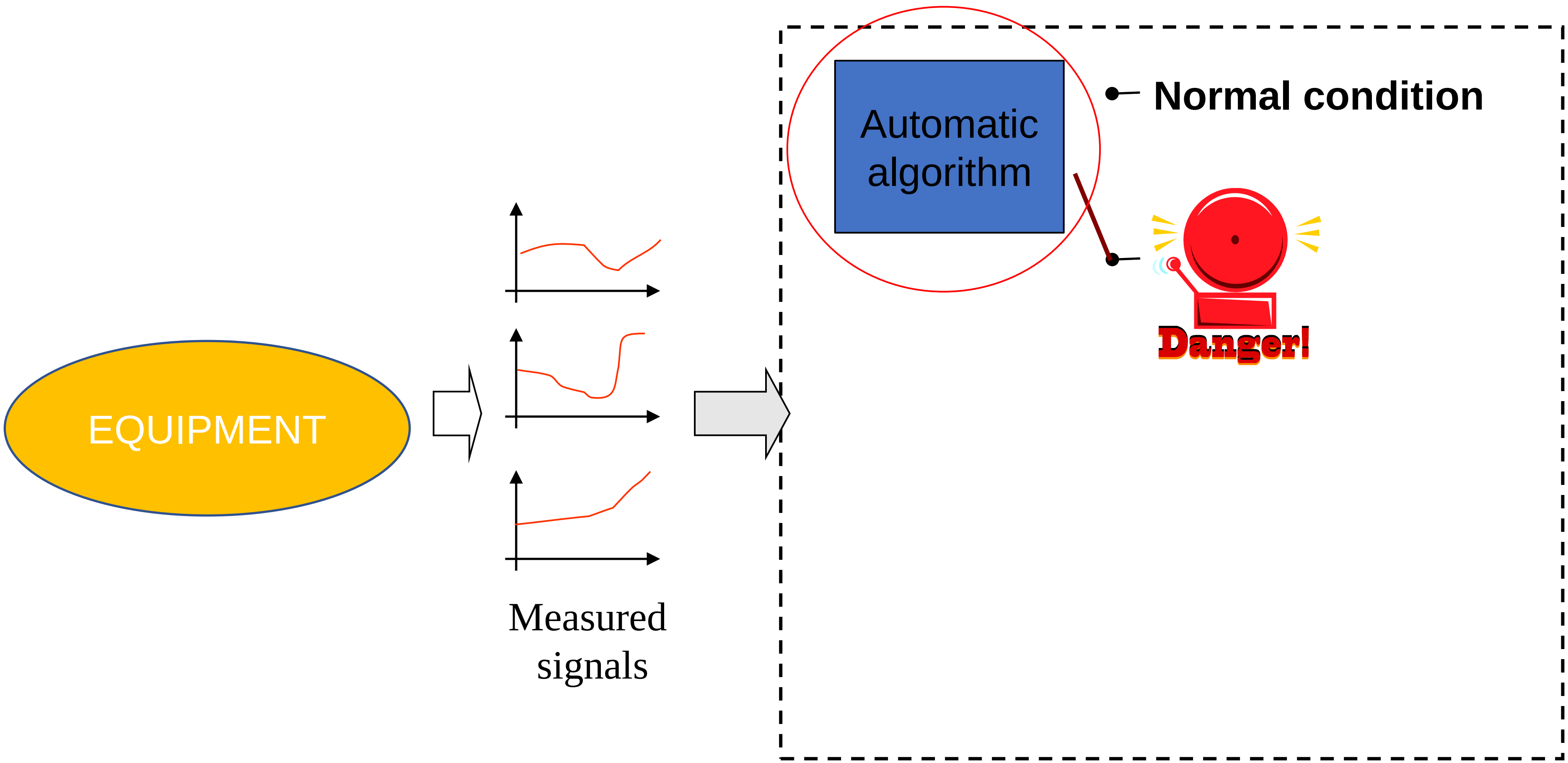
Predict

Remaining Useful Life (RUL)

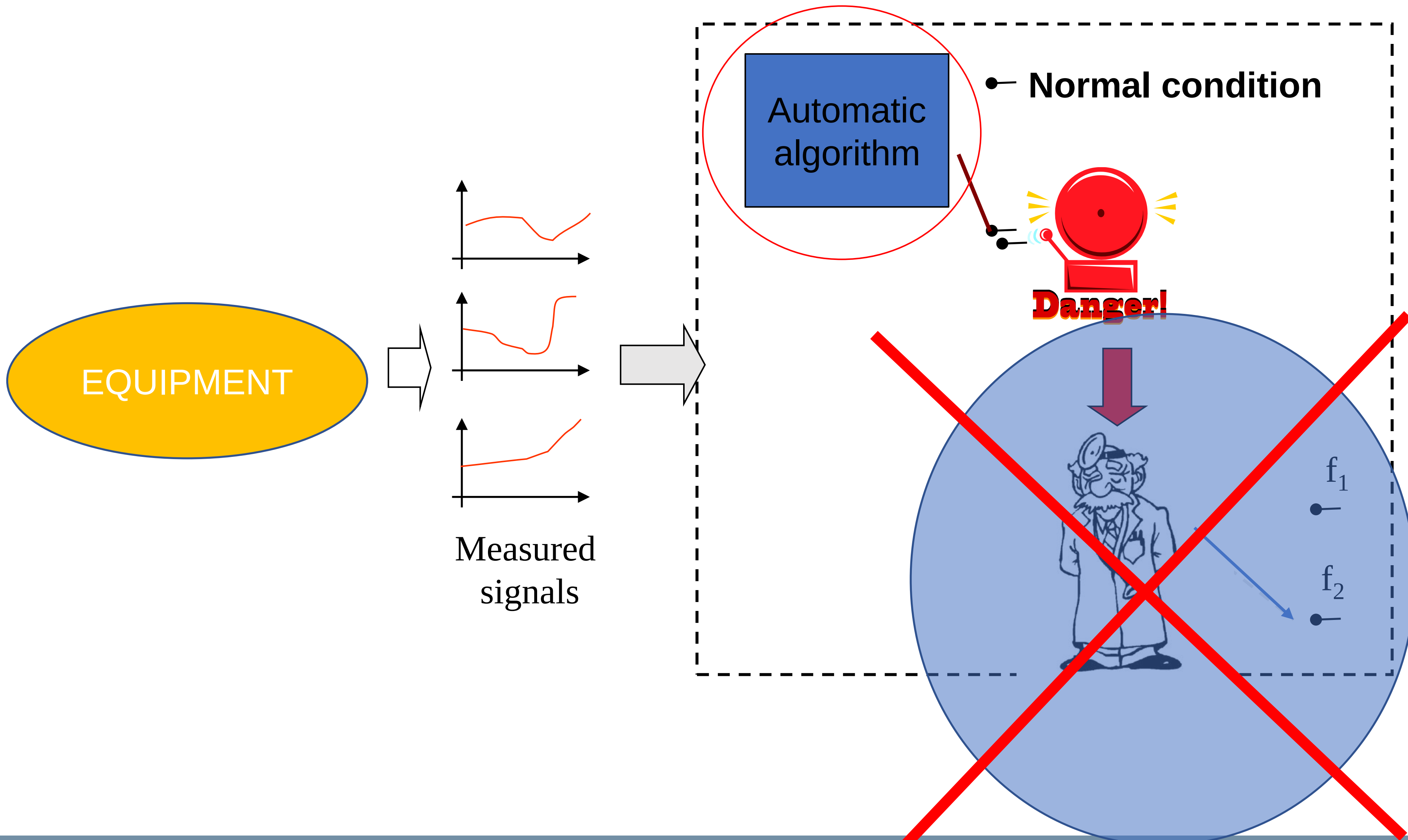
Fault Detection: What is?



Fault Detection: What is?



Fault Detection: What is not?



Fault Detection: Methods

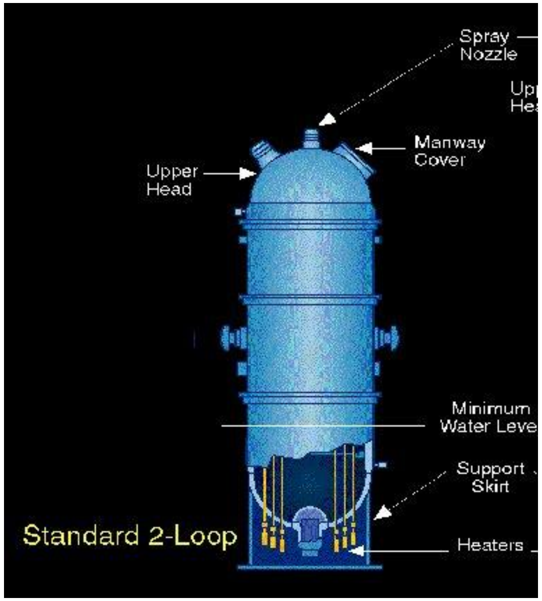
- Limit-based
- Model-based
- Data-driven

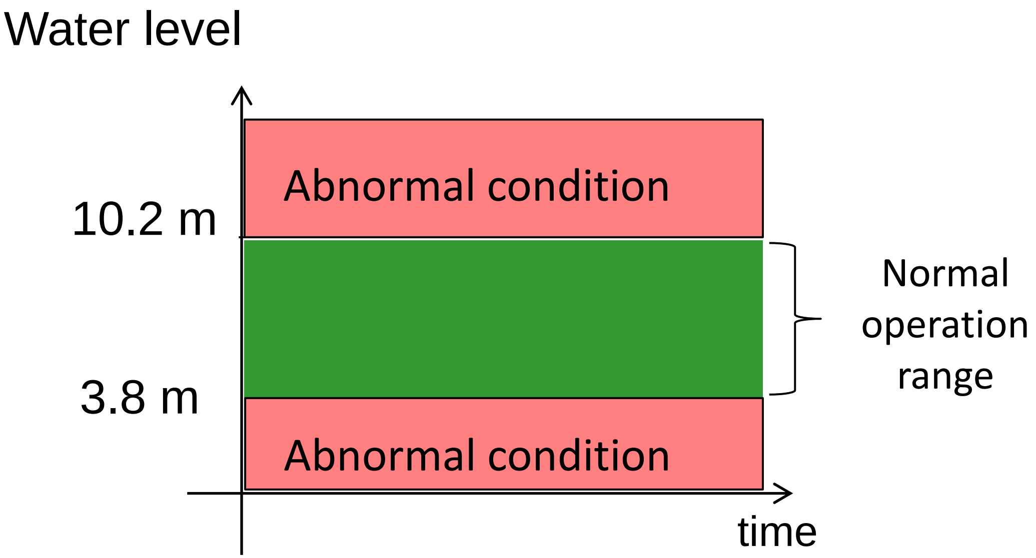
Limit-based fault detection: data & information

- Normal operation ranges of key signals

Example:

Pressurizer of a nuclear reactor





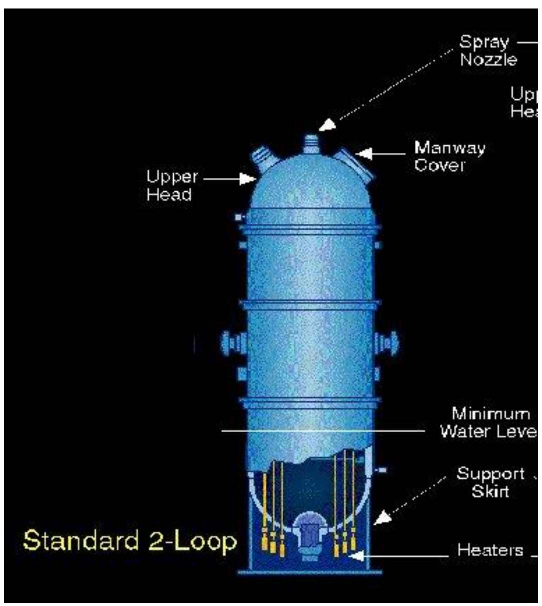
Limit-based fault detection: the method

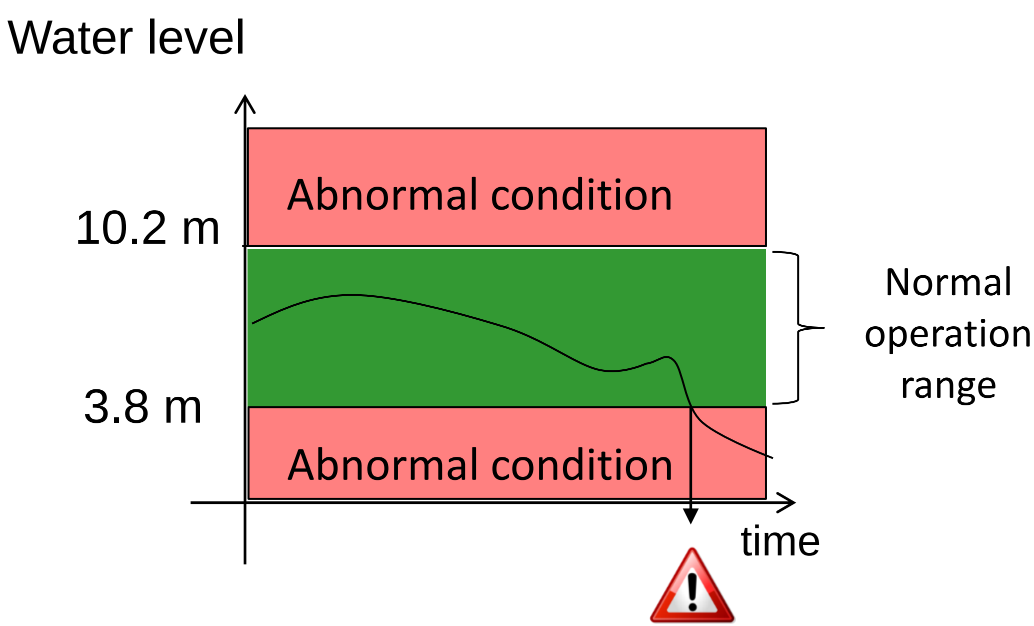
- Normal operation ranges of key signals

- Limit Value-Based Fault Detection

Example:

Pressurizer of a nuclear reactor





Limit-based fault detection: Limitations

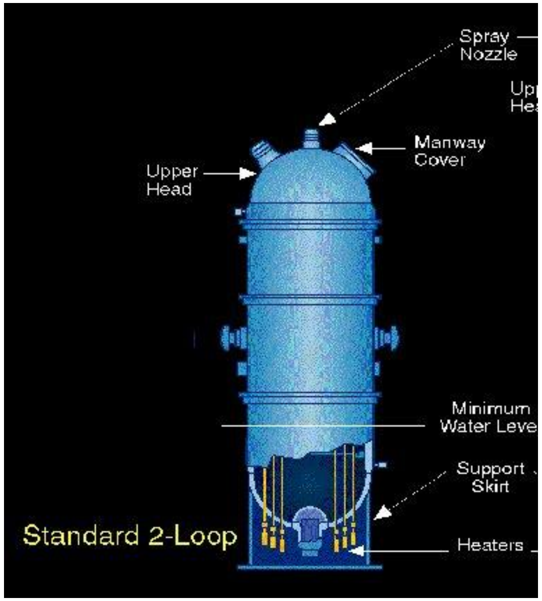
- Normal operation ranges of key signals

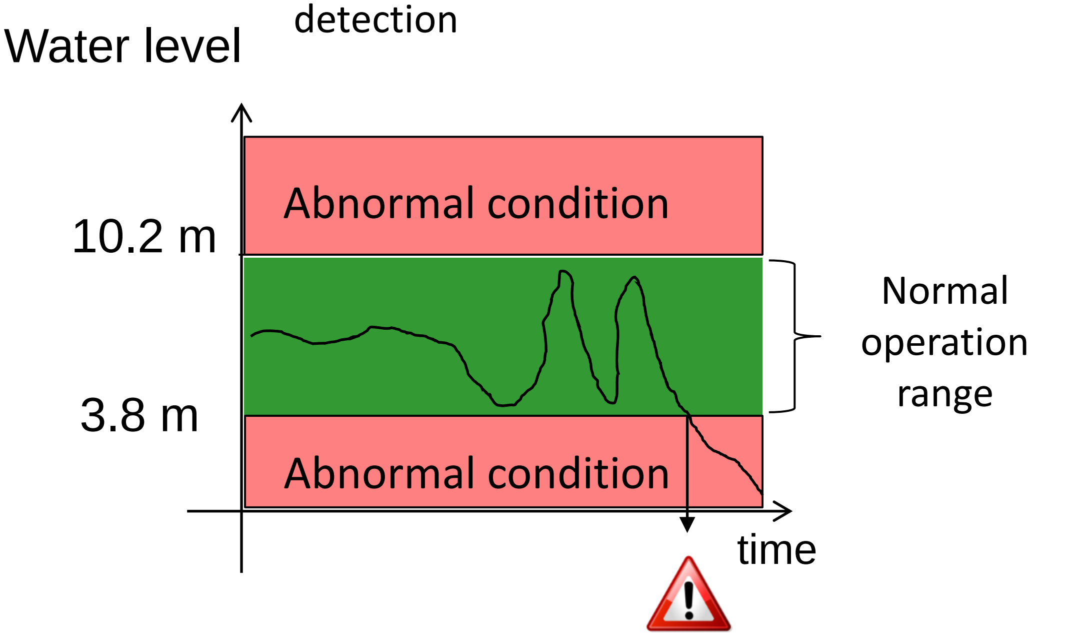
- Limit Value-Based Fault Detection

Limitations:
- No early detection
- Not applicable to fault detection during operational transients
- Control systems operations may hide small anomalies (the signal remains in the normal range although there is a process anomaly)
- Considering signal individually can delay detection

Example:

Pressurizer of a PWR nuclear reactor

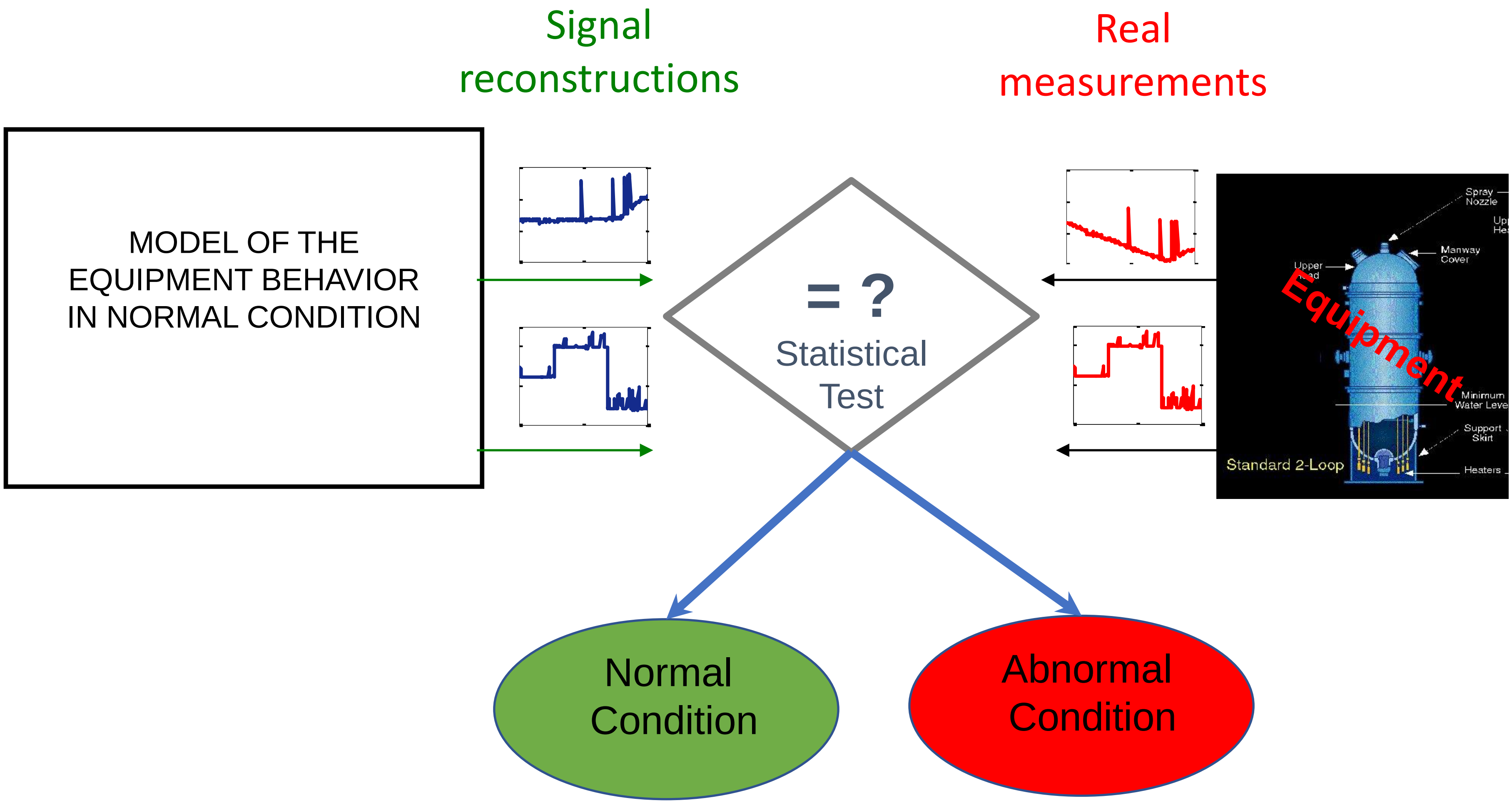




Fault Detection: Approaches

- Limit-based
- Model-based
- Data-driven

Model-based & Data-driven fault detection: basic idea



Fault Detection: Approaches

- Limit-based
- Model-based
- Data-driven

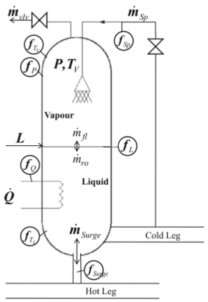
Model-based fault detection: data & information

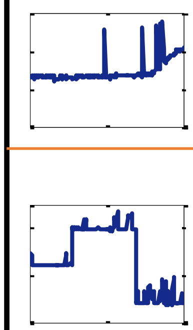
- Physics-based model of the process (used to reproduce the expected behavior of the signals in normal condition)

Example:

Signal reconstructions

Pressurizer model





Model-based fault detection: limitations

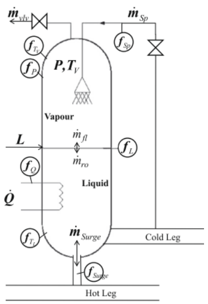
- Physics-based model of the process (used to reproduce the expected behavior of the signals in normal condition)

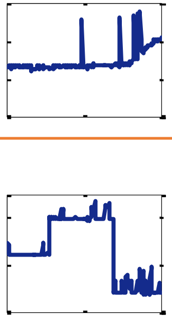
Example:

Signal reconstructions

Real measurements

Pressurizer model





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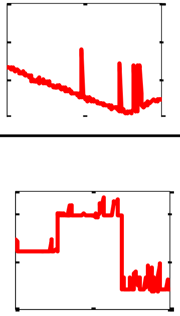
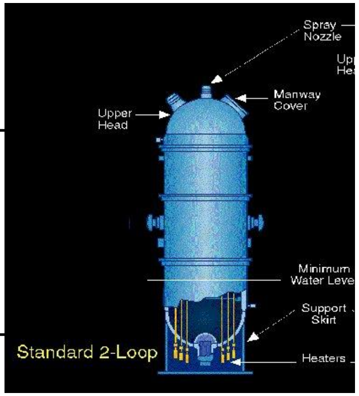




- Typically not available for complex systems
- Long computational time

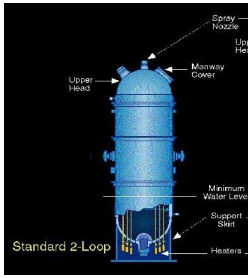
Abnormal Condition

Fault Detection: Approaches

- Limit-Based
- Model Based
- Data-driven

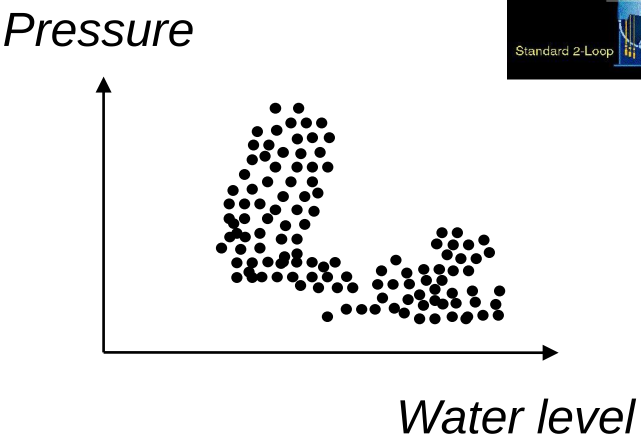
Data-driven fault detection: data & information

- Historical signal measurements in normal operation



Example:

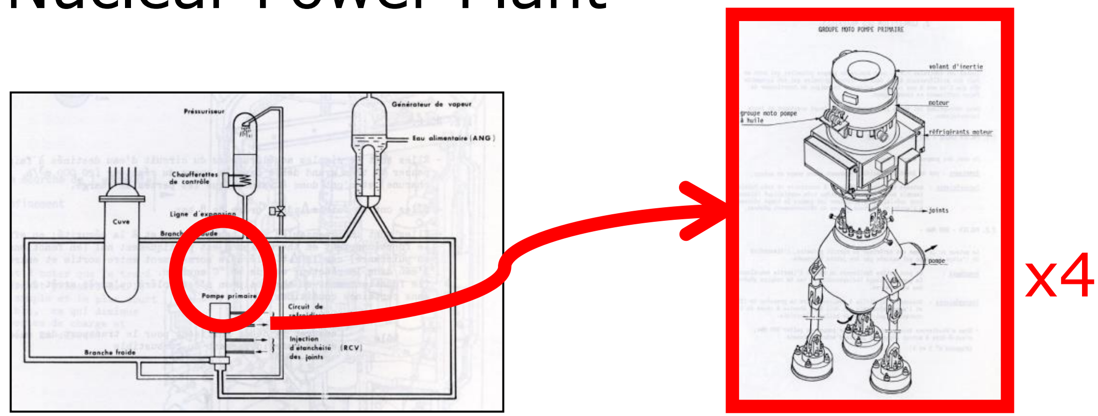
Pressure	Liquid temperature	Steam temperature	Spray flow	Surge line flow	Heaters power	Level
150.2	321	362	539	244	0	7.2
150.4	322	363	681	304	0	7.5
150.3	323	364	690	335	1244	7.7
…	…	…	…	…	…	…



Example of Application 1*

COMPONENT TO BE MONITORED

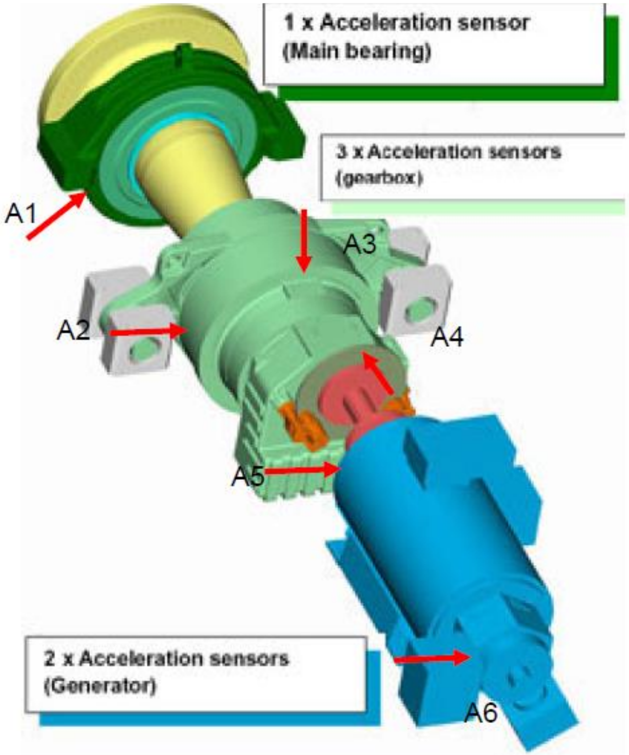
Reactor Coolant Pump of PWR Nuclear Power Plant



Measured signals	48 (Temperatures, pressures, flows,…)
Available data	Historical signal measurements in normal plant condition [1 year, frequency=1/30 Hz]

* Work developed with EDF-R&D

Example of Application 2



Measured signals	6 vibration signals measured by accelerometers
Available data	Historical signal measurements in normal plant condition [3 years, frequency=5 kHz]

Data-driven fault detection: possible methods

- Statistical Approaches:
  - AutoAssociative Kernel Regression (AAKR)
  - Principal Component Analysis (PCA)-based
  - …
- Artificial Intelligence (AI)-based
  - Feedforward Neural Networks (FNNs)
  - AutoAssociative Neural Networks (AANNs)
  - AutoEncoders (AEs)
  - Self Organizing Maps
  - …

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  - 1B) Principal Component Analysis (PCA)
- Part 2: Statistical Test
  - 2A) Thresholds-Based
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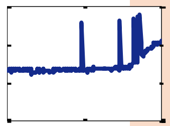
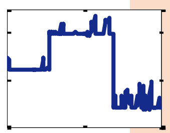
Part 1

MODEL OF THE EQUIPMENT BEHAVIOR IN NORMAL CONDITION

Signal reconstructions

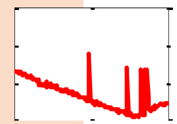


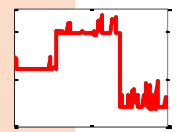


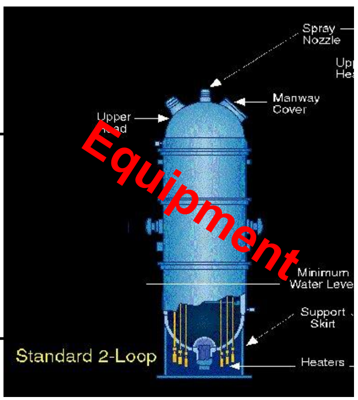
Part 2

= ?
Statistical Test

Real measurements







Normal Condition

Abnormal Condition

PART 1: Model of the Equipment Behaviour in Normal Condition

- Auto Associative Kernel Regression (AAKR)
- Principal Component Analysis (PCA)

What do we need?

Data in normal conditions

Signal

$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \cdots & x_{1j} & \cdots & x_{1n}^{obs-nc} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{k1} & \cdots & x_{kj} & \cdots & x_{kn} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{N1}^{obs-nc} & \cdots & x_{Nj} & \cdots & x_{Nn}^{obs-nc} \end{bmatrix}$$

Observation

obs-nc = observation in normal condition

Training set, input and output

- Training patterns:
  Historical signal measurements
  in normal condition

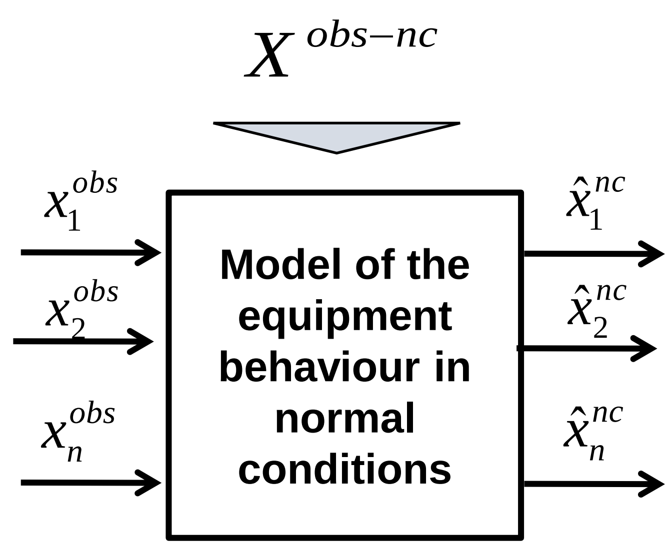
Signal

$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \cdots & x_{1j} & \cdots & x_{1n}^{obs-nc} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{k1} & \cdots & x_{kj} & \cdots & x_{kn} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{N1}^{obs-nc} & \cdots & x_{Nj} & \cdots & x_{Nn}^{obs-nc} \end{bmatrix}$$

Observation

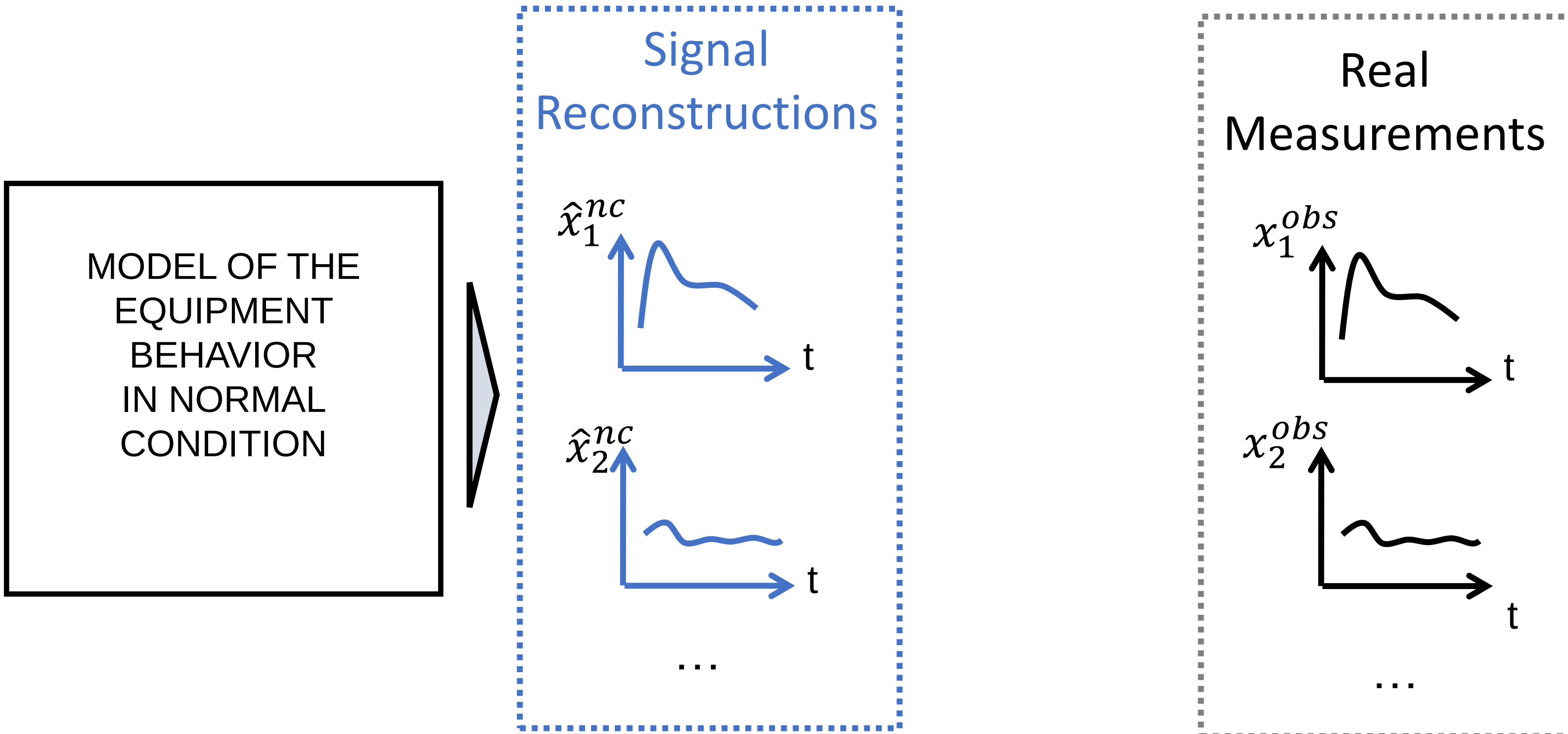
- Test input: $\vec{x}^{obs} = (x_1^{obs}, \ldots, x_n^{obs})$
  Signals measured at current time

- Test Output: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \ldots, \hat{x}_n^{nc})$
  Signal reconstructions
  (expected values of the signals
  in normal condition)



Requirement I

- Equipment is in normal condition

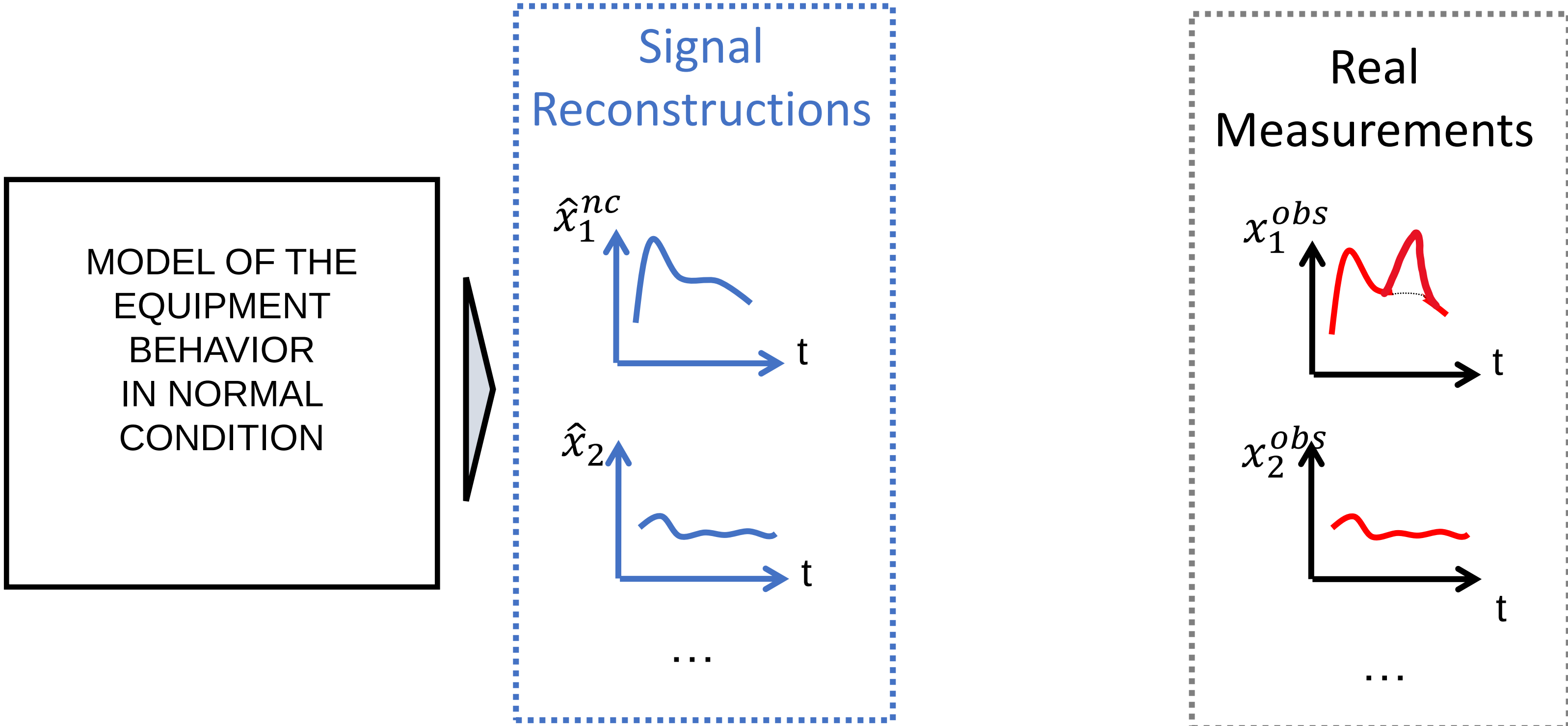


$$\vec{\hat{x}}^{nc} \cong \vec{x}^{obs}$$

$\hat{}$ =reconstruction
nc= normal condition

Requirement II

- Equipment is in abnormal condition



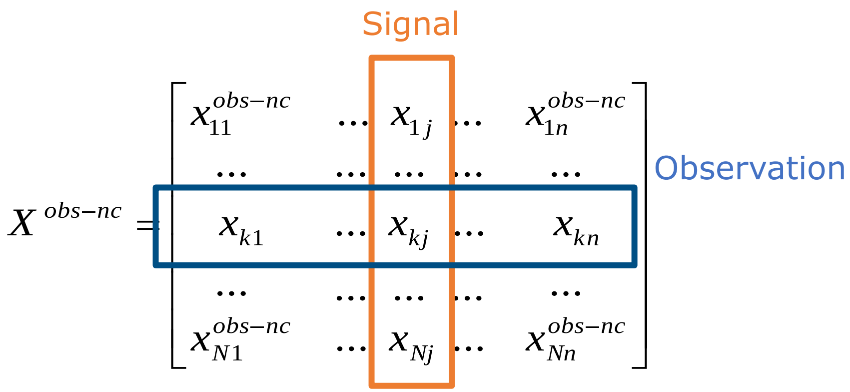
$$\vec{\hat{x}}^{nc} \neq \vec{x}^{obs}$$

PART 1: Model of the Equipment Behaviour in Normal Condition

- **1A) Auto Associative Kernel Regression (AAKR)**
- 1B) Principal Component Analysis (PCA)

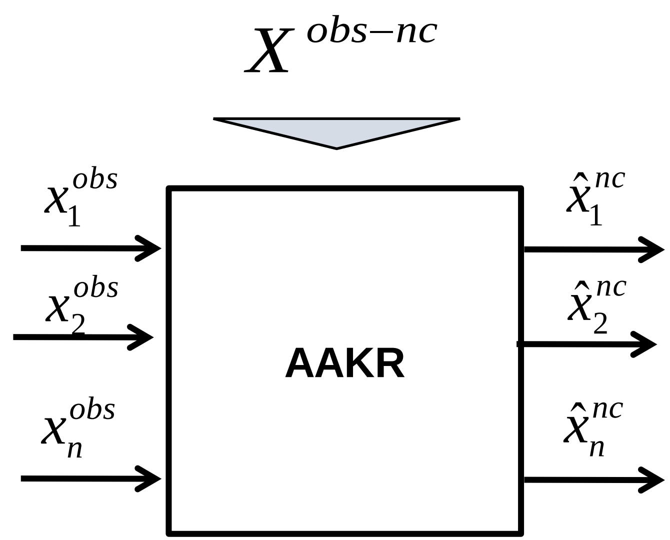
AAKR: Training set, input and output = Slide 26

- Training patterns:
  Historical signal measurements
  in normal condition



- Test input: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$
  Signals measured at current time

- Test Output: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$
  Signal reconstructions
  (expected values of the signals
  in normal condition)

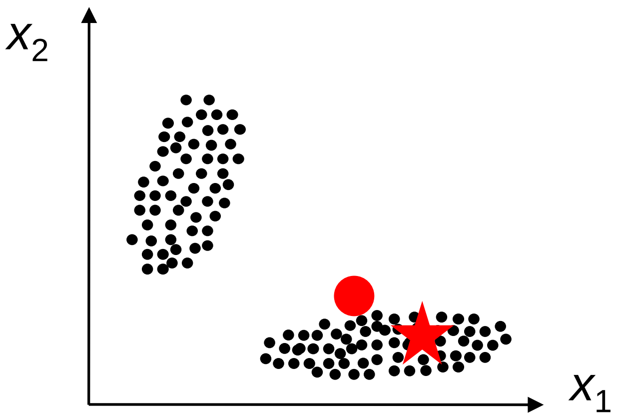


AAKR: the algorithm (1)

- Training patterns:

$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$$

- Test input ●: measured signals at current time $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$
- Test output ★: weighted sum of the training patterns $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$



On all the training pattern

$$\hat{x}_j^{nc} = \frac{\sum_{k=1}^{N} w(k) \cdot x_{kj}^{obs-nc}}{\sum_{k=1}^{N} w(k)}$$

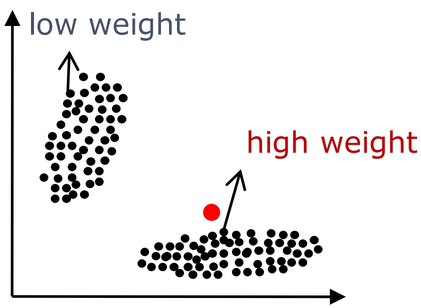
AAKR: the algorithm (2)

- Output $\hat{\vec{x}}^{nc} = (\hat{x}_1^{nc}, \ldots, \hat{x}_n^{nc})$ = weighted sum of the training patterns:

On all the training pattern

$$\hat{x}_j^{nc} = \frac{\sum_{k=1}^{N} w(k) \cdot x_{kj}^{obs-nc}}{\sum_{k=1}^{N} w(k)}$$

- weights *w*(*k*) = similarity measures between $\vec{x}^{obs}$ and $\vec{x}_k^{obs-nc}$ (the test and the *k*-th training pattern):



$$w(k) = \frac{1}{\sqrt{2\pi h}} e^{-\frac{d^2(k)}{2h^2}}$$

h = bandwidth parameter (it controls the decay speed)

AAKR: parameter *h* setting

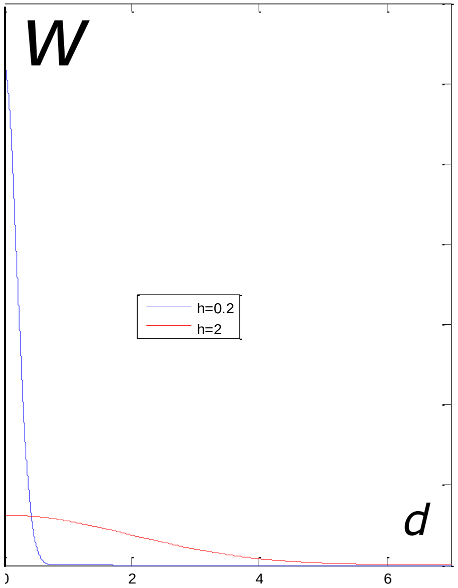
d=0 → w=0.40/h

d=h → w=0.24/h

d=2h → w=0.05/h

d=3h → w=0.004/h

$$\frac{w(d=0)}{w(d=3h)} = \frac{0.40}{0.004} = 100$$



AAKR: the algorithm (2)

- Output $\hat{\vec{x}}^{nc} = (\hat{x}_1^{nc}, ..., \hat{x}_n^{nc})$ = weighted sum of the training patterns:

$$\hat{x}_j^{nc} = \frac{\sum_{k=1}^{N} w(k) \cdot x_{kj}^{obs-nc}}{\sum_{k=1}^{N} w(k)}$$

- weights *w*(*k*) = similarity measures between $\vec{x}^{obs}$ and $\vec{x}_k^{obs-nc}$ (the test and the *k*-th training pattern):

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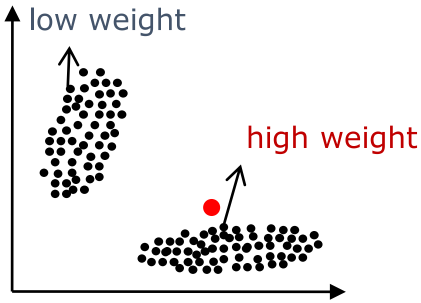
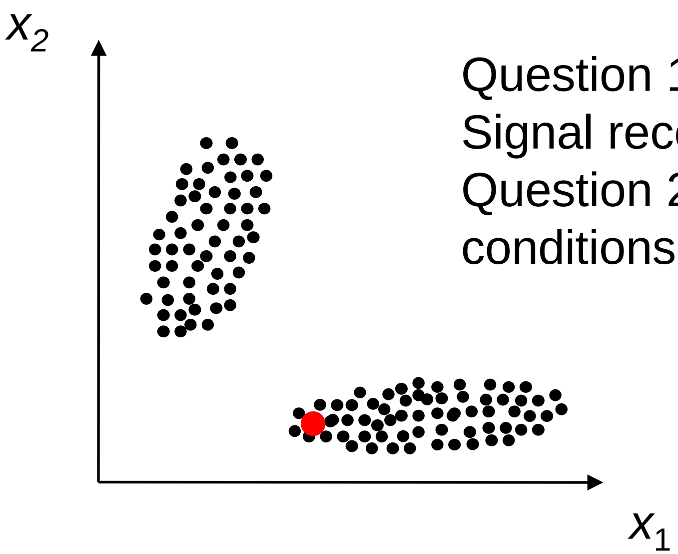


- with $d^2(k) = \sum_{j=1}^{n} (x_j^{obs} - x_{kj}^{obs-nc})^2$ Euclidean distance between $\vec{x}^{obs}$ and $\vec{x}_k^{obs-nc}$

AAKR: Exercise 1

- Signal values at current time: $\vec{x}^{obs} = (x_1^{obs}, \ldots, x_n^{obs})$ ●
- Historical signal measurements in normal plant condition: •

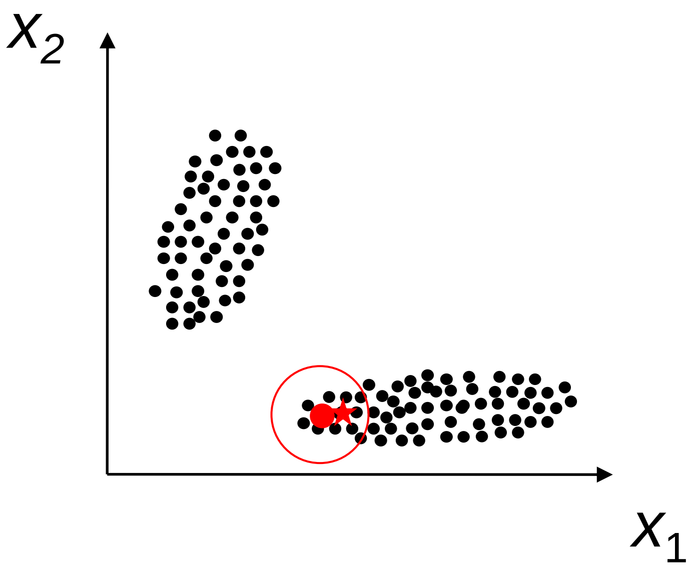


Question 1) Where do you expect to be the Signal reconstruction ★ $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \ldots, \hat{x}_n^{nc})$?
Question 2) Is the plant in normal or abnormal conditions?

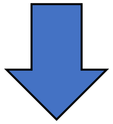
Exercise 1: Solution

•Signal values at current time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$ ●

•Signal reconstructions: $\hat{\vec{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$ ★ based on the available historical signal measurements in normal plant condition •



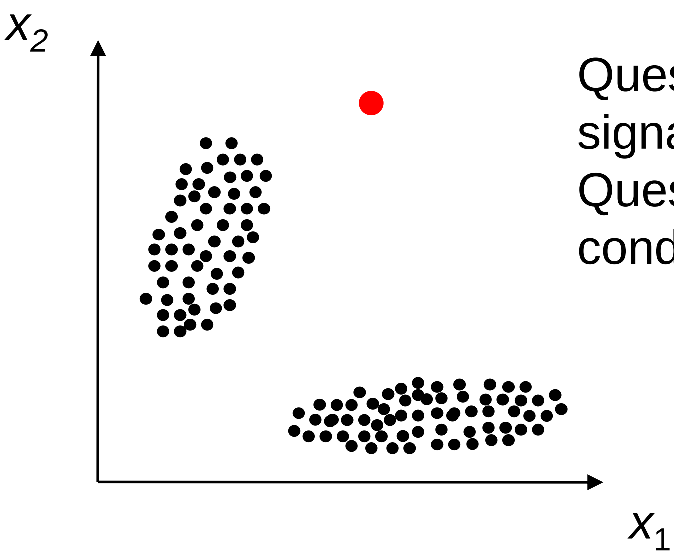
$$\vec{x}^{obs} \cong \hat{\vec{x}}^{nc}$$



normal condition

AAKR: Exercise 2

- Signal values at current time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$ ●
- Historical signal measurements in normal plant condition: •

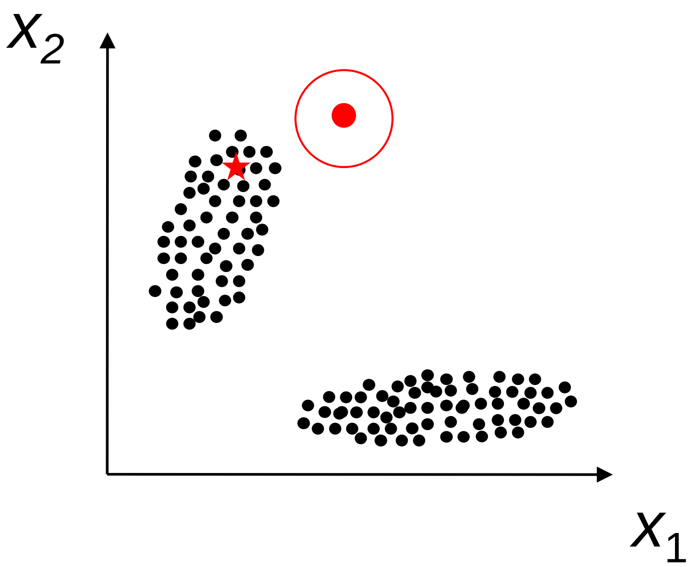


Question 1) Where do you expect to be the signal reconstruction ★ $\hat{\vec{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$?
Question 2) Is the plant in normal or abnormal conditions?

Exercise 2: Solution

•Signal values at current time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$ •

•Signal reconstructions: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \dots, \hat{x}_n^{nc})$ ★ based on the available historical signal measurements in normal plant condition •



$\vec{x}^{obs} \neq \vec{\hat{x}}^{nc}$

abnormal condition

•available historical signal measurements in normal plant condition

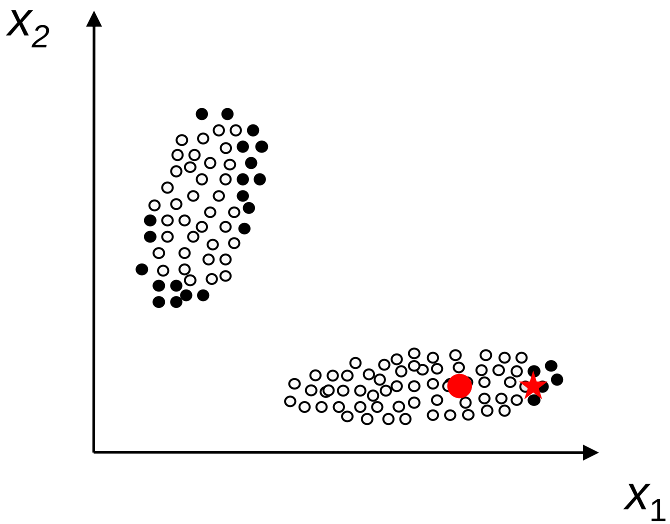
AAKR remarks: Computational Time

- Computational time:
  - No training of the model
  - Test:
    computational time depends on
    a) the number of training patterns N;
    b) the number of signals n.

$$d^2(k) = \sum_{j=1}^{n} (x_j^{obs} - x_{kj}^{obs-nc})^2$$

AAKR remarks: Accuracy

- Accuracy:
  - depends on the training set:
    - $\uparrow N \rightarrow \uparrow$ Accuracy



Few patterns and not well distributed in the training space

Inaccurate reconstruction

AAKR remarks

Pros:

- No need of hypothesis on data distribution (e.g. linearity)

Cons

- Performance related to number of training observations

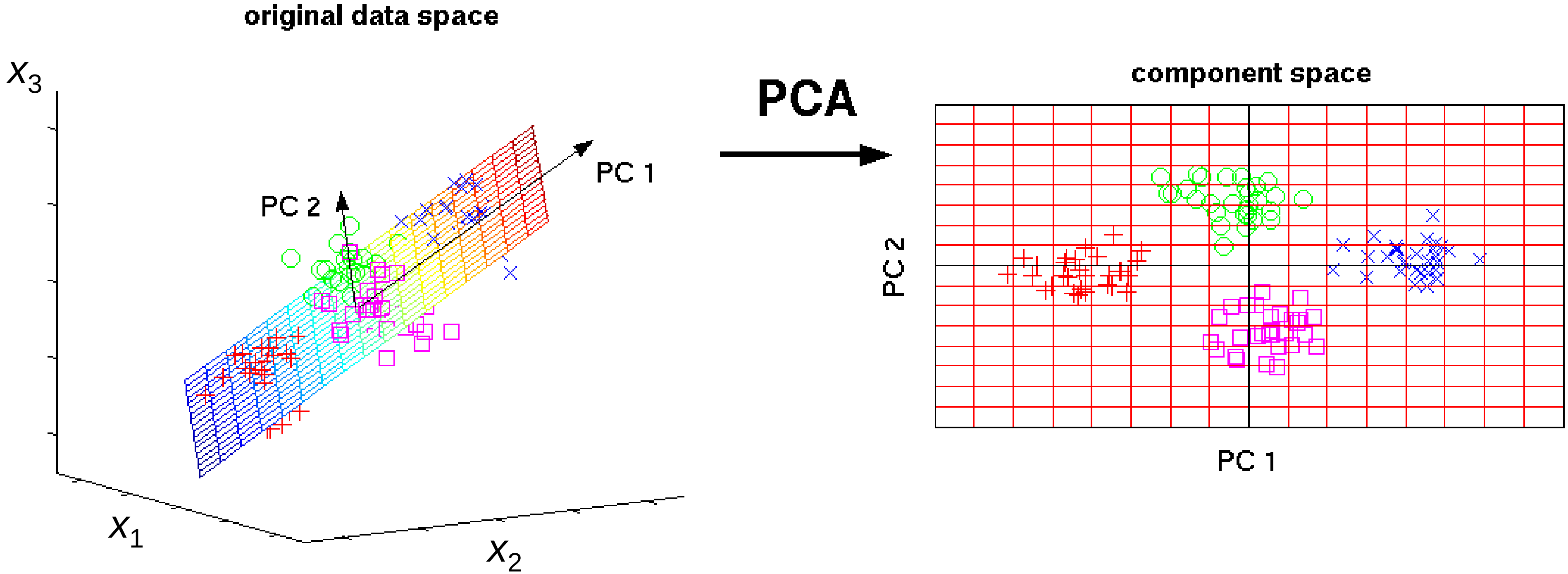
PART 1: Model of the Equipment Behaviour in Normal Condition

- 1A) Auto Associative Kernel Regression (AAKR)
- **1B) Principal Component Analysis (PCA)**

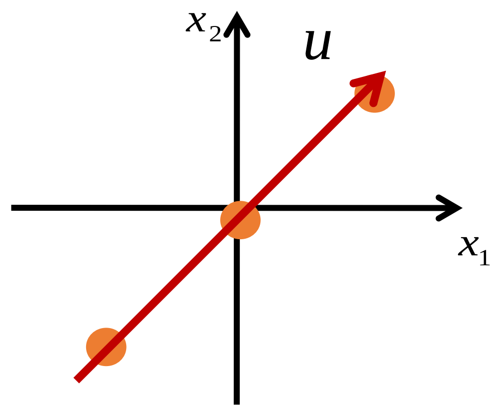
PCA: What is it?

PCA:

- Space transformation
- From an *n*-dimensional space to a l-dimensional space ($l < n$)
- Retaining most of the information (loosing the least information)

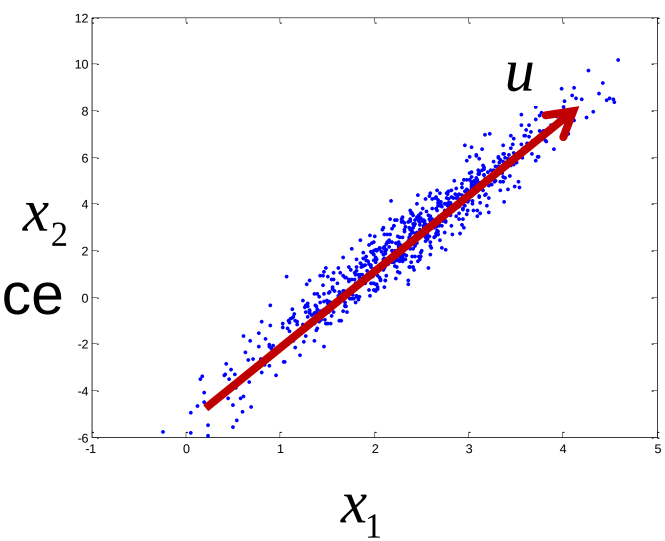


- Two signals are highly correlated or dependent
  ➔One is enough!



$$X = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ -1 & -1 \end{bmatrix}$$

- Key underlying phenomena
  ➔ Areas of variance in data
  ➔ Focus on directions along which the observations have largest variance



PCA: Training set, input and output = Slide 26 and Slide 27

- Training patterns:
  Historical signal measurements
  in normal condition

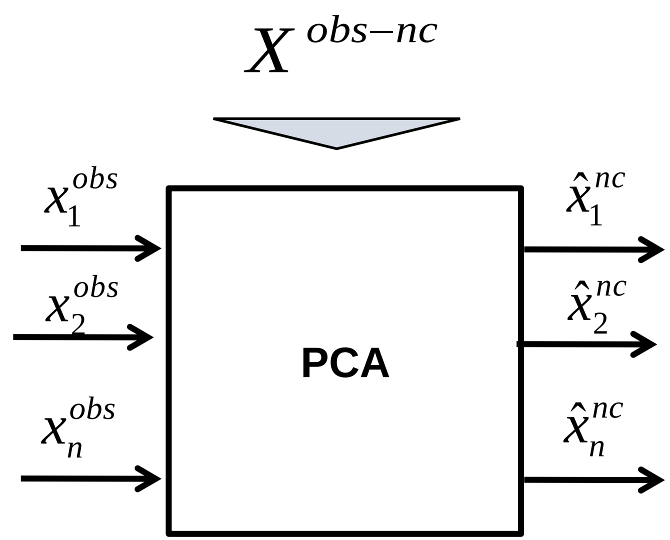
Signal

$$X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \cdots & x_{1j} & \cdots & x_{1n}^{obs-nc} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{k1} & \cdots & x_{kj} & \cdots & x_{kn} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{N1}^{obs-nc} & \cdots & x_{Nj} & \cdots & x_{Nn}^{obs-nc} \end{bmatrix}$$

Observation

- Test input: $\vec{x}^{obs} = (x_1^{obs}, \ldots, x_n^{obs})$
  Signals measured at current time

- Test Output: $\vec{\hat{x}}^{nc} = (\hat{x}_1^{nc}, \ldots, \hat{x}_n^{nc})$
  Signal reconstructions
  (expected values of the signals
  in normal condition)



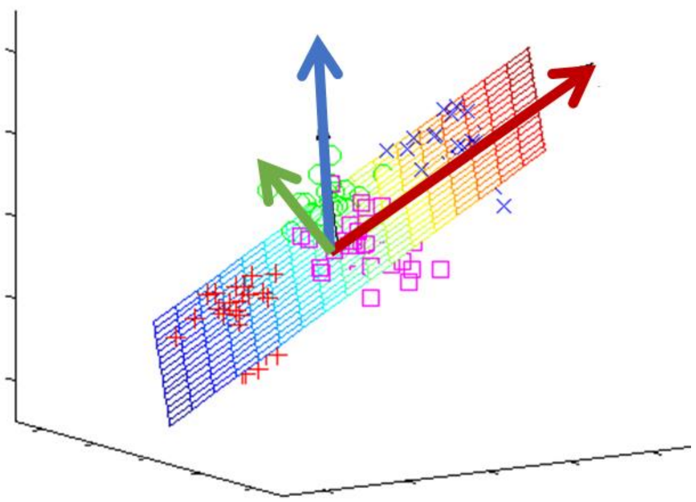
PCA for fault detection: operational steps (1)

Step 1: find Principal Components (PCs) in the training set X^{obs-nc}:

1) PC1 → is the direction of maximum variance

2) PC2 → is **orthogonal** to PC1 and describes the maximum residual variance

3) PC3 → is **orthogonal** to PC1 and PC2 and describes the maximum residual variance



Step 1: Mathematical details (1A)

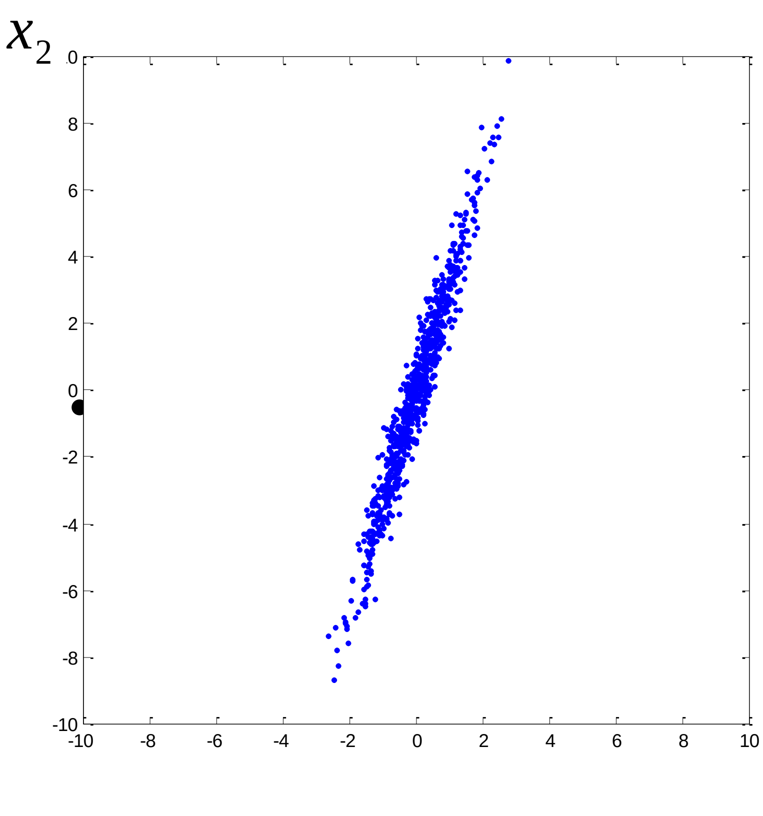
Objective: find principal components

Procedure:

- Compute V = covariance matrix of X^{obs-nc}

$$V = \left(X^{obs-nc} - \bar{X}^{obs-nc}\right)^T \left(X^{obs-nc} - \bar{X}^{obs-nc}\right)$$

Empirical mean matrix



$$\sum_{i=1}^{N}\left(x_{i1} - \bar{x}_i\right)^2$$

$$\sum_{i=1}^{N}\left(x_{i1} - \bar{x}_1\right)\left(x_{i2} - \bar{x}_2\right)$$

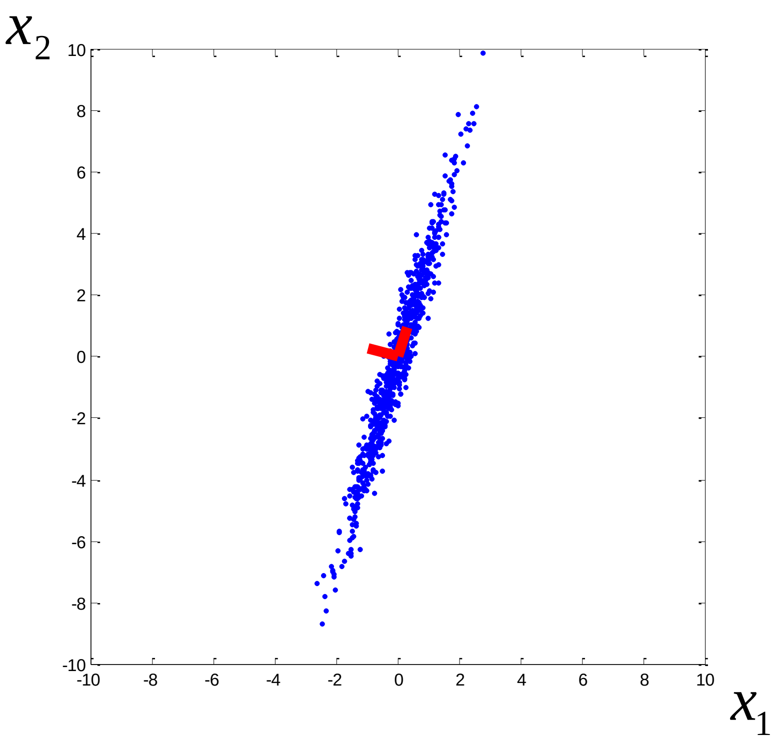
$$V = \begin{pmatrix} 0.79 & 2.55 \\ 2.55 & 8.75 \end{pmatrix}$$

Step 1: Mathematical Details (1B)

Objective: find principal components

Procedure:

- Compute V = covariance matrix of X^{obs-nc}
- Find the n eigenvectors $\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n$ of V and the corresponding eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$



$$V\vec{p} = \lambda\vec{p}$$

λ_1=9.50 largest

λ_2=0.04 smallest

$$\vec{p}_1 = [0.28 \quad 0.96]^T$$

$$\vec{p}_2 = [-0.96 \quad 0.28]^T$$

Step 1: Properties of the PCs (I)

$$P = [\vec{p}_1, \vec{p}_2] = \begin{bmatrix} 0.28 & -0.96 \\ 0.96 & 0.28 \end{bmatrix}$$

- *P* is an orthonormal basis:

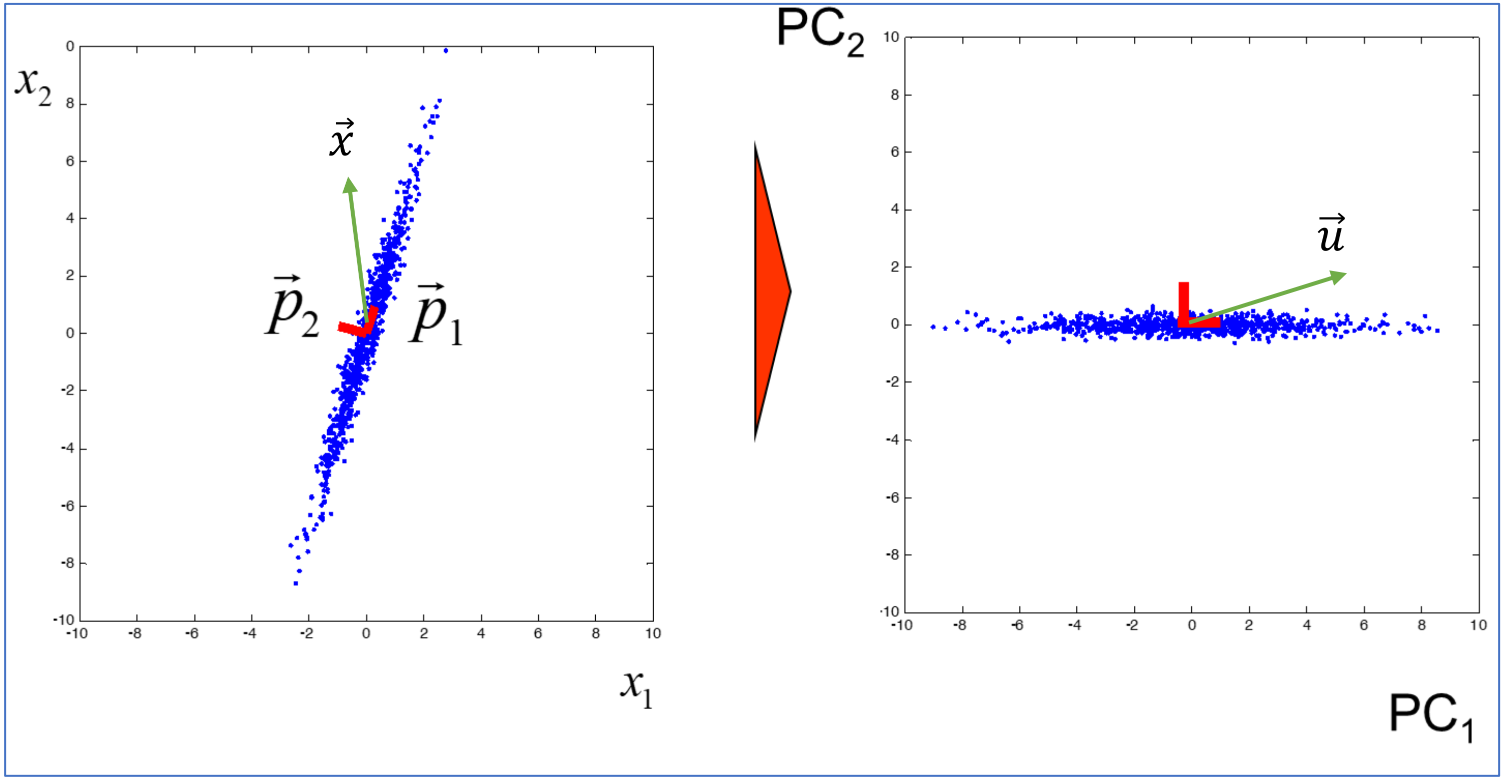
$$\vec{p}_i \perp \vec{p}_j \qquad \forall i \neq j$$

$$|\vec{p}_i| = 1$$

$$PP^T = I$$

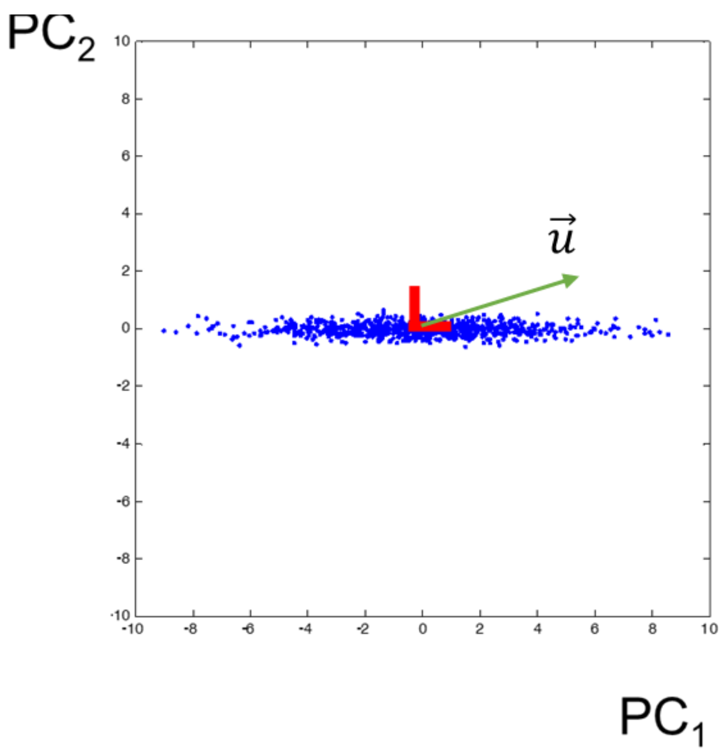
Step 1: Properties of the PCs (II)

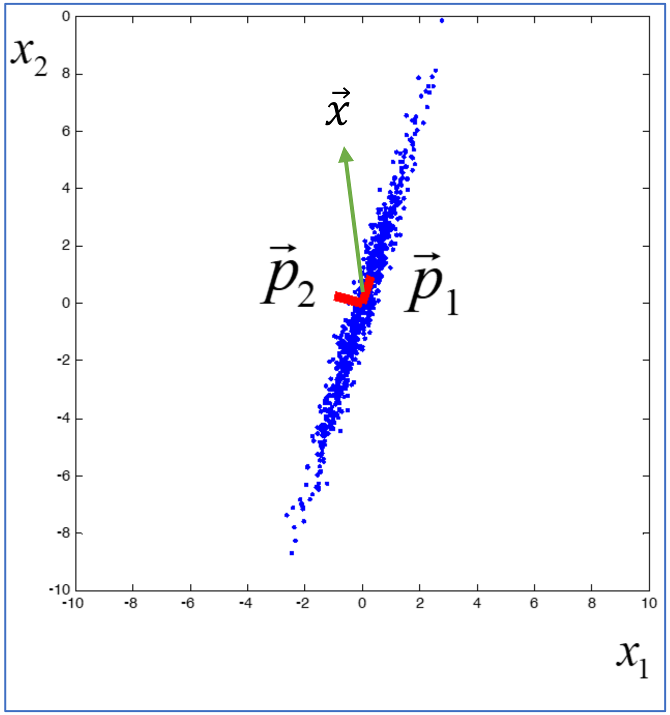
- *P* is an orthonormal basis:
- Data can be transformed from the original to the transformed bases and viceversa without any loss of information (multiplication for *P* and P^T)
  - $\vec{u}$ = the projection of $\vec{x}$ on the new basis is given by: $\vec{u} = \vec{x} \cdot P$



Step 1: Properties of the PCs (III)

- *P* is an orthonormal basis:
- Data can be transformed from the original to the transformed bases and viceversa without any loss of information (multiplication for *P* and P^T)
  - $\vec{u}$ = the projection of $\vec{x}$ on the new basis is given by: $\vec{u} = \vec{x} \cdot P$
  - $\vec{x}$ can be obtained from $\vec{u}$ by: $\vec{x} = \vec{u} \cdot P^T$





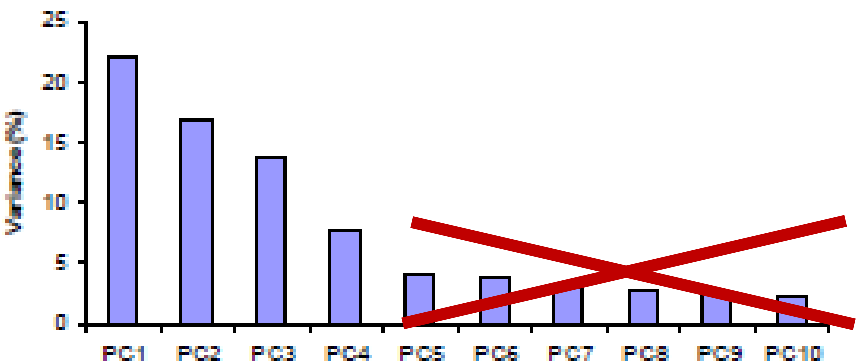
Step 1: Properties of the PCs (III)

- P is an orthonormal basis:
- Data can be transformed from the original to the transformed bases and viceversa without any loss of information (multiplication for P and P^T)
  - $\vec{u}$ = the projection of $\vec{x}$ on the new basis is given by: $\vec{u} = \vec{x} \cdot P$
  - $\vec{x}$ can be obtained from $\vec{u}$ by: $\vec{x} = \vec{u} \cdot P^T$
- The percentage of variance retained by the *i*-th principal component is:

$$\%Var\left(PC_i\right) = \frac{\lambda_i}{\sum_{i=1,\dots,n} \lambda_i}$$

PCA for fault detection: operational steps (2)

Step 2 [PCA approximation]: ignore the PCs of lower significance.



- Lost small information
- Reduce the number of dimensions from n=10 to = 4

PCA for fault detection: operational steps (2)

- Step 2 [PCA approximation]: ignore the PCs of lower significance.

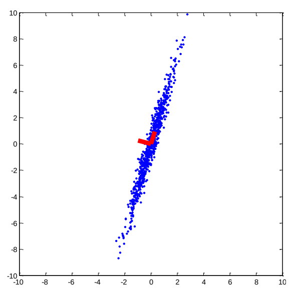
map the observation $\vec{x}^{obs}$ in a subspace $\Re^l \subset \Re^n$ identified by the first l< n eigenvectors $\vec{p}_1, \dots, \vec{p}_l$:

$$\vec{x}^{obs} P_l \quad \text{with } P_l = [\vec{p}_1, \dots, \vec{p}_l]$$

Example of application to the normal condition data: X^{obs-nc}

X^{obs-nc}

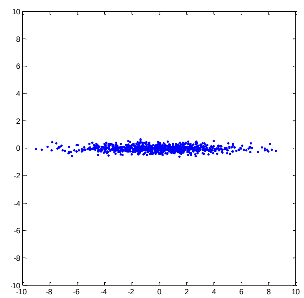
x_2



x_1

$X^{obs-nc}P$

PC2



PC1

$X^{obs-nc}P_l$

PC1

We consider only the directions that are most meaningful in normal condition (directions of maximum variance)

PCA for fault detection: operational steps (3)

- Step 2 [PCA approximation]: ignore the PCs of lower significance.

map the observation $\vec{x}^{obs}$ in a subspace $\Re^{l} \subset \Re^{n}$ identified by the first $l < n$ eigenvectors $\vec{p}_1, \ldots, \vec{p}_l$:

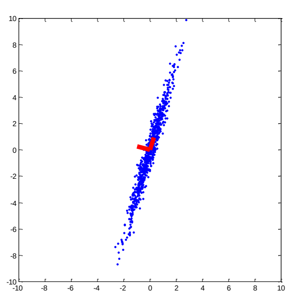
$$\vec{x}^{obs} P_l \quad \text{with } P_l = [\vec{p}_1, \ldots, \vec{p}_l]$$

- Step 3: [Antitransformation]: signal reconstructions $\hat{\vec{x}}^{nc} = \vec{x}^{obs} P_l \, P_l^T$

Example of application to the normal condition data: X^{obs-nc}

X^{obs-nc}

x_2



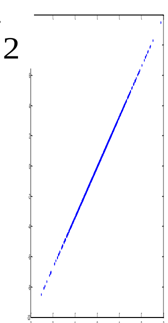
x_1

$P_\lambda = [\vec{p}_1, \ldots, \vec{p}_\lambda]$

$X^{obs-nc} P_\lambda$

PC1

x_2



x_1

We loose the noise in the space of the measured signals

PCA for fault detection: Summary

- Historical data $X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$ ➤ Find P_l from X^{obs-nc}

PCA for fault detection: Summary

- Historical data $X^{obs-nc} = \begin{bmatrix} x_{11}^{obs-nc} & \dots & x_{1j} & \dots & x_{1n}^{obs-nc} \\ \dots & \dots & \dots & \dots & \dots \\ x_{k1} & \dots & x_{kj} & \dots & x_{kn} \\ \dots & \dots & \dots & \dots & \dots \\ x_{N1}^{obs-nc} & \dots & x_{Nj} & \dots & x_{Nn}^{obs-nc} \end{bmatrix}$ → Find P_l from X^{obs-nc}
- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, \dots, x_n^{obs})$

    - Transform and project $\vec{x}^{obs} P_l$

I'm looking at the measurements considering only the directions that are most meaningful in normal condition (directions of maximum variance)

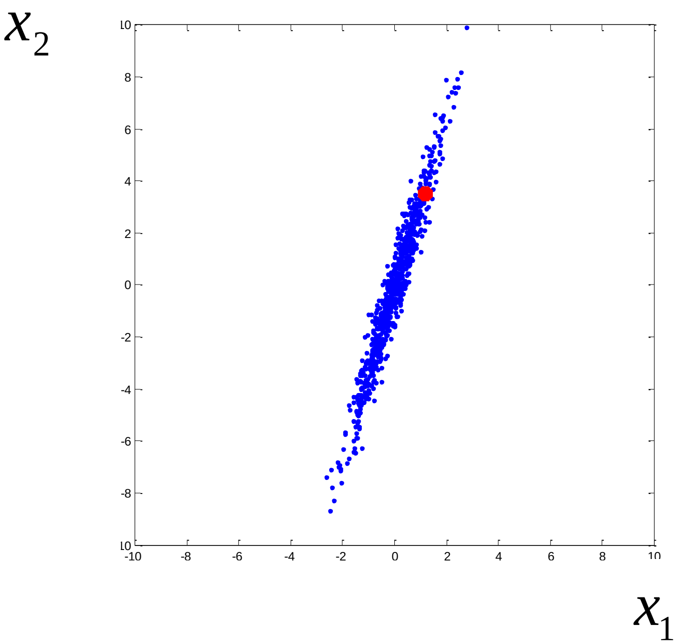
- Antitrnansform $\hat{\vec{x}}^{nc} = \vec{x}^{obs} P_l P_l^T$ Signal reconstructions

$\hat{\vec{x}}^{nc} \cong \vec{x}^{obs}$ → normal condition *I loose only the irrelevant noise*

$\hat{\vec{x}}^{nc} \neq \vec{x}^{obs}$ → abnormal condition *The process is changed*

Exercise 1

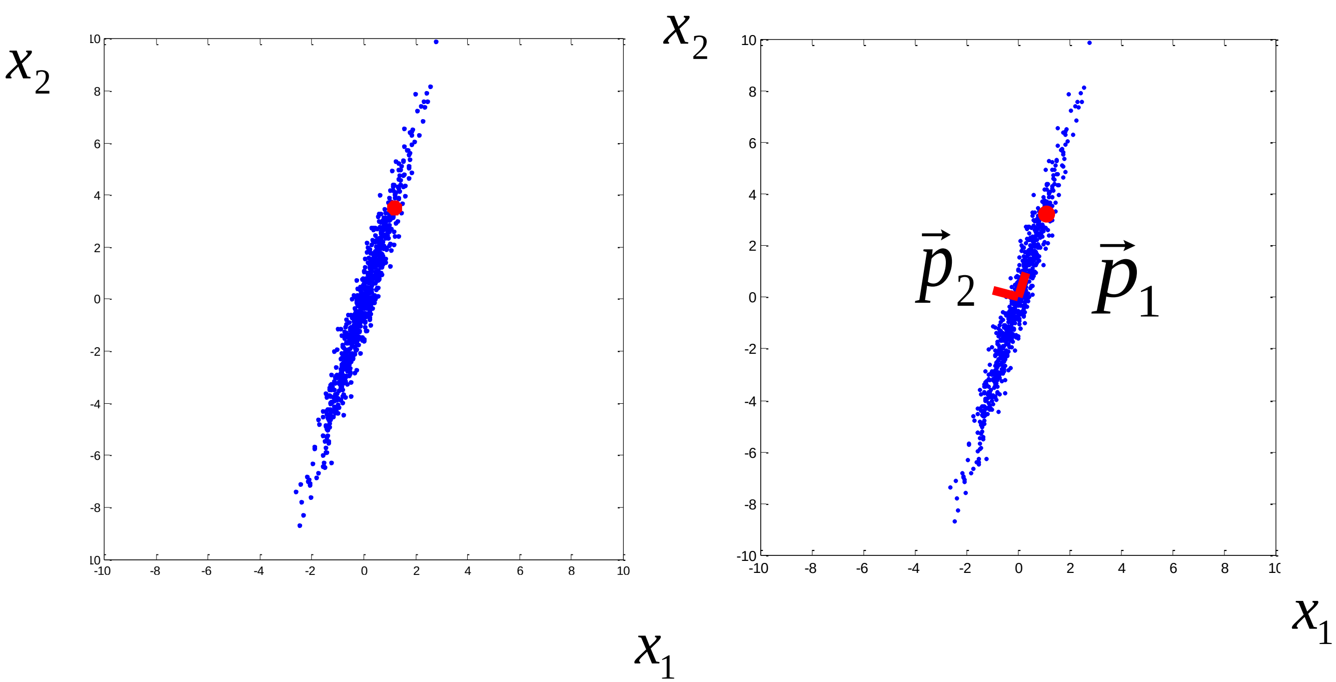
- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ •
- Signal reconstructions?
- Normal or abnormal condition?



- available historical signal measurements in normal plant condition

Exercise 1: Solution

- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ ●
- Step 1: find principal components: $\vec{p}_1$, $\vec{p}_2$



- available historical signal measurements in normal plant condition

Exercise 1: Solution

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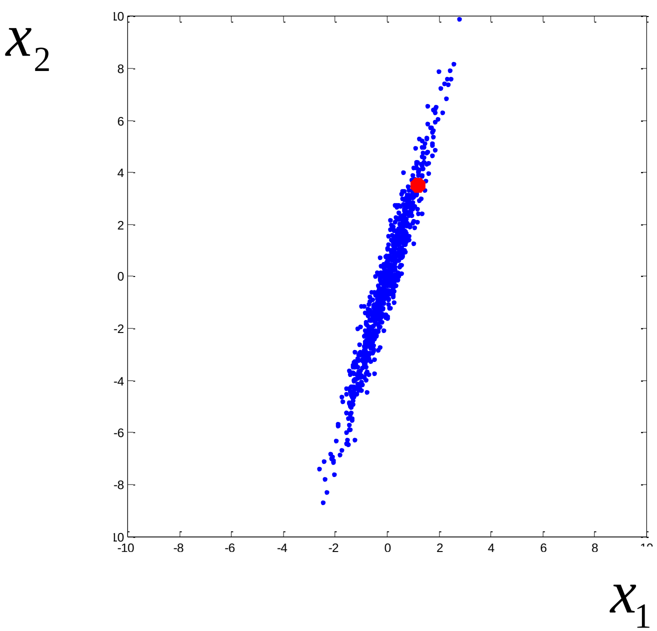
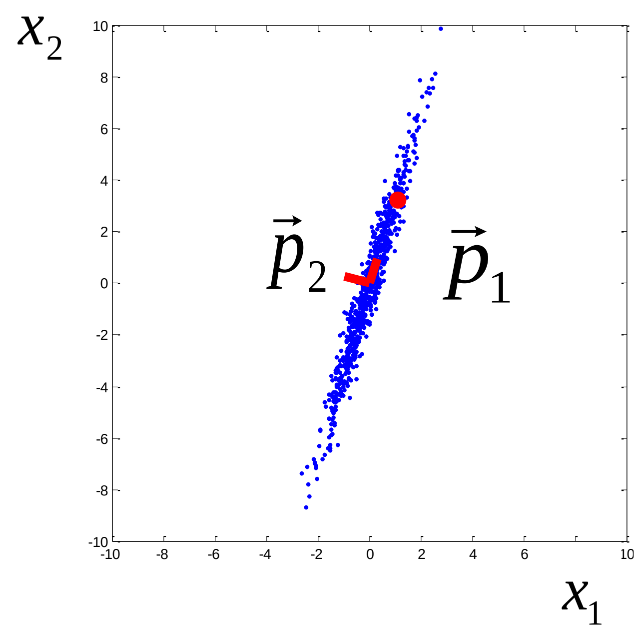
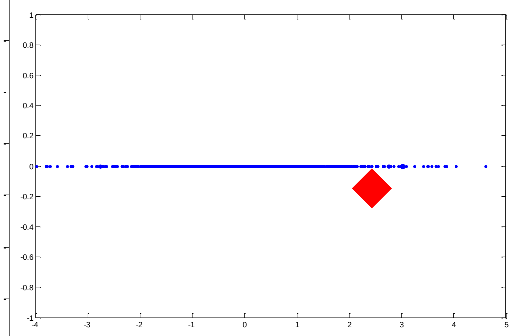
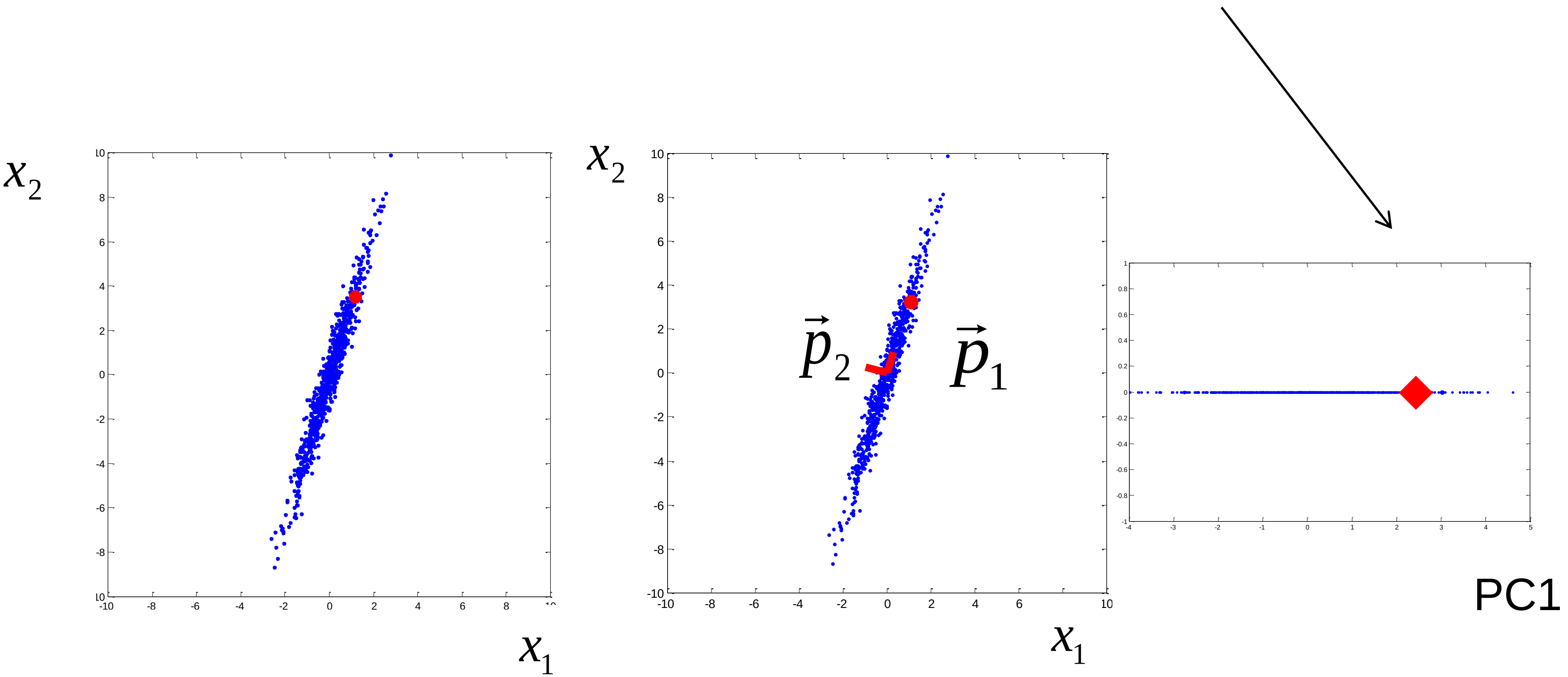






Exercise 1: Solution

- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ •
- Step 1: find principal components $\vec{p}_1, \vec{p}_2$
- Step 2 (PCA approximation): keep only 1 PC of • ,i.e. $\vec{x}^{obs} \cdot \vec{p}_1$



Exercise 1: Solution

- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$
- Step 1: find principal components
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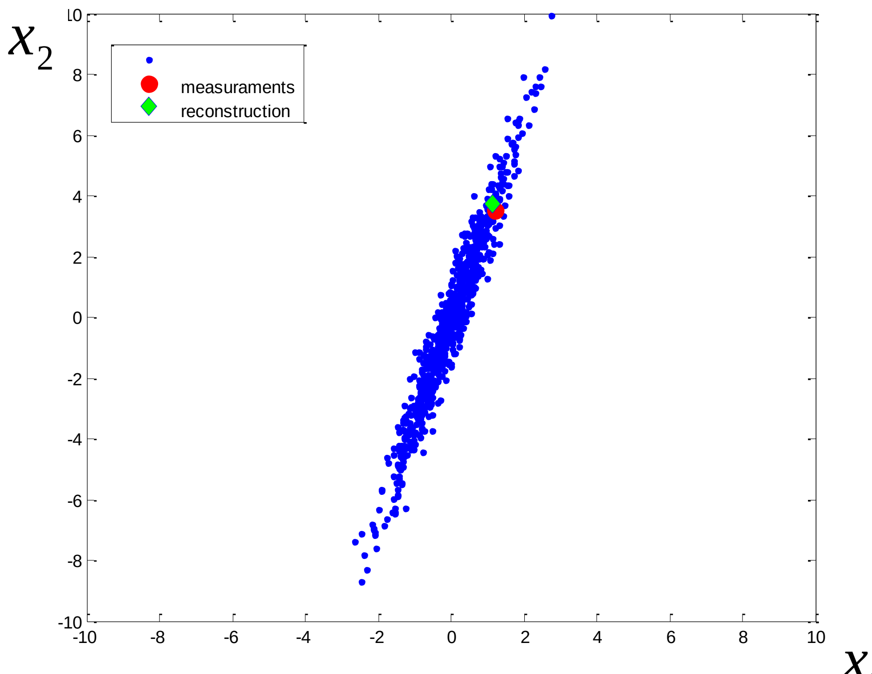
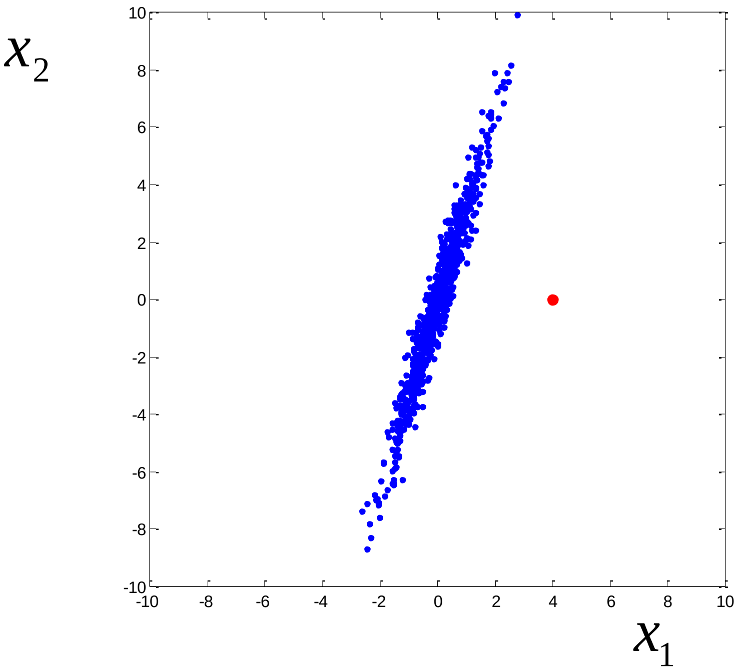
- Step 3 (antitransform): $\hat{\vec{x}}^{nc} = \vec{x}^{obs} P_l P_l^T$



$\vec{x}^{obs} \cong \hat{\vec{x}}^{nc}$ normal condition

Exercise 2

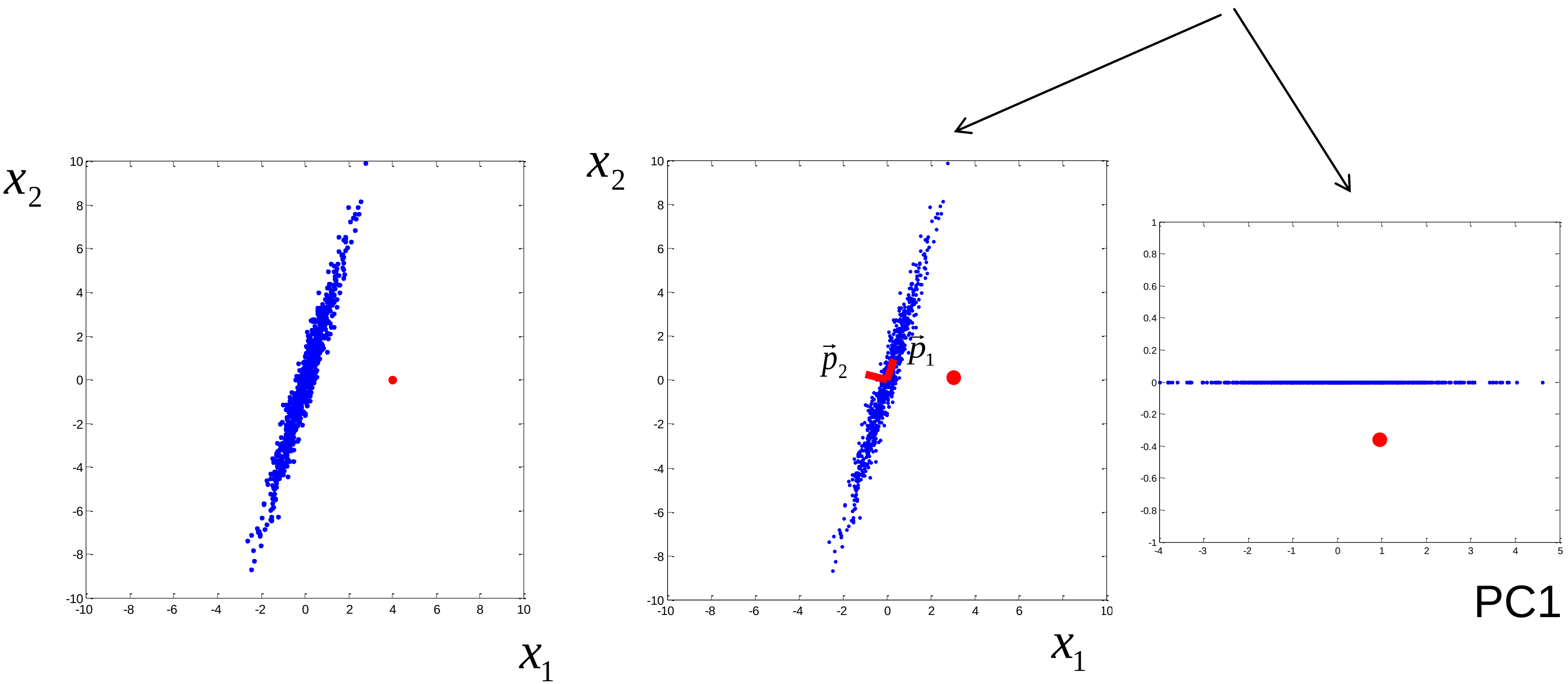
- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ ●
- Signal reconstructions?
- Normal or abnormal condition?



- available historical signal measurements in normal plant condition

Exercise 2: Solution

- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$
- Step 1: find principal components $\vec{p}_1\ \vec{p}_2$
- Step 2 (PCA approximation): keep only 1 PC of , i.e. $\vec{x}^{obs} \cdot \vec{p}_1$



Exercise 2: Solution

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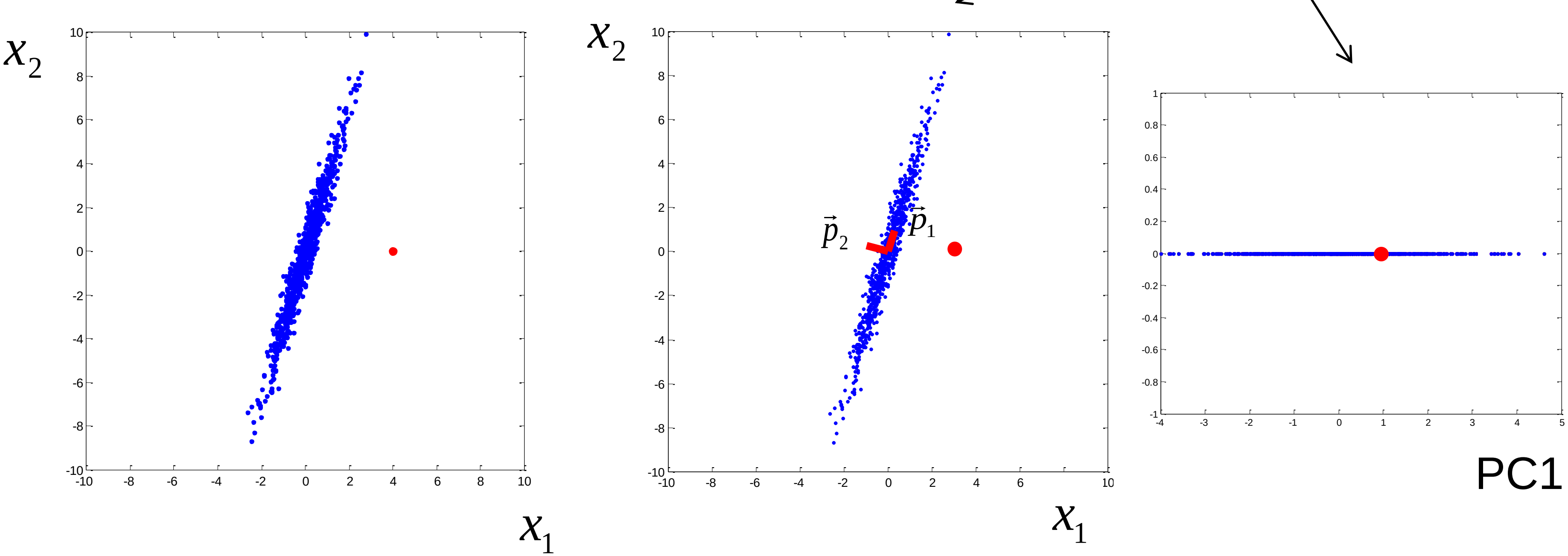
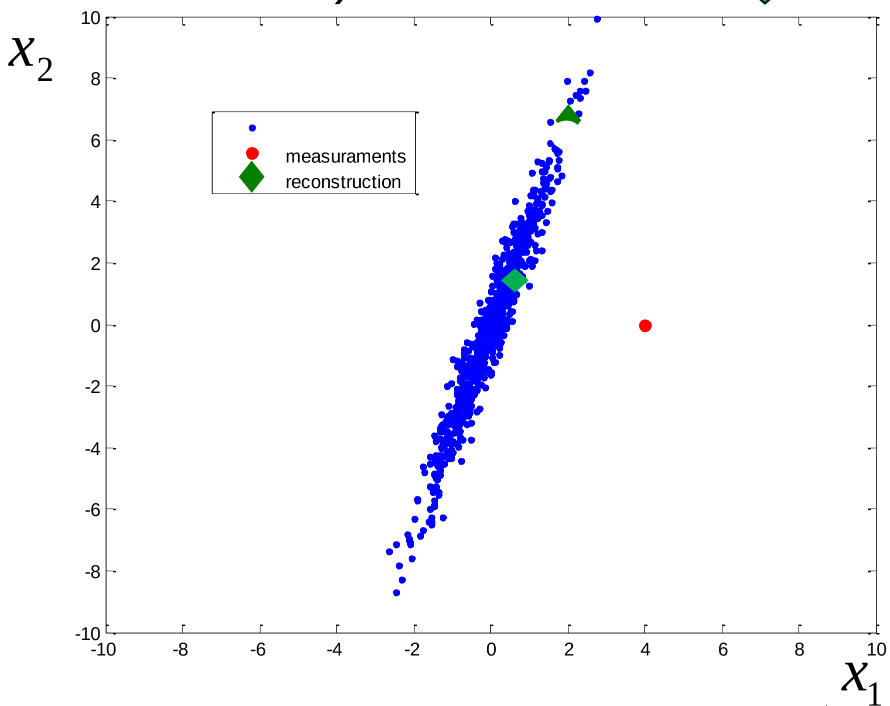


Exercise 2: Solution

- Measured signals at present time: $\vec{x}^{obs} = (x_1^{obs}, x_2^{obs})$ ●
- Step 1: find principal components
- Step 2 (PCA approximation): keep only 1 PC of ● ,i.e. $\vec{x}^{obs} \cdot \vec{p}_1$
- Step 3 (antitransform): $\hat{\vec{x}}^{nc} = \vec{x}^{obs} P_l P_l^T$ ◆



$\vec{x}^{obs} \neq \hat{\vec{x}}^{nc}$ ➡ Abnormal condition

PCA remarks: computational time

Computational time:

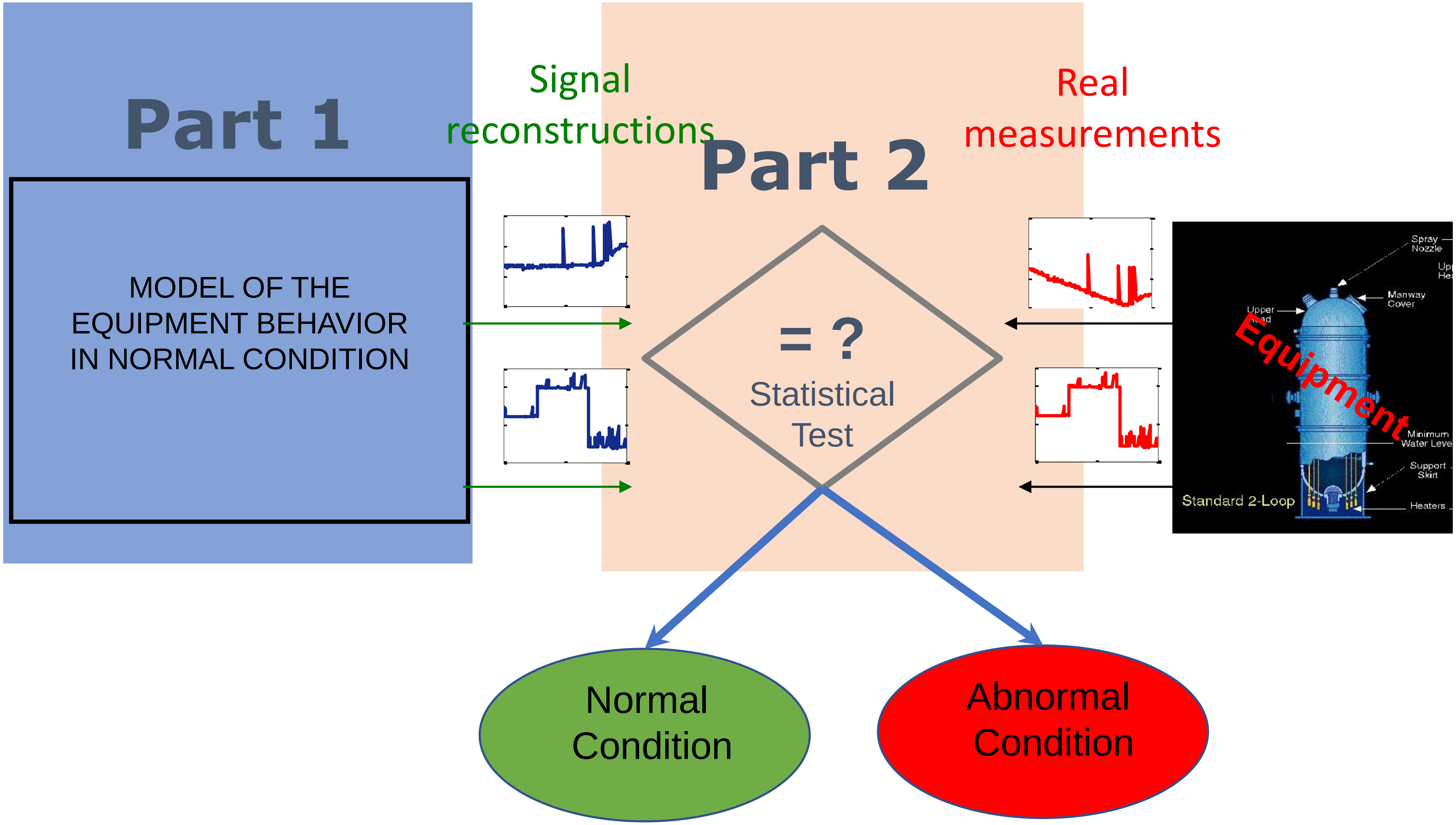
- Training time = computational time necessary to find the Principal Components is proportional to the number of measured signals n
- Execution time: very short (only 2 matrix multiplications) → OK for online applications

PCA remarks: complex structure

Performance:

Unsatisfactory for dataset characterized by highly non-linear relationships

Part 2: Statistical Test



In This Lecture

- Part 1: Model of the Equipment Behavior in Normal Condition
  - 1A) Auto Associative Kernel Regression (AAKR)
  - 1B) Principal Component Analysis (PCA)
- **Part 2: Statistical Test**
  - 2A) Thresholds-Based
  - 2B) Q-Statistics
  - 2C) Sequential Probability Ratio Test (SPRT)

PART 2: Statistical Test

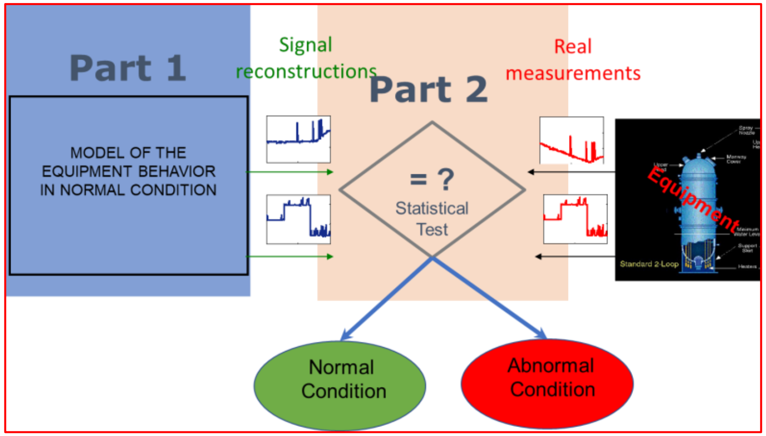
- Thresholds-based
- Q Statistics
- Sequential Probability Ratio Test (SPRT)

Abnormal condition detection: decision

- Basics of the decision: residual analysis

$$\vec{r} = \vec{x}^{obs} - \vec{\hat{x}}^{nc}$$

$$\begin{cases} \vec{r} \approx 0 \rightarrow & \text{Normal condition} \\ \vec{r} \neq 0 \rightarrow & \text{Abnormal condition} \end{cases}$$

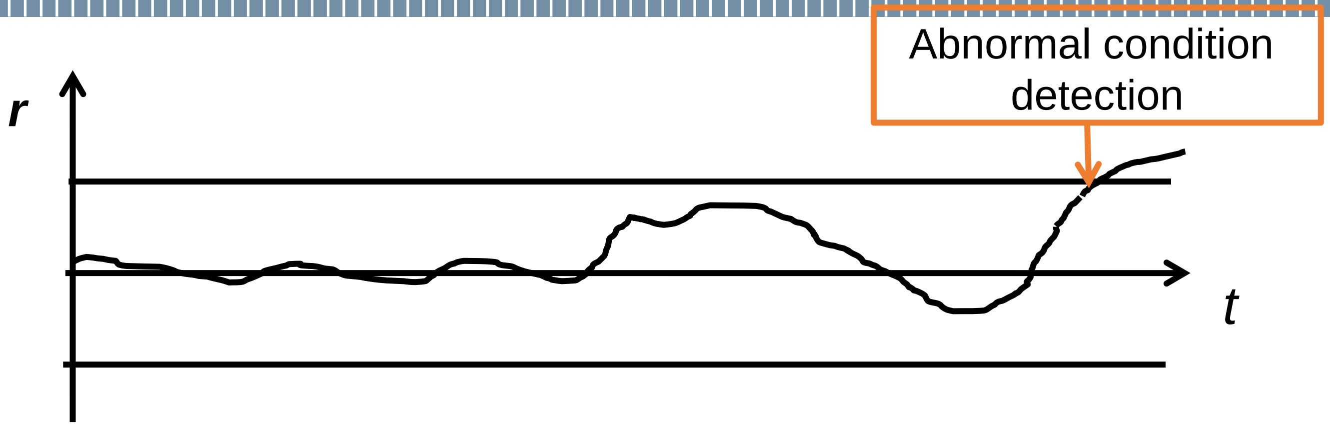


- Methods
  - Thresholds-based approach
  - Stochastic approaches:
    - Q Statistics
    - Sequential Probability Ratio Test (SPRT)

PART 2 A: Statistical Test

- **Thresholds-based**
- Q Statistics
- Sequential Probability Ratio Test (SPRT)

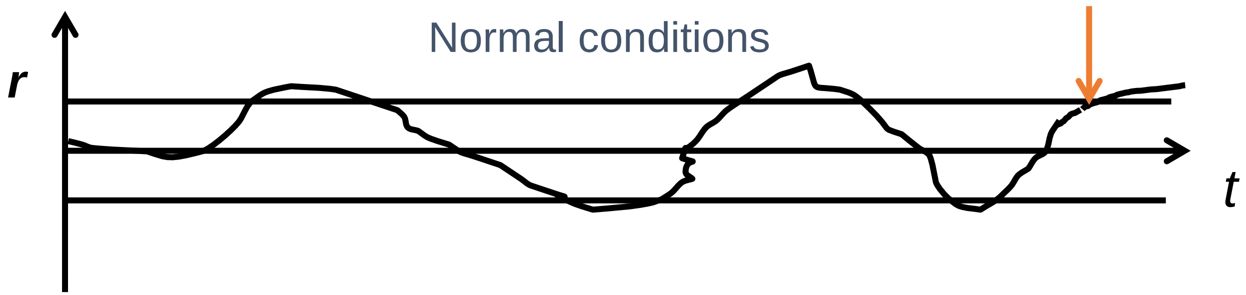
Thresholds-based



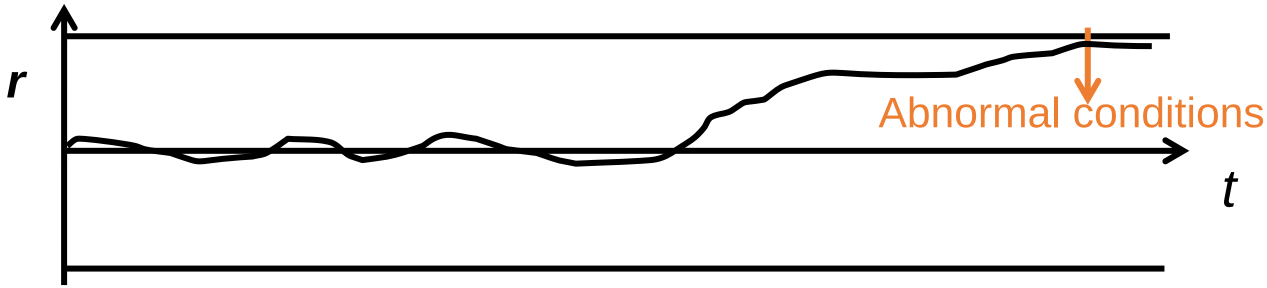
Thresholds-Based: Remarks

- Easy to apply
- Thresholds setting is difficult and error-prone

Too small thresholds → high false alarm rates (α)



Too large thresholds → high missing alarm rates (β)



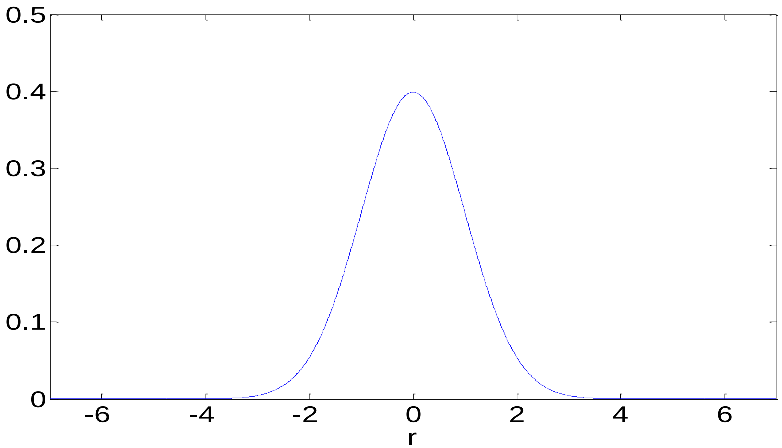
PART 2 B: Statistical Test

- Thresholds-based
- **Q-Statistics**
- Sequential Probability Ratio Test (SPRT)

Stochastic approaches

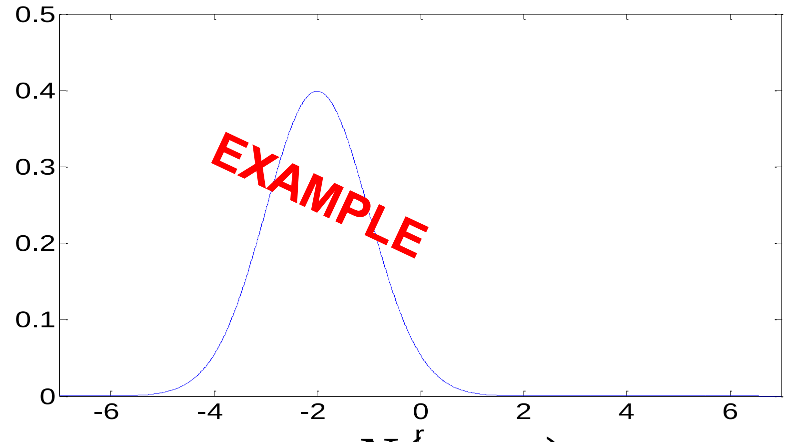
- Residual (*r*)= **random variable** described by a probability law
- The probability law is different in case of normal/abnormal condition

Normal condition



$$r \sim N(0, \sigma)$$

Abnormal condition



$$r \sim N(\mu_1, \sigma)$$

Q statistics

- Assuming the signal reconstructions at time t are:
  $\vec{\hat{x}}^{nc}(t) = (\hat{x}_1\ (t), \hat{x}_2\ (t), \dots, \hat{x}_n\ (t))$,

  then the Q-stat at time t is:

$$Q(t) = \vec{r}(t)^T \cdot \vec{r}(t) = \left(\vec{x}^{obs}(t) - \vec{\hat{x}}^{nc}(t)\right)\left(\vec{x}^{obs}(t) - \vec{\hat{x}}^{nc}(t)\right)^T =$$

$$= \sum_{i=1}^{n} (x_i^{obs}(t) - \hat{x}_i^{nc}(t))^2$$

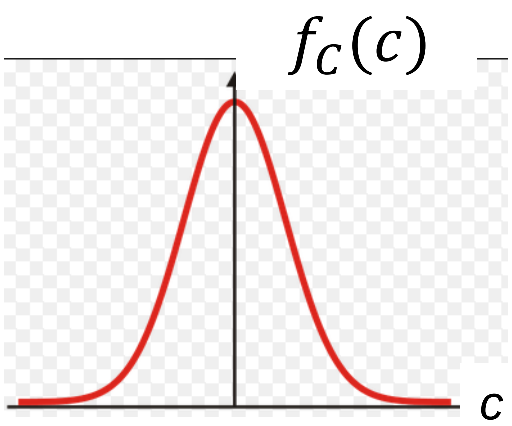
The Q-statistics (squared prediction error) is a metric that accounts for the amount of variance that is not captured by the **chosen** l - dimensional PCA model, which represents the "normal behaviour" of the signals.

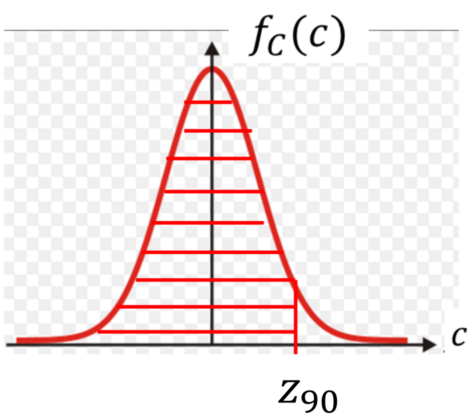
Q Statistics

- Let $\vartheta_i = \sum_{k=l+1}^{n} \lambda_k^i$, $i = 1,2,3$; $h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}$

Theorem ,see Appendix 1, J.E. Jackson and G.S. Mudholkar, Technometrics, 21(3), (1979), 341-349

- $c = \frac{\theta_1\left[\left(\frac{Q}{\theta_1}\right)^{h_0} - 1 - \frac{\theta_2 h_0 (h_0 - 1)}{\theta_1^2}\right]}{\sqrt{2\theta_2 h_0^2}}$ is approximately $N(0,1)$





$$F_C(c) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{c} e^{-\frac{1}{2}\xi^2} d\xi$$

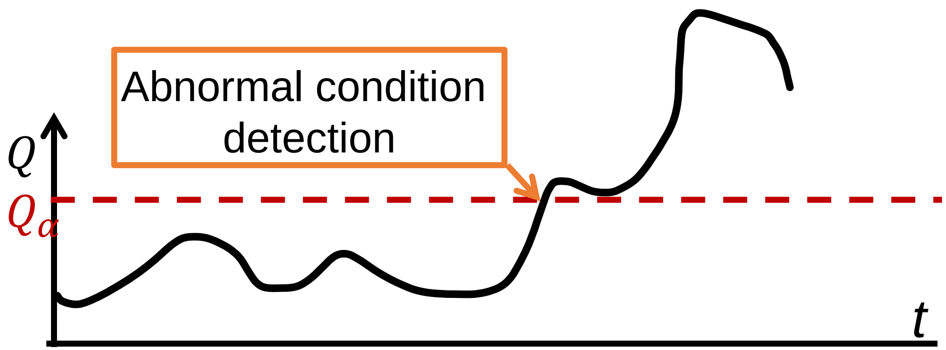
from tables

Equipment in normal condition → $P(c \le z_{90})$=0.9

Q Statistics

$$Q_\alpha = \vartheta_1 \left(\frac{h_0 z_\alpha \sqrt{2\theta_2}}{\theta_1} + 1 + \frac{h_0 \theta_2 (h_0 - 1)}{\theta_1^2} \right)^{1/h_0}$$

→ $P(Q \leq Q_{90})$=0.9



$P(abnormal\ condition | normal\ condition)$=0.1

FALSE POSITIVE

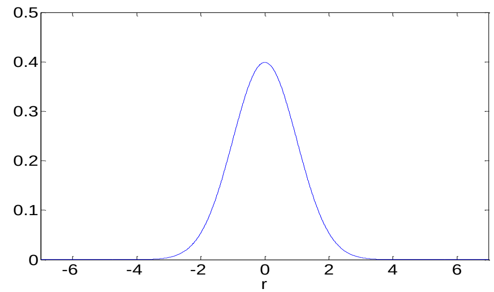
PART 2 C: Statistical Test

- General Idea
- Q Statistics
- **Sequential Probability Ratio Test (SPRT)**

SPRT

- $R_T = \{r^{(1)}, \dots, r^{(T)}\}$ sequence of residuals at time $t = 1, \dots T$, where $r^{(t)} = x^{obs}(t) - \hat{x}^{nc}(t)$
- Binary hypothesis test:
  - Null hypothesis (H_0) ≡ Normal condition

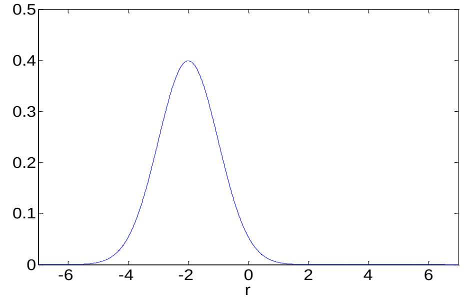
$$r^{(t)} \sim \mathcal{N}(0, \sigma), \forall t$$



$$f_0(r) \equiv N(0, \sigma)$$

  - Alternative hypothesis (H_1) ≡ Abnormal condition

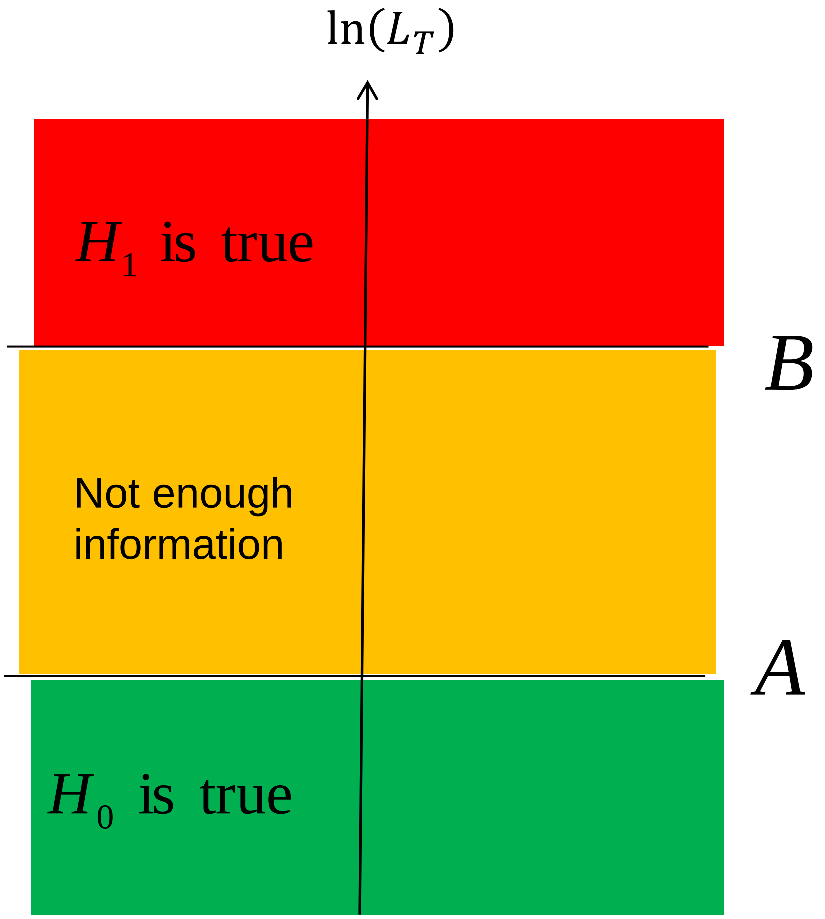
$$r^{(t)} \sim \mathcal{N}(\mu_1, \sigma), \forall t$$



$$f_1(r) \equiv N(\mu_1, \sigma)$$

SPRT: the decision

$$L_T = \frac{P\{R_T|H_1 \text{is true}\}}{P\{R_T|H_0 \text{is true}\}} = \frac{f_1(r^{(1)}) \cdot f_1(r^{(2)}) \cdot \ldots \cdot f_1(r^{(T)})}{f_0(r^{(1)}) \cdot f_0(r^{(2)}) \cdot \ldots \cdot f_0(r^{(T)})}$$



SPRT Theorem

$$B = \ln \frac{1-\beta}{\alpha}$$

$$A = \ln \frac{\beta}{1-\alpha}$$

False alarm

$$P\{D_1 \mid H_0 \text{ is true}\} \leq \alpha$$

$$P\{D_0 \mid H_1 \text{ is true}\} \leq \beta$$

Missing alarm

SPRT for the positive mean test

- Null hypothesis (H_0) ≡ Normal condition $r^{(t)} \sim \mathcal{N}(0, \sigma)$
- Alternative hypothesis (H_1) ≡ Abnormal condition $r^{(t)} \sim \mathcal{N}(\mu_1, \sigma)$

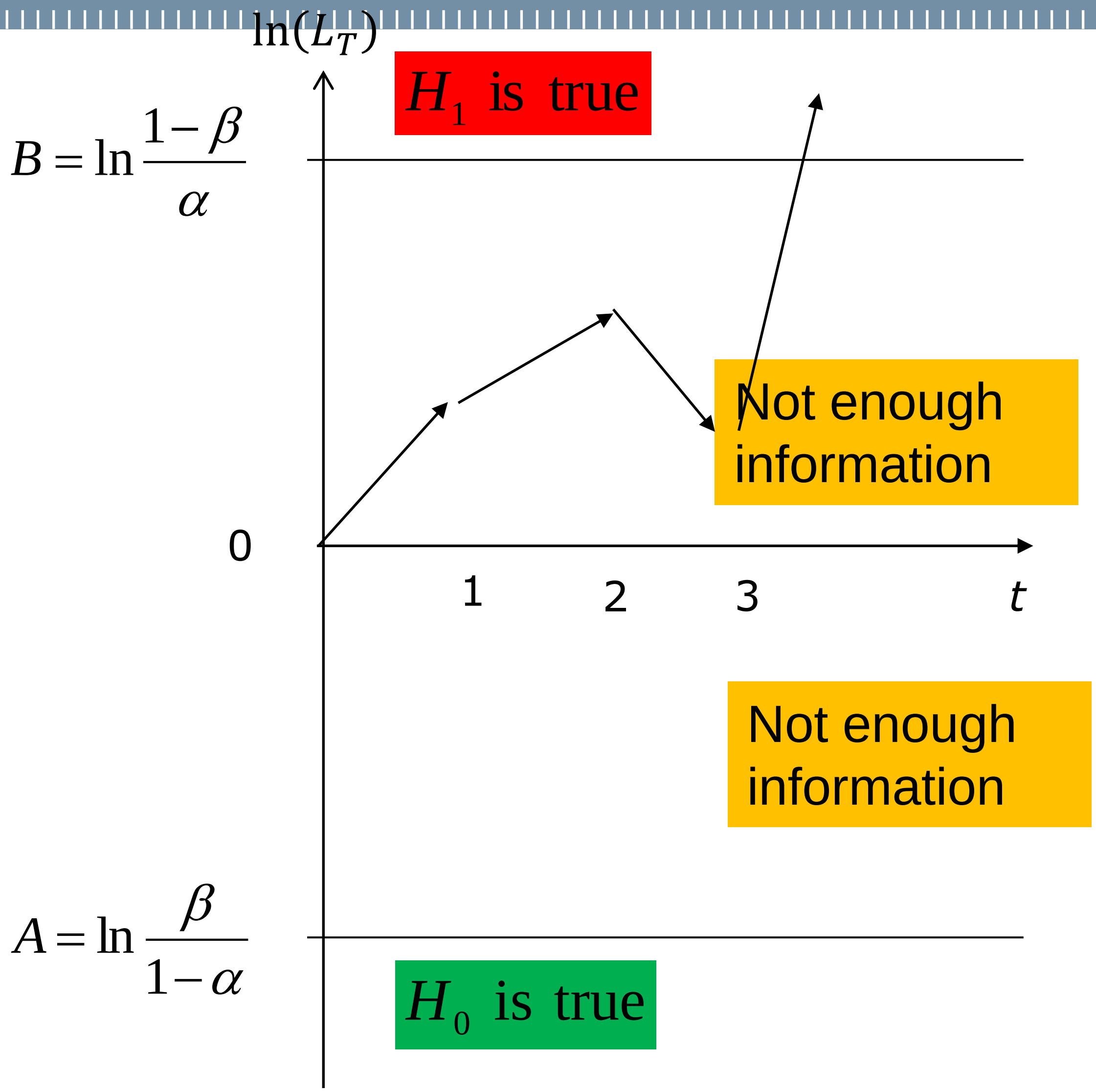
$$L_T = \frac{P(r^{(1)}, \dots, r^{(T)} | H_1)}{P(r^{(1)}, \dots, r^{(T)} | H_0)} = e^{-\frac{1}{2\sigma^2}\sum_{t=1}^{T} \mu_1(\mu_1 - 2r^{(t)})} = e^{\frac{\mu_1}{\sigma^2}\sum_{t=1}^{T}\left(r^{(t)} - \frac{\mu_1}{2}\right)}$$

$$\ln(L_T) = \frac{\mu_1}{\sigma^2}\sum_{t=1}^{T}\left(r^{(t)} - \frac{\mu_1}{2}\right) = \frac{\mu_1}{\sigma^2}\sum_{t=1}^{T-1}\left(r^{(k)} - \frac{\mu_1}{2}\right) + \frac{\mu_1}{\sigma^2}\left(r^{T} - \frac{\mu_1}{2}\right)$$
$$= \ln(L_{T-1}) + \frac{\mu_1}{\sigma^2}\left(r^{(T)} - \frac{\mu_1}{2}\right)$$

Sequential Formula!

SPRT: Example

$$L_0 = 1 \rightarrow \ln(L_0) = 0$$
$$\ln(L_1) = \ln(L_0) + \frac{\mu_1}{\sigma^2}\left(r^{(1)} - \frac{\mu_1}{2}\right)$$
$$\ln(L_2) = \ln(L_1) + \frac{\mu_1}{\sigma^2}\left(r^{(2)} - \frac{\mu_1}{2}\right)$$

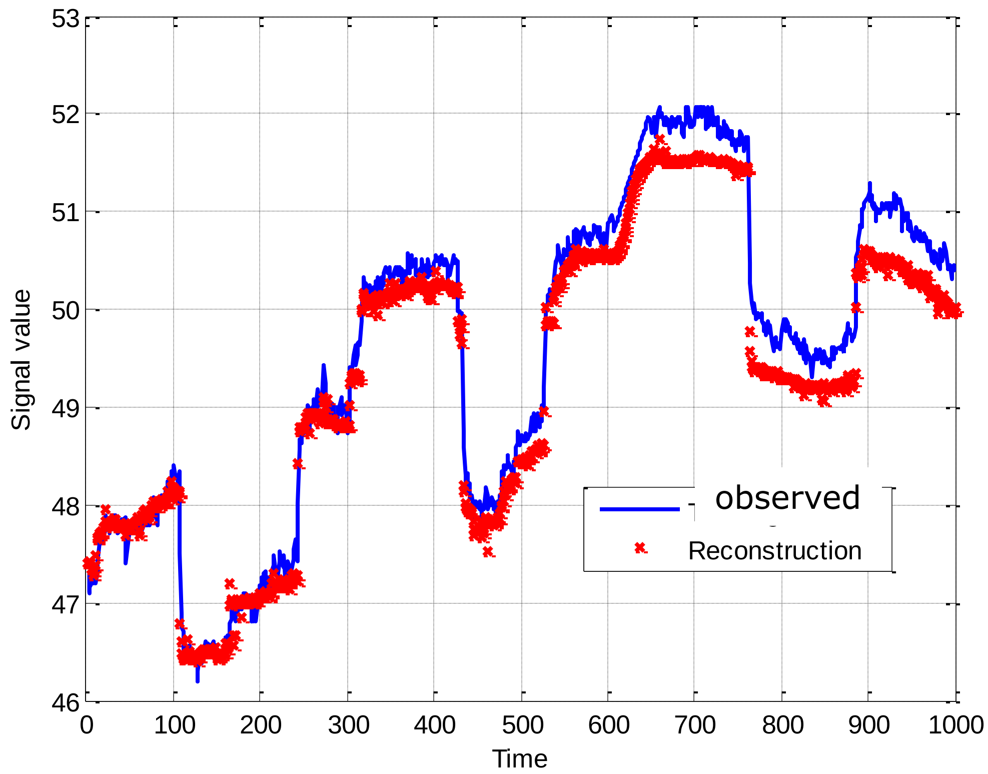


SPRT: parameters to be set

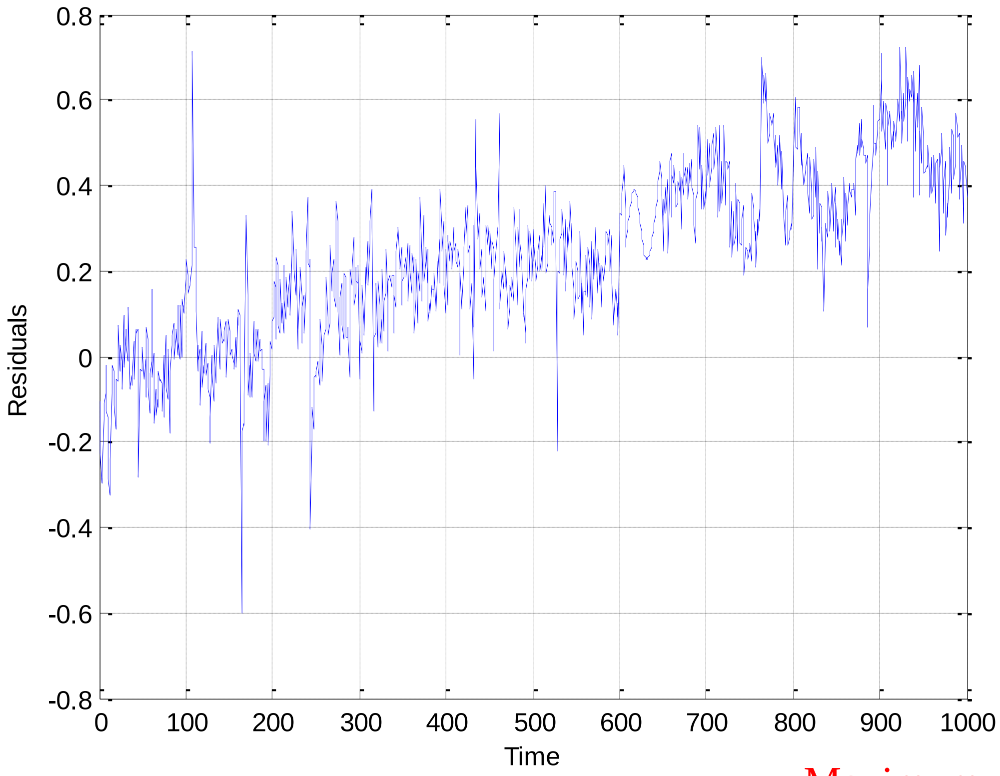
- the residual variance in normal condition (σ^2)
- the expected offset amplitude (μ_1)
- the maximum acceptable false alarm rate (α)
- the maximum acceptable missing alarm rate (β)

Example

Time interval	Simulated Offset
[0-200]	No
[201-400]	Yes (amplitude = 0.11)
[401-600]	Yes (amplitude = 0.23)
[601-800]	Yes (amplitude = 0.34)
[801-1000]	Yes (amplitude = 0.46)



Example: residuals



No offset

Maximum offset

Example: SPRT

H_1 is true

Parameter	Value
α	0.01
B	0.01
μ_0	0
μ_1	0.46
σ^2	0.12

$$B = \ln \frac{1-\beta}{\alpha}$$

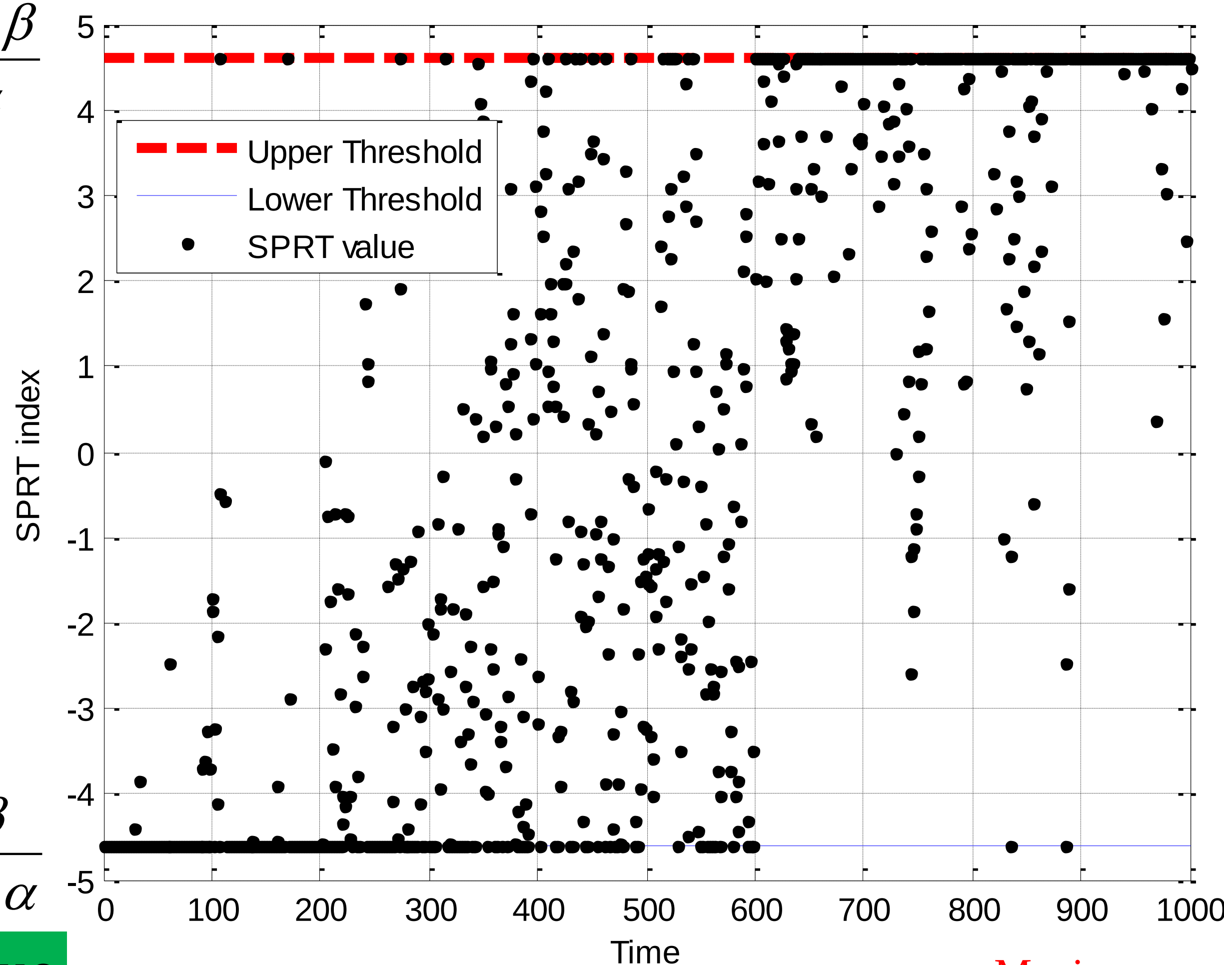
$$L_0 = 1 \rightarrow \ln(L_0) = 0$$

$$\ln(L_1) = \ln(L_0) + \frac{\mu_1}{\sigma^2}\left(r^{(1)} - \frac{\mu_1}{2}\right)$$

$$\ln(L_2) = \ln(L_1) + \frac{\mu_1}{\sigma^2}\left(r^{(2)} - \frac{\mu_1}{2}\right)$$

$$A = \ln \frac{\beta}{1-\alpha}$$

H_0 is true



No offset

Maximum offset

SPRT: performance

Average Sample Number (ASN) needed to deliver a decision

Time interval	Offset	Estimated ASN	Number of times in which a normal condition has been detected	Number of times in which an abnormal condition has been detected
[0-200]	No	1.2	150	2
[201-400]	amplitude = 0.11	1.9	70	5
[401-600]	amplitude = 0.23	2.4	15	17
[601-800]	amplitude = 0.34	2.1	0	94
[801-1000]	amplitude = 0.46	1.2	2	142

Challenges?

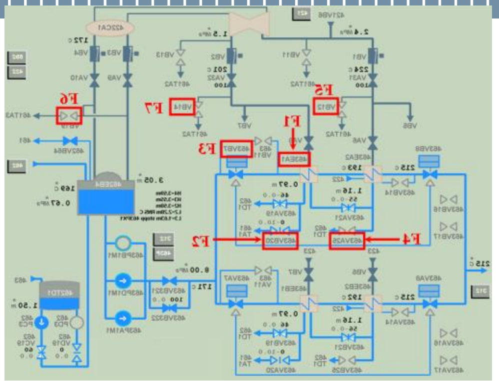
Fault Detection - Challenges

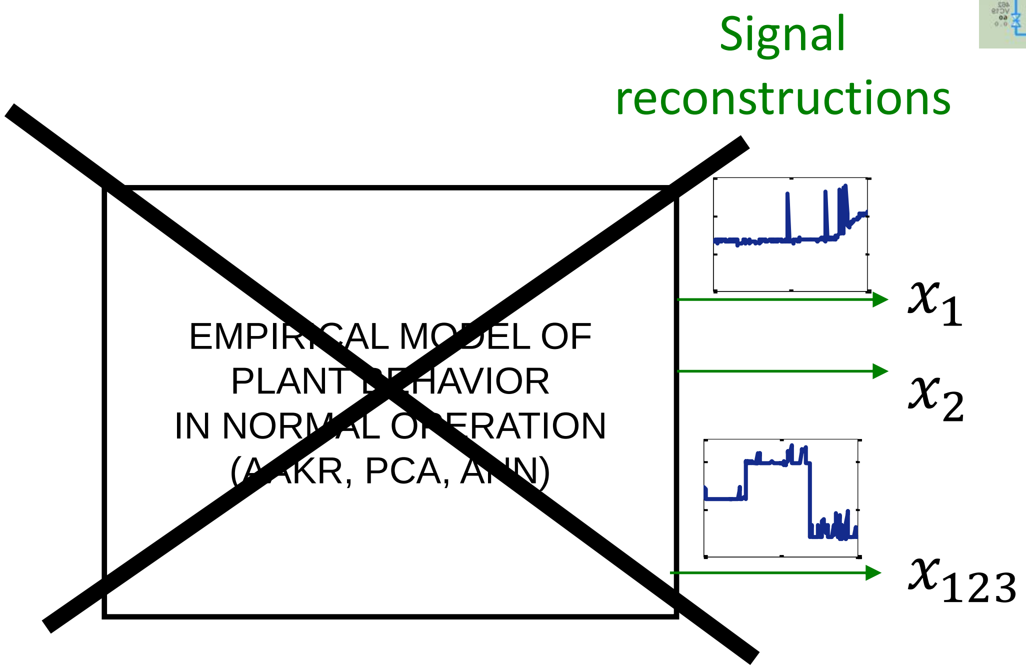
- Hundreds of Signals are Monitored
- Evolving Environment
- Robustness

Data-Driven Fault Detection: Challenge I

1	#312KA031 VALUE	kg/s	sum feedwater flows
2	#312KA032 VALUE	kg/s	sum feedwater flows
3	#312KA033 VALUE	kg/s	sum feedwater flows
4	#312KA201 VALUE	MPa	pressure difference over VA23
5	#312KA301 VALUE	kg/s	flow train A
6	#312KA302 VALUE	kg/s	flow train A
7	#312KA303 VALUE	kg/s	flow train A
8	#312KA306 VALUE	kg/s	flow low power (fløde låglast)
9	#312KA502 VALUE	C	feedwater temp train A
10	#312KA503 VALUE	C	temp after VA8
11	#312KC301 VALUE	kg/s	flow train C
12	#312KC302 VALUE	kg/s	flow train C
13	#312KC303 VALUE	kg/s	flow train C
14	#312KC502 VALUE	C	feedwater temp line C
15	#313KA511 VALUE	C	temp PA1 (main circulation pump)
16	#313KA512 VALUE	C	temp PA2 (main circulation pump)
17	#313KA711 VALUE	mm/s	vibration PA1 radial
18	#313KA712 VALUE	mm/s	vibration PA2 radial
19	#313KA721 VALUE	mm/s	vibration PA1 tangential
20	#313KA722 VALUE	mm/s	vibration PA2 tangential
21	#313KA731 VALUE	rpm	rotation speed PA1
22	#313KA732 VALUE	rpm	rotation speed PA2
23	#313KB511 VALUE	C	temp PB1 (main circulation pump)
24	#313KB512 VALUE	C	temp PB2 (main circulation pump)
25	#313KB711 VALUE	mm/s	vibration PB1 radial
26	#313KB712 VALUE	mm/s	vibration PB2 radial
27	#313KB721 VALUE	mm/s	vibration PB1 tangential
28	#313KB722 VALUE	mm/s	vibration PB2 tangential
29	#313KB731 VALUE	rpm	rotation speed PB1
…	…	…	…
123	#313KC511 VALUE	C	temp PC1 (main circulation pump)

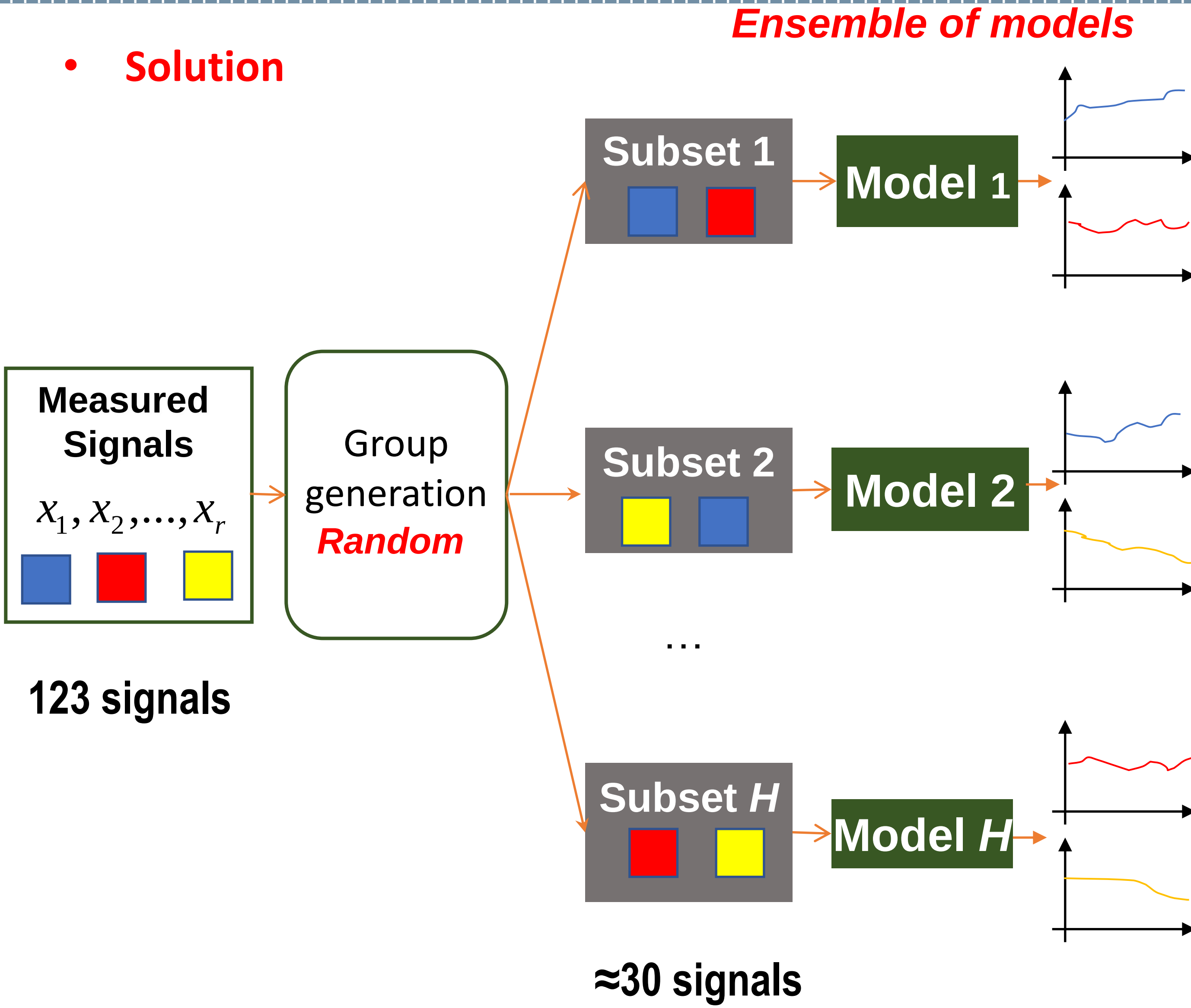
Hundreds of signals are monitored





DIFFICULT TO DEVELOP

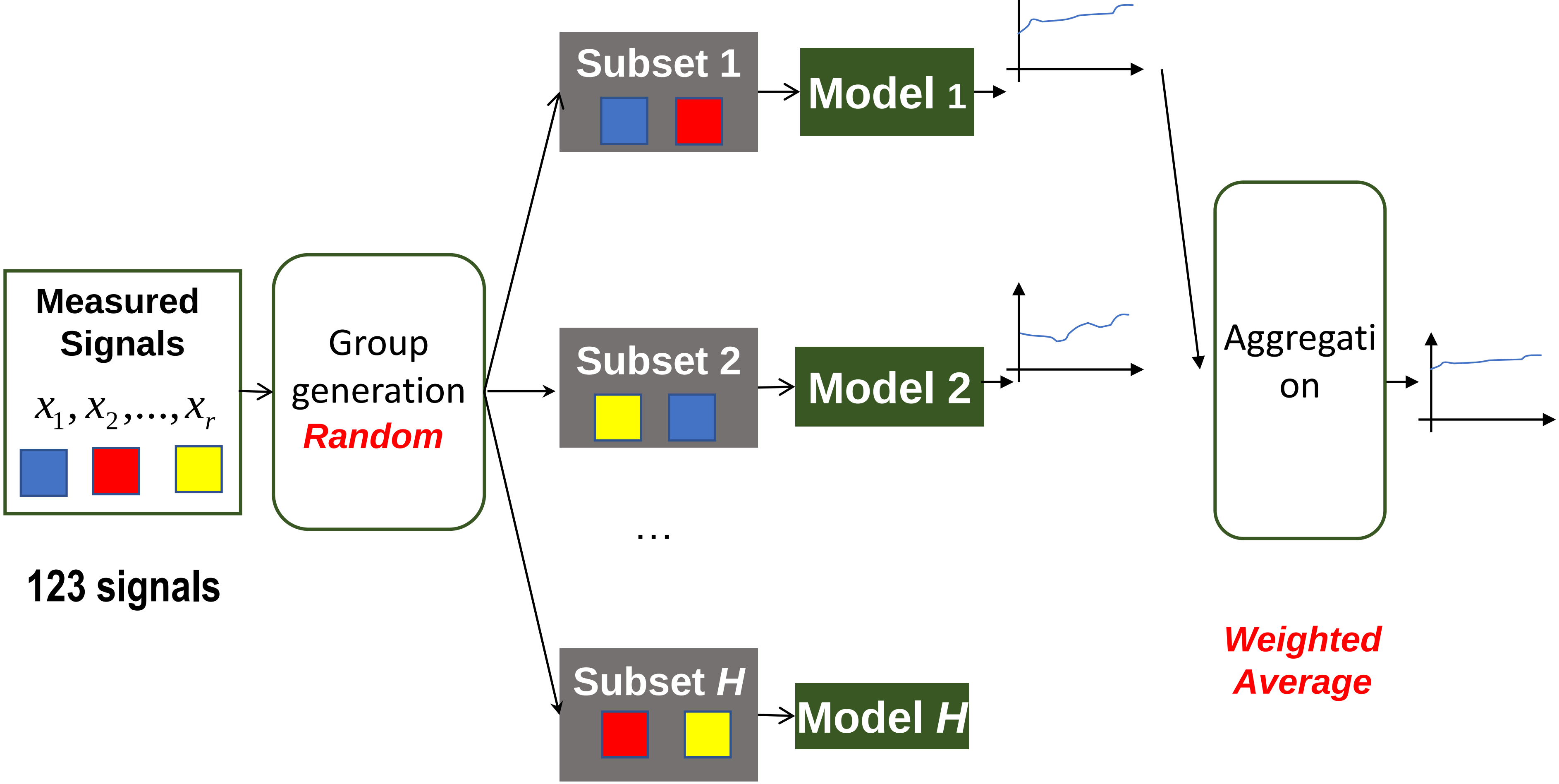
- **Solution**


Ensemble of models
Measured Signals
$x_1, x_2, \ldots, x_r$
123 signals
Group generation
Random
Subset 1
Model 1
Subset 2
Model 2
…
Subset H
Model H
≈30 signals

Fault Detection: Challenge I

Ensemble of models

- **Solution**

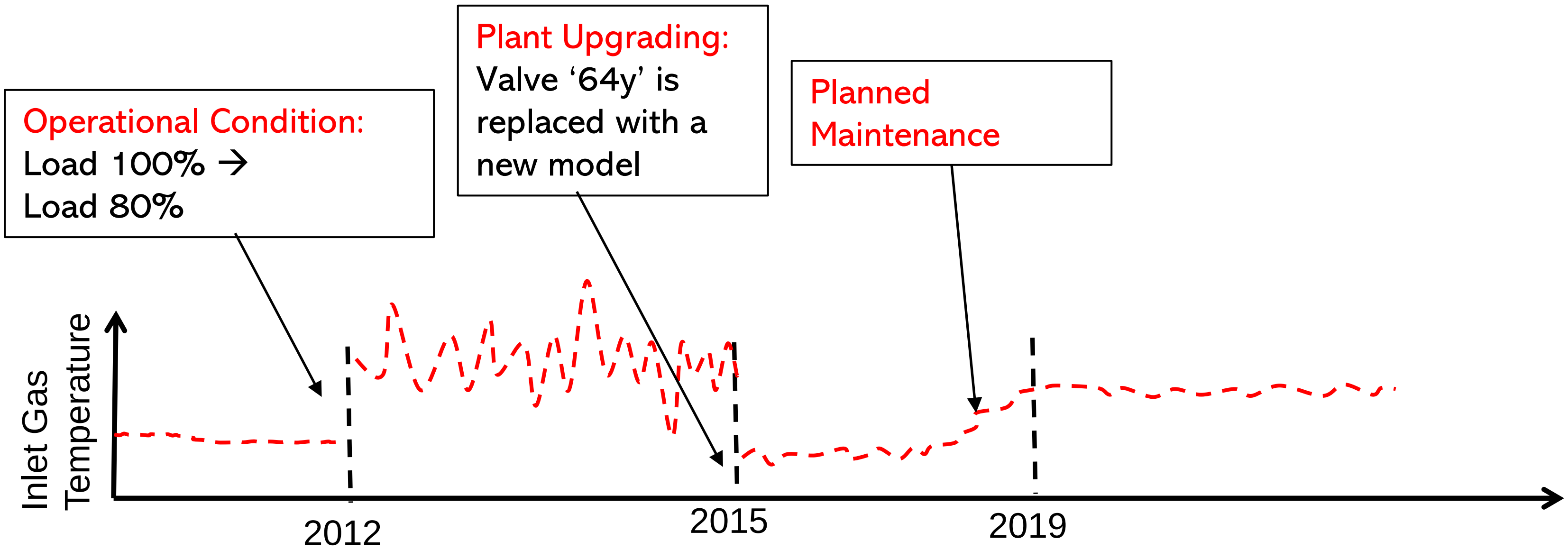


123 signals

≈20 signals

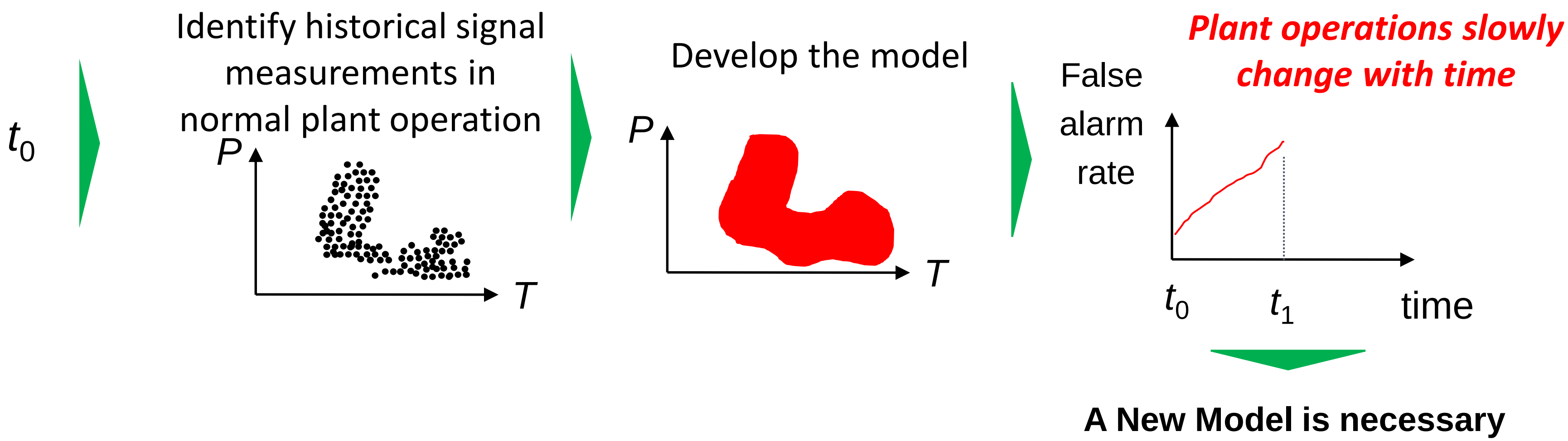
Fault Detection: Challenge II

Evolving Environment



Fault Detection: Challenge II

Example: monitoring the turbine of an electric power plant

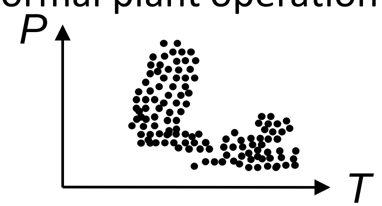


Fault Detection: Challenge II

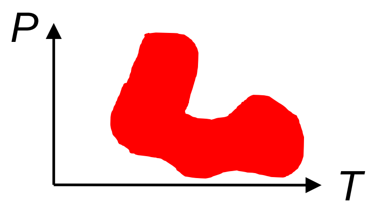
Example: monitoring the turbine of an electric power plant

t_0

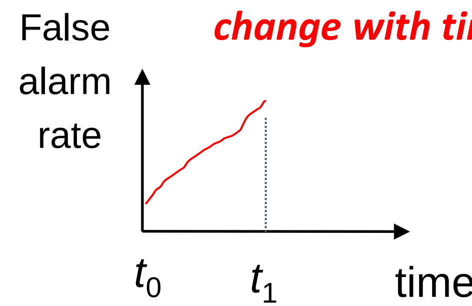
Identify historical signal measurements in normal plant operation



Develop the model



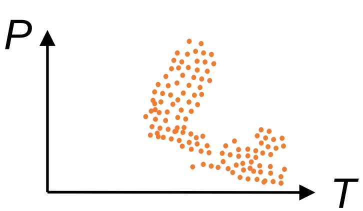
Plant operations slowly change with time



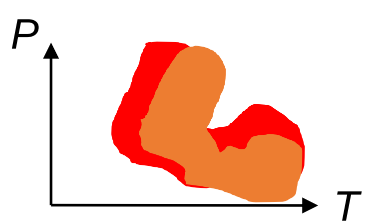
A New Model is necessary

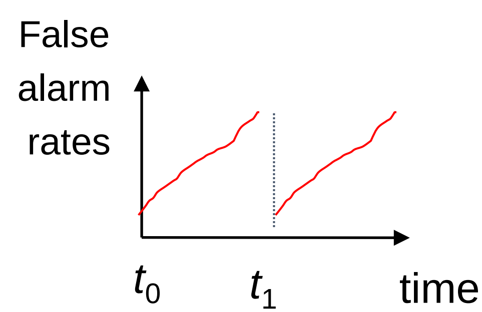
t_1

Identify recent historical signal measurements in normal plant operation



Develop the new model

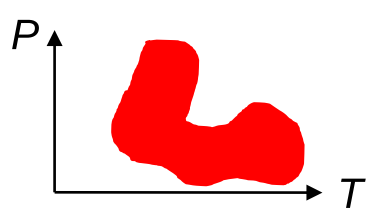


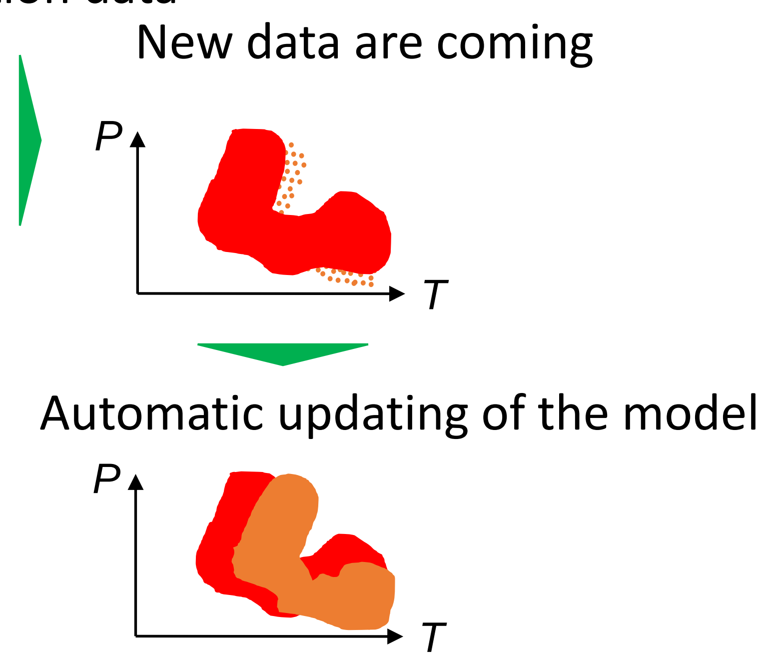


Periodic Human Interventions for developing new models! → high costs!

Fault Detection: Challenge II

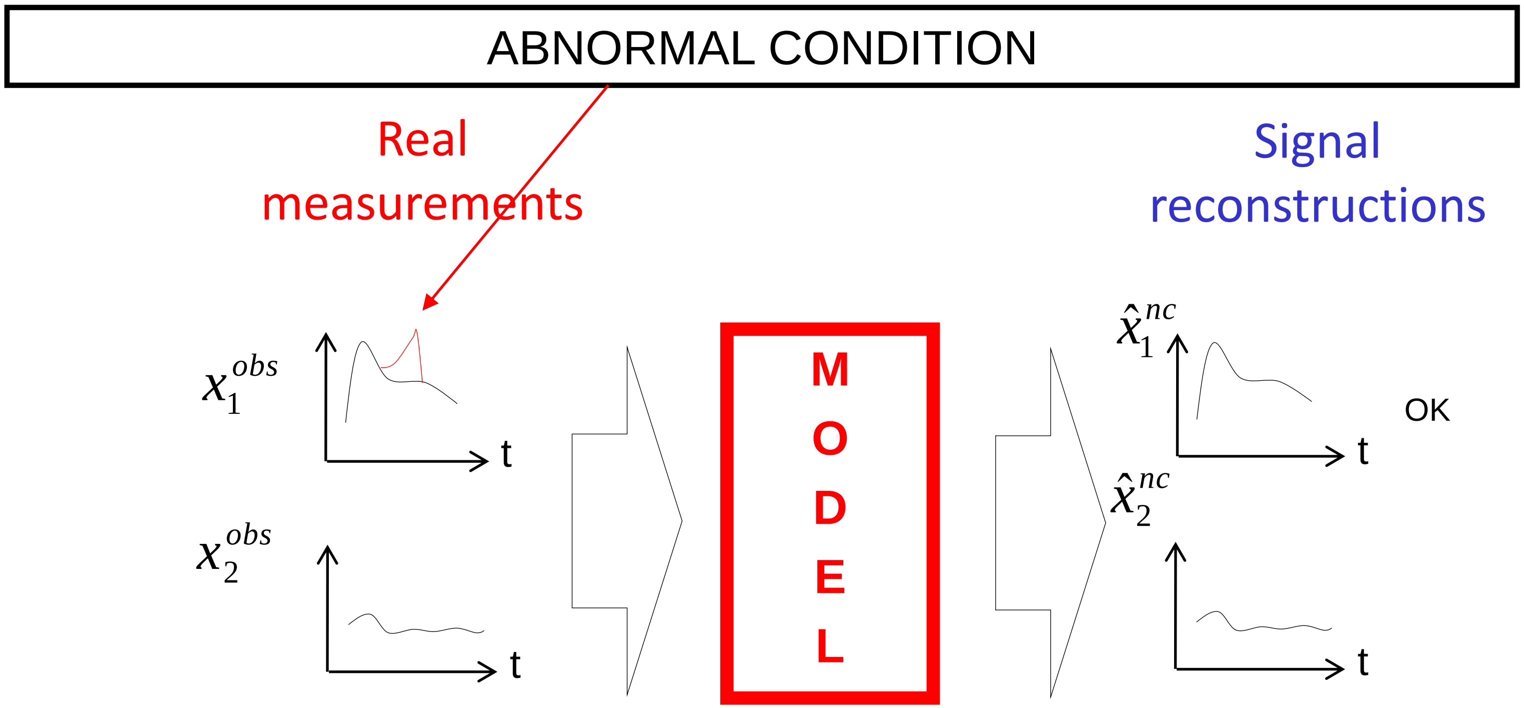
- The detection model should be able to follow the process changes:
- Incremental learning of the new data that gradually becomes available
- No necessity of human intervention for:
  - selecting recent normal operation data
  - building the new model





Fault Detection: Challenge III

Robustness of the Reconstruction

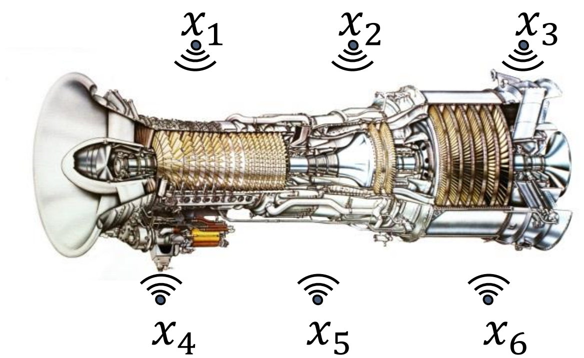


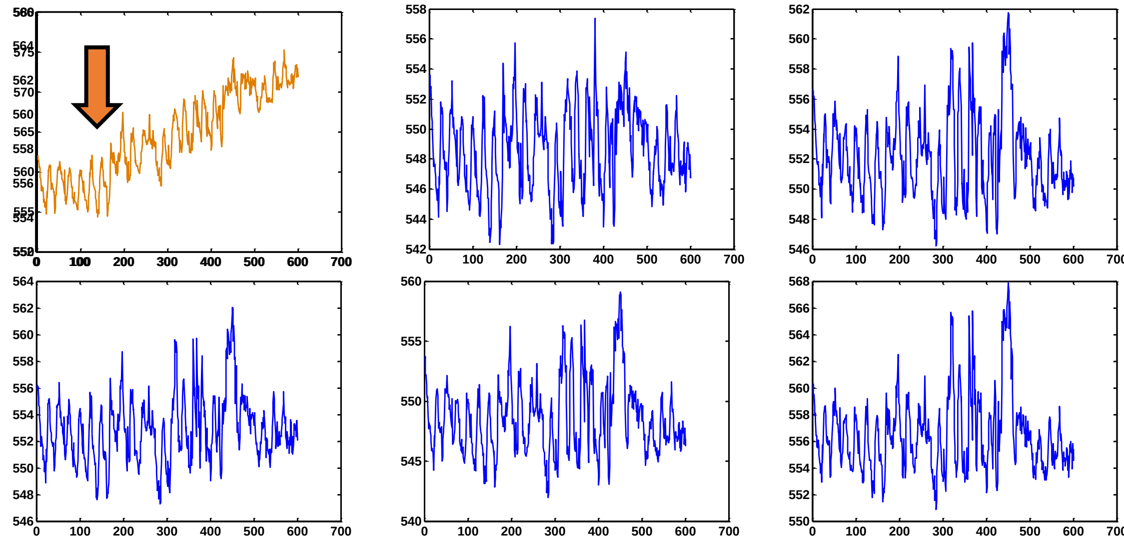
ROBUSTNESS:

$$\vec{\hat{x}}^{nc} \cong \vec{x}^{obs-nc}$$

Example1:
Monitoring a Turbine for Energy Production

6 Temperature Sensors in different position

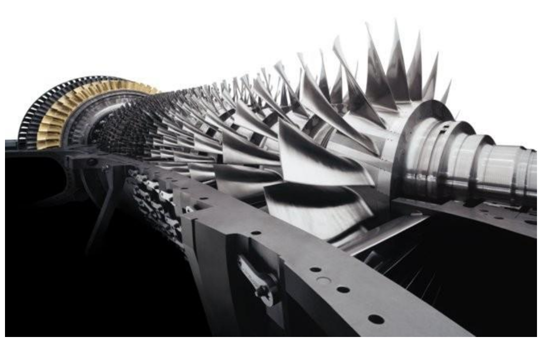




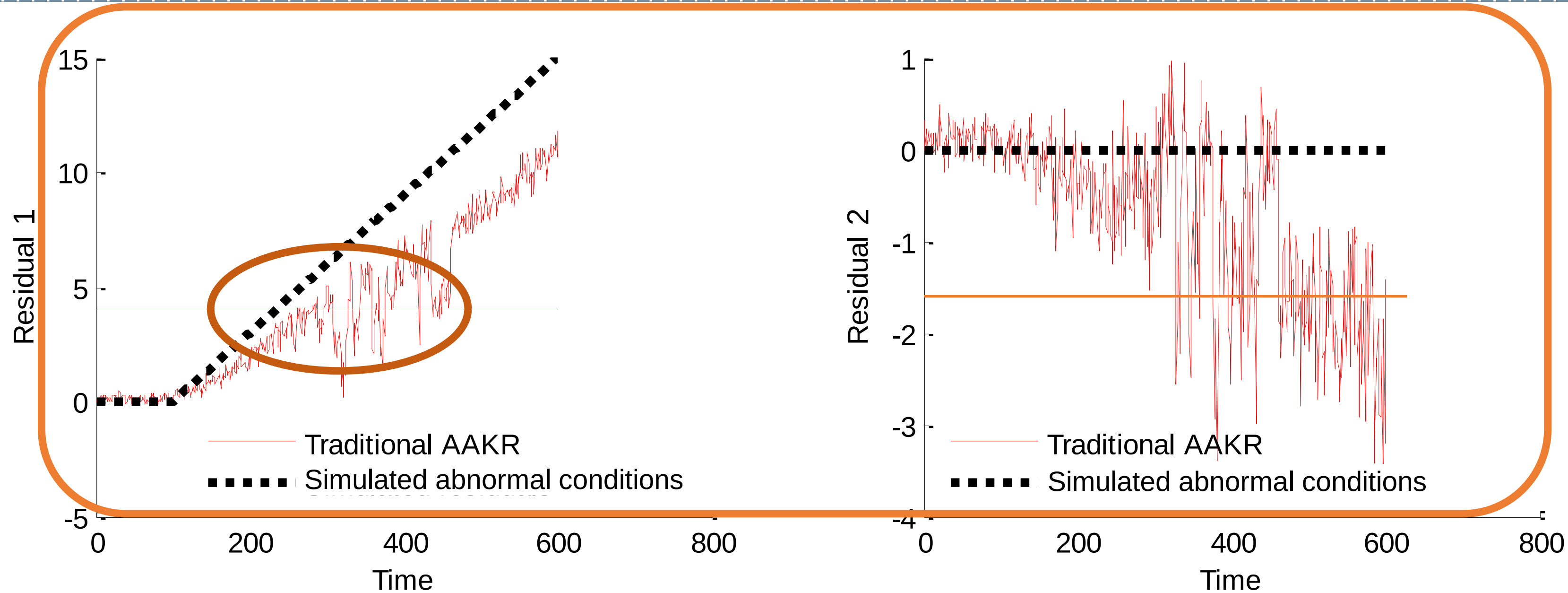
Abnormal condition of the first signal

Highly Correlated Signals

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1	0.97	0.98	0.98	0.99	0.98
x_2	0.97	1	0.95	0.99	0.98	0.96
x_3	0.98	0.95	1	0.96	0.99	0.99
x_4	0.98	0.99	0.96	1	0.98	0.97
x_5	0.99	0.98	0.99	0.98	1	0.99
x_6	0.98	0.96	0.99	0.97	0.99	1

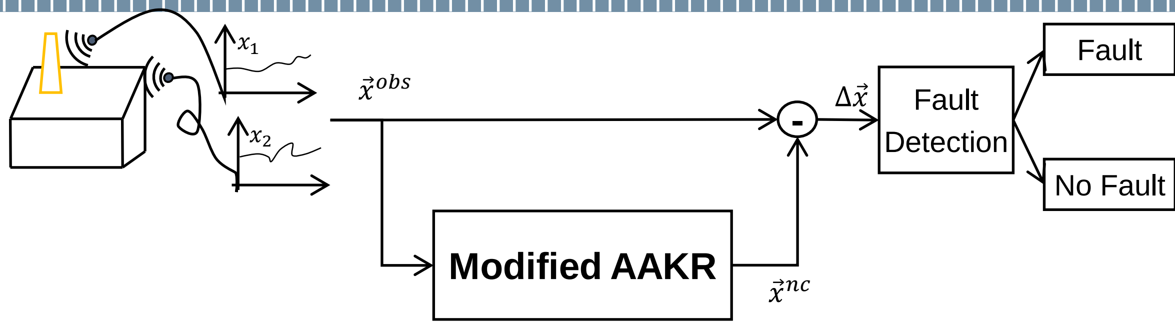


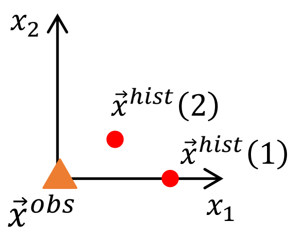
Example: Traditional AAKR



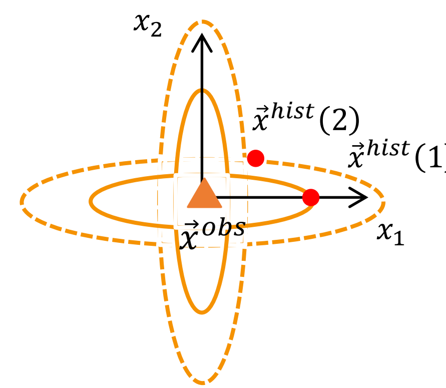
- **DELAY IN THE DETECTION**
- **IMPOSSIBILITY TO IDENTIFY THE SIGNALS TRIGGERING THE ABNORMAL BEHAVIOR**

Our Contribution:
A modified AAKR method



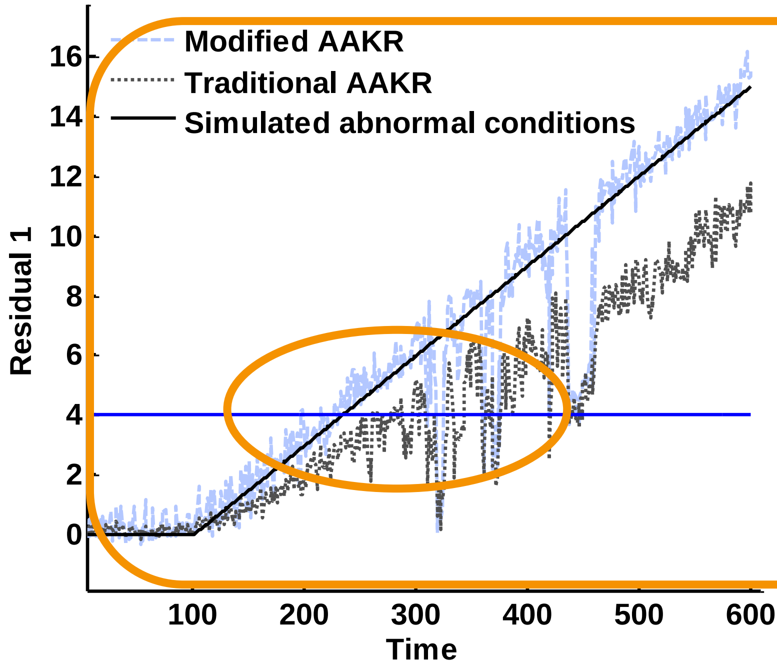


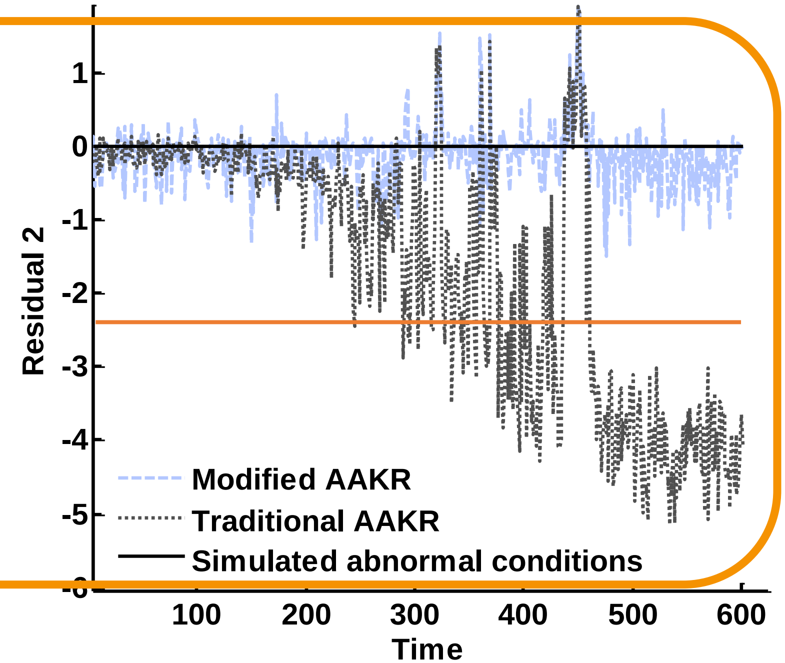
Malfunctions causing variations of a small number of signals are more frequent than those causing variations of a large number of signals



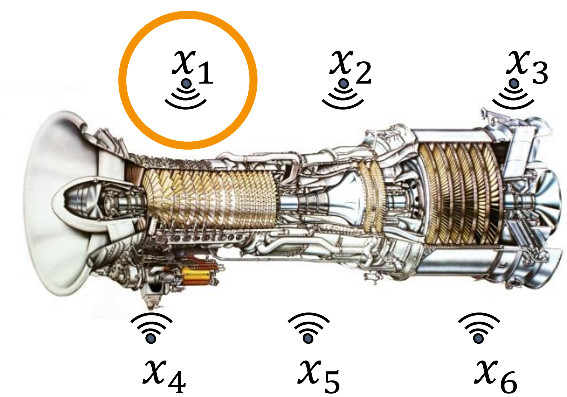
Modification of the loci of equisimilarity points

Results

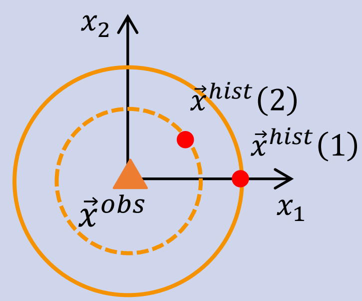
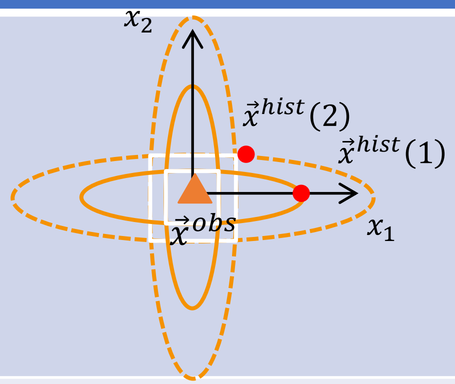




- **Early Detection**
- **Correct Diagnosis of the signal that triggers the alarms**
- **More Accurate**



Modified AAKR

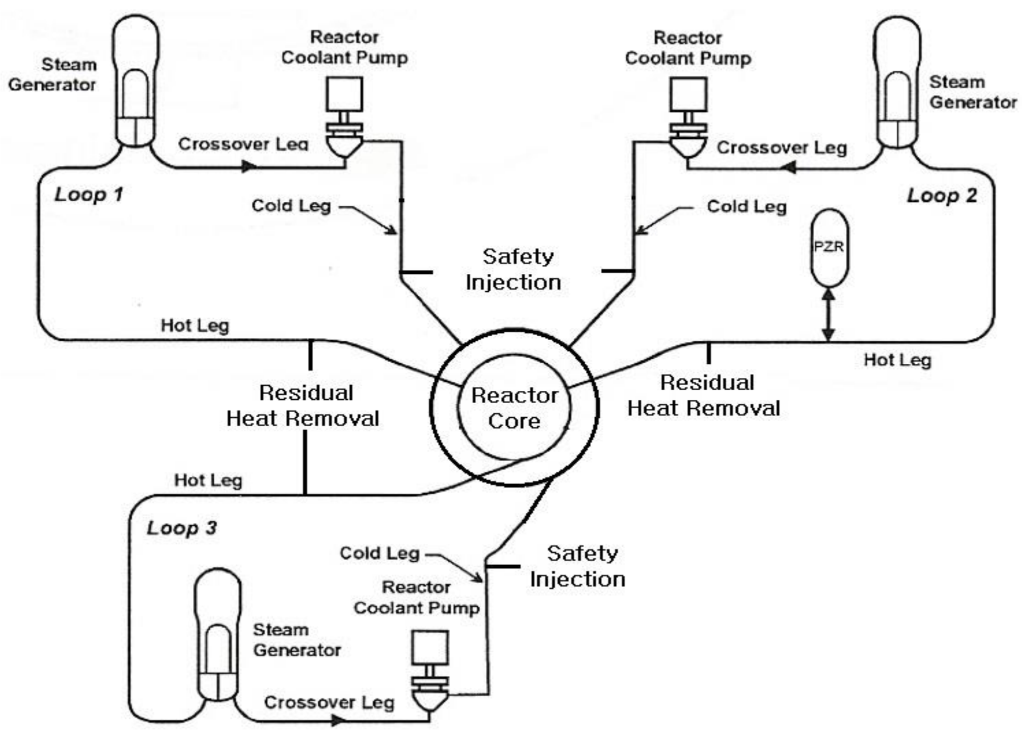
	Traditional AAKR	Modify AAKR
Loci of equisimilarity points		
Accuracy	**OK!**	**OK!**
Robustness	**NO!** Especially with correlated signals	1. Robust reconstruction of the values expected in normal conditions 2. Correct identification of signals affected by abnormal condition 3. Good performance with correlated signals

Example2:
Monitoring Reactor coolant System of a Nuclear Power Plant

6 Sensors of reactor coolant system (RCS) measured during startup transient

S1 (Cold leg temperature) **S2** (Core exit temperature) **S3** (Hot leg temperature)

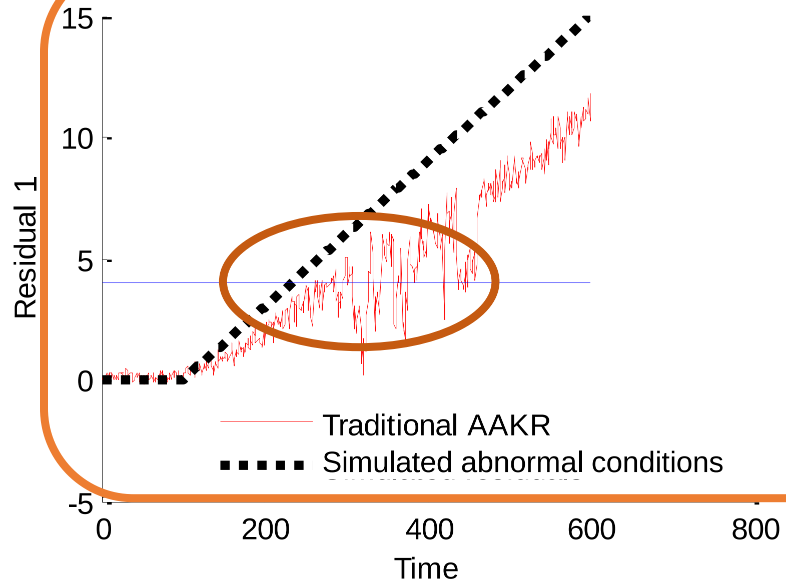
S4 (Safety injection flow) **S5** (Residual heat removal flow) **S6** (Sub-cooling margin temperature)

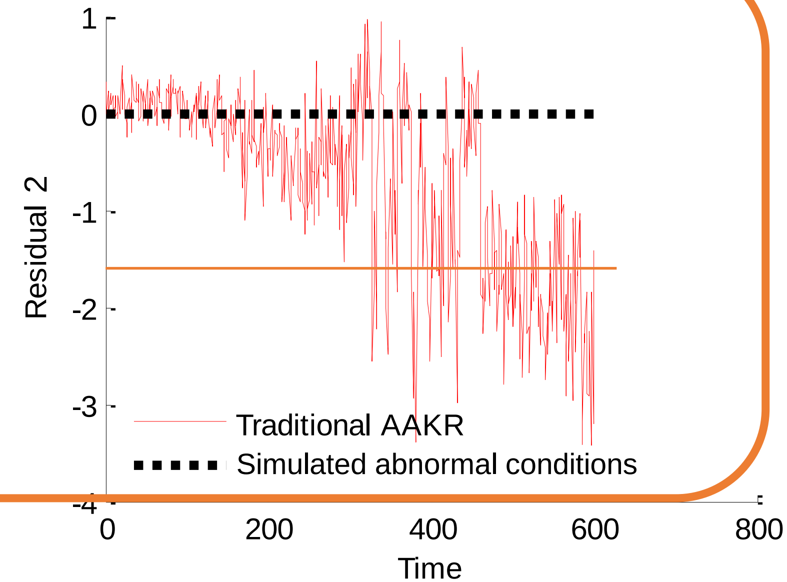


Correlations

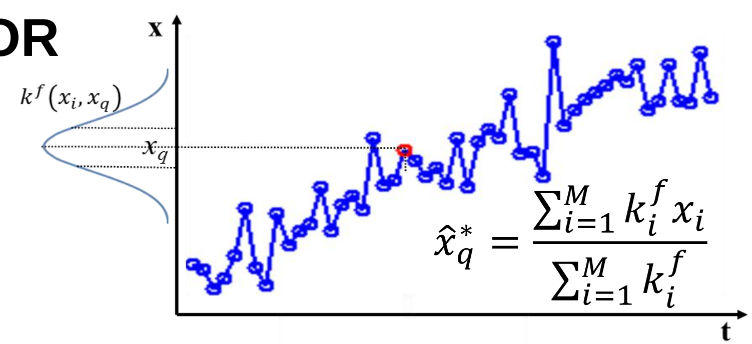
	S1	S2	S3	S4	S5	S6
S1	1	0.99	0.99	-0.16	0.39	-0.98
S2	0.99	1	0.99	-0.18	0.39	-0.98
S3	0.99	0.99	1	-0.12	0.42	-0.97
S4	-0.16	-0.18	-0.12	1	0.65	0.31
S5	0.39	0.39	0.42	0.65	1	-0.23
S6	-0.98	-0.98	-0.97	0.31	-0.23	1

Example2: Traditional AAKR

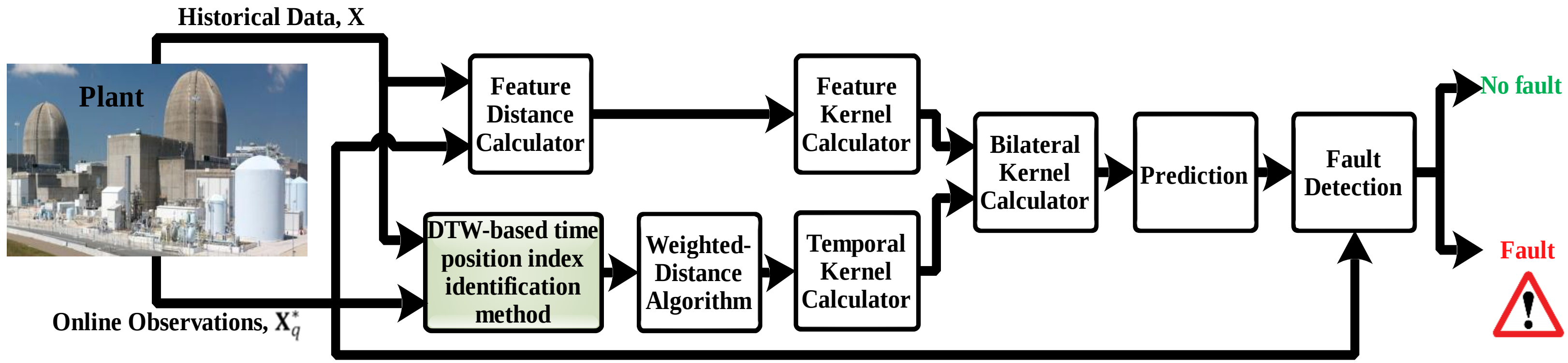


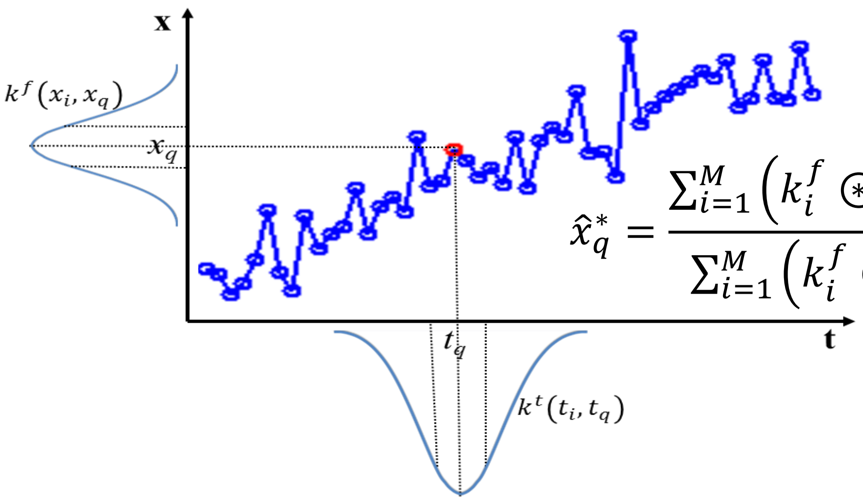


- **DELAY IN THE DETECTION**
- **IMPOSSIBILITY TO IDENTIFY THE SIGNALS TRIGGERING THE ABNORMAL BEHAVIOR**
- **LACKS TEMPORAL INFORMATION**



Our Contribution: AABKR method – Aggregating Bilateral Directions

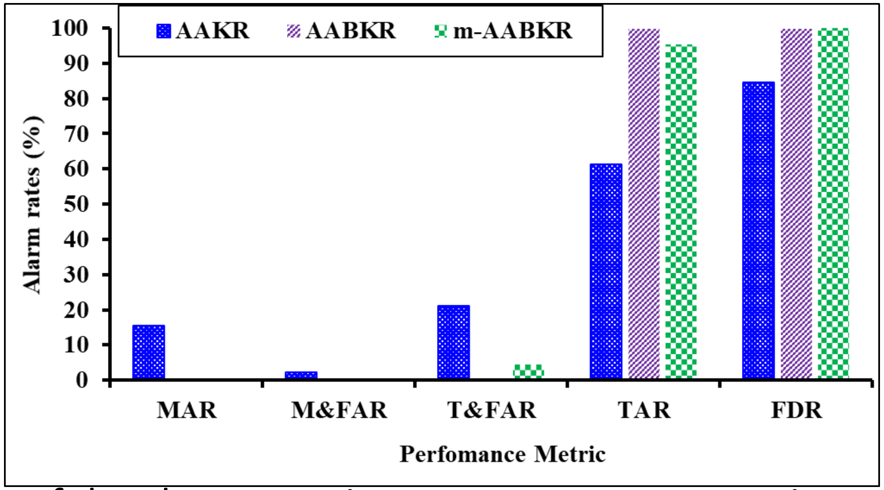




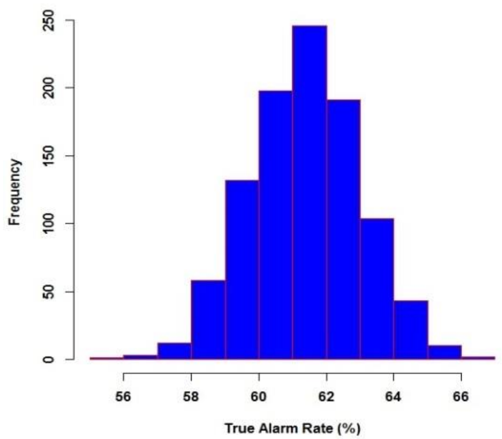
$$\hat{x}_q^* = \frac{\sum_{i=1}^{M}\left(k_i^f \circledast k_i^t\right) x_i}{\sum_{i=1}^{M}\left(k_i^f \circledast k_i^t\right)}$$

Aggregating bilateral directions capturing both spatial and temporal dependencies

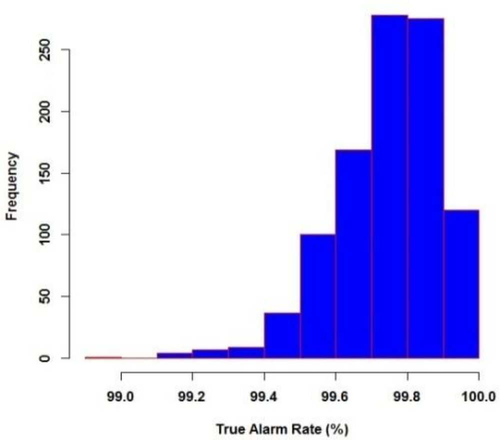
Results



Means of the alarm rates in start-up process operating condition



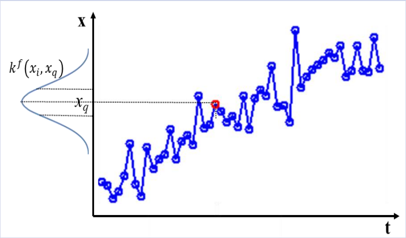
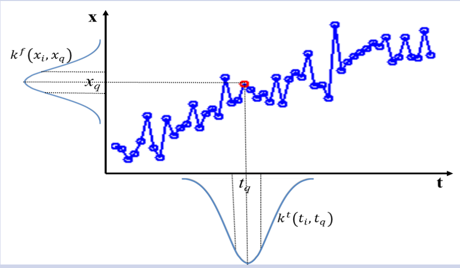
(a) AAKR



(b) AABKR

Distributions of the TARs for a thousand-run Monte Carlo in start-up

Modified AAKR (AABKR)

	Traditional AAKR	Modify AAKR (AABKR)
Information captured		
Accuracy	**OK!**	**OK!**
Robustness	**NO!** Especially with correlated signals and normal transient data	1. Robust reconstruction of the values expected in normal conditions 2. Correct identification of signals affected by abnormal condition 3. Good performance with normal transient monitoring