POLITECNICO MILANO 1863

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Markov Reliability and Availability Analysis Part II: Continuous-Time Discrete State Markov Processes

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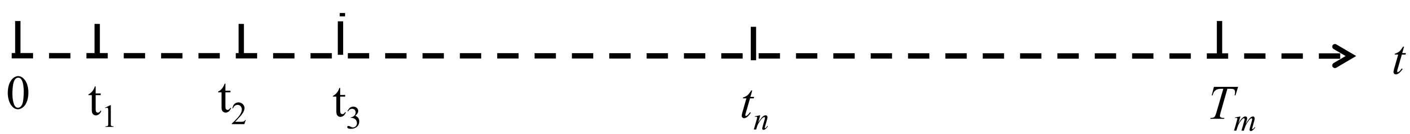
Continuous Time Discrete State Markov Processes

- The **stochastic process** may be **observed** at:
  - Discrete times

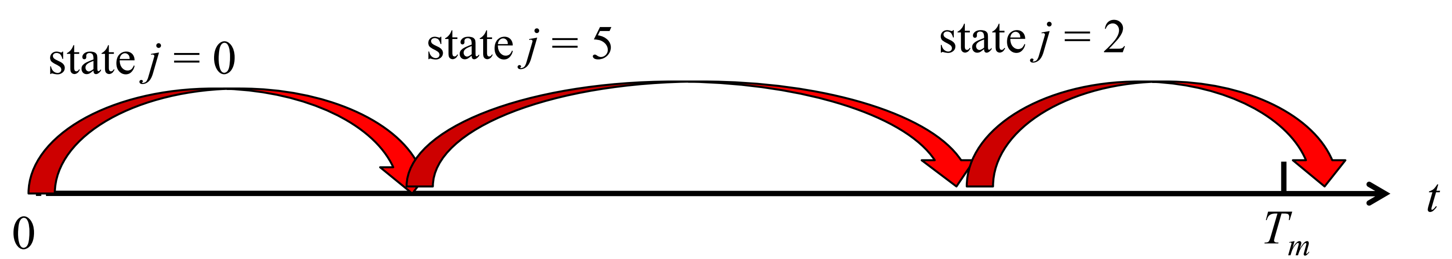
→ DISCRETE-TIME DISCRETE-STATE MARKOV PROCESSES



  - Continuously

→ CONTINUOUS-TIME DISCRETE-STATE MARKOV PROCESS

- The stochastic process is **observed continuously** and **transitions** are assumed to **occur continuously in time**



The conceptual model: Finite State Space

- The random process of system transition between states in time is described by a **stochastic process** $\{X(t);\ t \geq 0\}$
- $X(t) :=$ system state at time t
  - $X(3.6) = 5$: the system is in state number 5 at time $t = 3.6$

OBJECTIVE:

Computing the probability that the system is in a given state as a function of time, for all possible states

$$P\left[X\left(t\right) = j\right], t \in \left[0, T_m\right], j = 0, 1, \ldots, N$$

Objective:

$$P[X(t) = j], t \in [0, T_m], j = 0,1, \dots, N$$

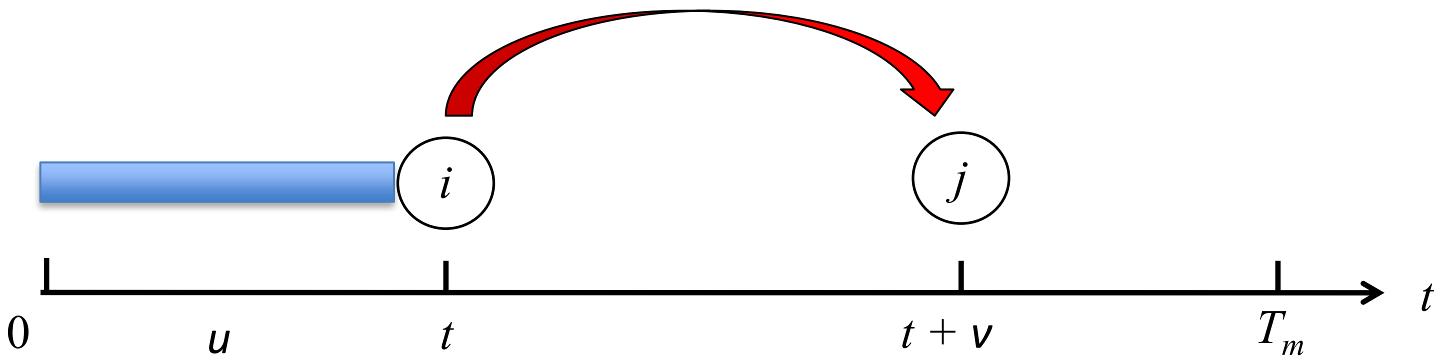
What do we need?

Transition Probabilities!

- **Transition probability** that the system moves to state j at time $t + v$ given that it is in state i at **current** time t and given **the previous system history**

$$P\left[X(t+v)=j \mid X(t)=i, X(u)=x(u), 0 \le u < t\right]$$

$$(i = 0, 1, \ldots, N, j = 0, 1, \ldots, N)$$



- **IN GENERAL STOCHASTIC PROCESSES:**

the **probability** of a **future** state of the system usually depends on its **entire life history**

$$P\left[X(t+v)=j \mid X(t)=i, X(u)=x(u), 0 \leq u < t\right]$$

$$(i = 0, 1, \ldots, N, j = 0, 1, \ldots, N)$$

- **IN MARKOV PROCESSES:**

the **probability** of a **future** state of the system **only** depends on its **present state**

$$P\left[X(t+v)=j \mid X(t)=i, \cancel{X(u)=x(u), 0 \leq u < t}\right]$$

$$=$$

$$P\left[X(t+v)=j \mid X(t)=i\right]$$

$$(i = 0, 1, \ldots, N, j = 0, 1, \ldots, N)$$

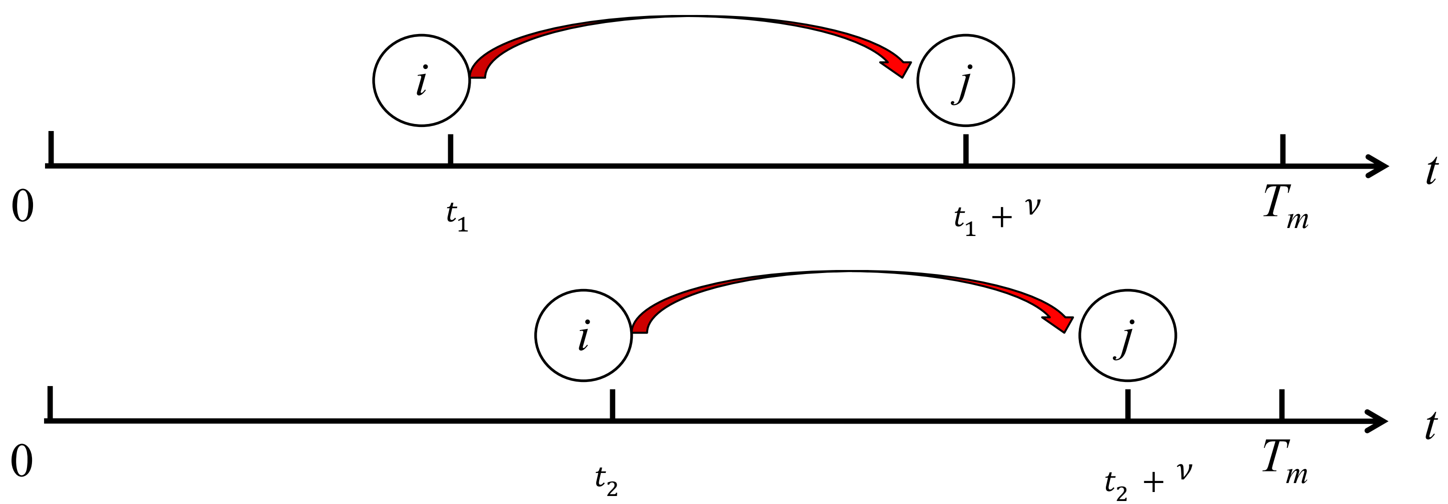
THE PROCESS HAS “NO MEMORY”

The Conceptual Model: homogeneous Markov process

If the **transition probability** depends on the **interval *ν*** and **not** on the **individual times** t and $t + \nu$

- the transition probabilities are **stationary**
- the Markov process is **homogeneous** in time

$$p_{ij}(t, t+\nu) = P\left[X(t+\nu) = j \mid X(t) = i\right] = p_{ij}(\nu)$$



The Conceptual Model

- Homogeneus process without memory

Transition time → Exponential distribution

The conceptual model: Transition Rates

HYPOTHESIS:

- The time interval $\nu = dt$ is **small** such that **only one** event (i.e., one **stochastic transition**) can occur within it

$$p_{ij}(dt) = P[X(t+dt) = j | X(t) = i] = 1 - e^{-\alpha_{ij} \cdot dt}$$

α_{ij} = **transition rate** from state *i* to state *j*

HYPOTHESIS:

- The time interval $\nu = dt$ is **small** such that **only one** event (i.e., one **stochastic transition**) can occur within it

$$p_{ij}(dt) = P[X(t+dt) = j | X(t) = i] = 1 - e^{-\alpha_{ij} \cdot dt}$$

$$= \quad \text{(Taylor } 1^{st} \text{ order expansion)}$$

$$\alpha_{ij} \cdot dt + \theta(dt), \quad \lim_{dt \to 0} \frac{\theta(dt)}{dt} = 0$$

$$p_{ii}(dt) = 1 - \sum_{j \neq i} p_{ij}(dt) = 1 - dt \cdot \sum_{j \neq i} \alpha_{ij} + \theta(dt)$$

Analogy with the Discrete Time Markov Chains

Discrete-time | **Continuous-time**

$$p_{ij} = P[X(n+1) = j | X(n) = i] \quad \Rightarrow \quad \alpha_{ij} = P[X(t+dt) = j | X(t) = i]$$

$$\underline{P}(n+1) = \underline{P}(n) \cdot \underline{\underline{A}} \quad \Rightarrow \quad \underline{P}(t+dt) = \underline{P}(t) \cdot \underline{\underline{A}}$$

$$\underline{\underline{A}} = \begin{pmatrix} p_{00} & p_{01} & \cdots & p_{0N} \\ p_{10} & p_{11} & \cdots & p_{1N} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N0} & p_{N1} & \cdots & p_{NN} \end{pmatrix} \quad \Rightarrow \quad \underline{\underline{A}} = \begin{pmatrix} 1 - dt \cdot \sum_{j=1}^{N} \alpha_{0j} & \alpha_{01} \cdot dt & \ldots & \alpha_{0N} \cdot dt \\ \alpha_{10} \cdot dt & 1 - dt \cdot \sum_{\substack{j=0 \\ j \neq 1}}^{N} \alpha_{1j} & \ldots & \alpha_{1N} \cdot dt \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

$$\left[P_0(t+dt)P_1(t+dt)\ldots P_N(t+dt)\right] = \left[P_0(t)P_1(t)\ldots P_N(t)\right]\cdot \begin{pmatrix} 1-dt\cdot\sum_{j=1}^{N}\alpha_{0j} & \alpha_{01}\cdot dt & \ldots & \alpha_{0N}\cdot dt \\ \alpha_{10}\cdot dt & 1-dt\cdot\sum_{\substack{j=0\\ j\neq 1}}^{N}\alpha_{1j} & \ldots & \alpha_{1N}\cdot dt \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

- First-equation:

$$P_0(t+dt) = \left[1-dt\sum_{j=1}^{N}\alpha_{0j}\right]P_0(t) + \alpha_{10}P_1(t)\cdot dt + \ldots + \alpha_{N0}P_N(t)dt$$

The conceptual model: the fundamental matrix equation (2)

$$P_0\left(t+dt\right)=\left[1-dt\sum_{j=1}^{N}\alpha_{0j}\right]P_0\left(t\right)+\alpha_{10}P_1\left(t\right)\cdot dt+\ldots+\alpha_{N0}P_N\left(t\right)dt$$

subtract $P_0(t)$ on both sides

$$P_0\left(t+dt\right)-P_0\left(t\right)=P_0\left(t\right)-P_0\left(t\right)-\sum_{j=1}^{N}\alpha_{0j}P_0\left(t\right)dt+\alpha_{10}P_1\left(t\right)dt+\ldots+\alpha_{N0}P_N\left(t\right)dt$$

divide by dt

$$\frac{P_0\left(t+dt\right)-P_0\left(t\right)}{dt}=-\sum_{j=1}^{N}\alpha_{0j}P_0\left(t\right)+\alpha_{10}P_1\left(t\right)+\ldots+\alpha_{N0}P_N\left(t\right)$$

let $dt\rightarrow 0$

$$\lim_{dt\rightarrow 0}\frac{P_0\left(t+dt\right)-P_0\left(t\right)}{dt}=\frac{dP_0}{dt}=-\sum_{j=1}^{N}\alpha_{0j}\cdot P_0\left(t\right)+\alpha_{10}\cdot P_1\left(t\right)+\ldots+\alpha_{N0}\cdot P_N\left(t\right)$$

$$\frac{dP_0}{dt} = -\sum_{j=1}^{N} \alpha_{0j} P_0(t) + \alpha_{10} P_1(t) + \cdots + \alpha_{N0} P_N(t) = [\ P_0(t), P_1(t), \dots, P_N(t)] \cdot \begin{bmatrix} -\sum_{j=1}^{N} \alpha_{0j} \\ \alpha_{10} \\ \dots \\ \alpha_{N0} \end{bmatrix}$$

- Extending to the other equations:

$$\frac{d\underline{P}}{dt} = \underline{P}(t) \cdot \underline{\underline{A}}^*, \quad \underline{\underline{A}}^* = \begin{pmatrix} \underbrace{-\sum_{j=1}^{N} \alpha_{0j}}_{\alpha_{00}} & \alpha_{01} & \dots & \alpha_{0N} \\ \alpha_{10} & \underbrace{-\sum_{\substack{j=0 \\ j \neq 1}}^{N} \alpha_{1j}}_{\alpha_{11}} & \dots & \alpha_{1N} \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

TRANSITION RATE MATRIX

It will be indicated as *A*

System of *N*+1 **linear**, **first-order differential equations** in the unknown state probabilities

$$P_j(t), \ j = 0, 1, 2, \dots, N, \ t \geq 0$$

Example 1: one component/one repairman-Markov Diagram and transition rate matrix

Consider a system made by one component which can be in two states: working or failed. Assume constant failure rate λ and constant repair rate μ. You are required to:

- Draw the Markov diagram
- Find the transition rate matrix, A

Example 2: system with $\boldsymbol{N}$ identical components and $\boldsymbol{N}$ repairman available

Consider a system made by $\boldsymbol{N}$ identical components which can be in two states: working or failed. Assume constant failure rate λ, that $\boldsymbol{N}$ repairman are available and that the single component repair rate is constant and equal to μ. You are required to:

- Draw the Markov diagram
- Find the transition rate matrix, *A*

Example 3: system with N identical components and 1 repairman available

Consider a system made by $\boldsymbol{N}$ identical components which can be in two states: working or failed. Assume constant failure rate λ, that 1 repairman is available and that the component repair rate is constant and equal to μ. You are required to:

- Draw the Markov diagram
- Find the transition rate matrix

Solution to the Fundamental Equation

Solution to the fundamental equation of the Markov process continuous in time

$$\begin{cases} \dfrac{d\underline{P}}{dt} = \underline{P}(t)\cdot \underline{\underline{A}} \\ \underline{P}(0) = \underline{C} \end{cases}$$

where

$$\underline{\underline{A}} = \begin{pmatrix} -\sum_{j=1}^{N} \alpha_{0j} & \alpha_{01} & \ldots & \alpha_{0N} \\ \alpha_{10} & -\sum_{\substack{j=0 \\ j\neq 1}}^{N} \alpha_{1j} & \ldots & \alpha_{1N} \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

System of N**+1 linear**, **first-order differential equations** in the unknown state probabilities

$$P_j(t),\ j = 0,1,2,\ldots,N,\ t \geq 0$$

USE LAPLACE TRANSFORM

Solution to the fundamental equation of of the Markov process continuous in time: the Lapace Transform Method

- Laplace Transform: $\tilde{P}_j(s) = L\left[P_j(t)\right] = \int_0^\infty e^{-st} P_j(t)\,dt,\; j = 0,1,\ldots,N$
- First derivative: $L\left(\frac{dP_j(t)}{dt}\right) = s \cdot \tilde{P}_j(s) - P_j(0), \quad j = 0,1,\ldots,N$
- Apply the Laplace operator to $\frac{d\underline{P}}{dt} = \underline{P}(t) \cdot \underline{\underline{A}}$

$$L\left[\frac{d\underline{P}(t)}{dt}\right] = L\left[\underline{P}(t) \cdot \underline{\underline{A}}\right]$$

First derivative ← $\boxed{s\underline{\tilde{P}}(s) - \underline{C}} = \boxed{\underline{\tilde{P}}(s) \cdot \underline{\underline{A}}}$ → Linearity

$$\underline{\tilde{P}}(s) = \underline{C} \cdot \left[s \cdot \underline{\underline{I}} - \underline{\underline{A}}\right]^{-1}$$

$\underline{P}(t)$ = **inverse transform of** $\underline{\tilde{P}}(s)$

Solution to the Fundamental Equation: Steady State Probabilities

- At steady state $\underline{P}(t) = \underline{\Pi}$

$$\frac{d\underline{P}(t)}{dt} = 0 \quad \color{red}{\Rightarrow} \quad \frac{d\underline{P}(t)}{dt} = \underline{P}(t)\cdot \underline{\underline{A}} = \underline{\Pi}\cdot \underline{\underline{A}} = 0$$

- Solve the (linear) system: $\begin{cases} \underline{\Pi}\cdot \underline{\underline{A}} = 0 \\ \sum_{j=0}^{N} \Pi_j = 1 \end{cases}$

Solution to the Fundamental Equation: Steady State Probabilities

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- Solve the (linear) system:

$$\begin{cases} \underline{\Pi}\cdot \underline{\underline{A}} = 0 \\ \sum_{j=0}^{N} \Pi_j = 1 \end{cases}$$

- It can be shown that:

$$\Pi_j = \frac{D_j}{\sum_{i=0}^{N} D_i} \qquad j = 0,1,2,\ldots,N$$

D_j = determinant of the **square matrix** obtained from $\underline{\underline{A}}$ by **deleting** the j-th row and column

Example 4: one component/one repairman – Solution to the fundamental equation (1)

Consider a system made by one component which can be in two states: working (‘0’) or Failed (‘1’). Assume constant failure rate λ and constant repair rate μ and that the component is working at $t = 0$: $\underline{C} = [1 \quad 0]$

- You are required to find the component steady state and instantaneous availability

Quantity of Interest

Frequency of departure from a state

- **Unconditional** probability of **arriving in state *j*** in the next dt **departing from state *i*** at time *t*: $P[X(t+dt)=j, X(t)=i]$

$$P[X(t+dt)=j, X(t)=i] = P[X(t+dt)=j|X(t)=i]\cdot P[X(t)=i] = p_{ij}(dt)P_i(t)$$

- **Frequency of departure from state *i* to state *j***:

$$v_{ij}^{dep}(t) = \lim_{dt\to 0}\frac{P[X(t+dt)=j, X(t)=i]}{dt} = \lim_{dt\to 0}\frac{p_{ij}(dt)P_i(t)}{dt} = \alpha_{ij}P_i(t)$$

- **Total frequency of <u>departure</u> from state *i* to any other state *j***:

$$v_i^{dep}(t) = \sum_{\substack{j=0\\ j\neq i}}^{N} v_{ij}^{dep}(t) = \sum_{\substack{j=0\\ j\neq i}}^{N} \alpha_{ij}\cdot P_i(t) = P_i(t)\sum_{\substack{j=0\\ j\neq i}}^{N}\alpha_{ij} = -\alpha_{ii}\cdot P_i(t)$$

Frequency of departure from a state at steady state

- **Unconditional** probability of **arriving in state *j*** in the next dt **departing from state *i*** at time *t*: $P[X(t+dt)=j, X(t)=i]$

$$P[X(t+dt)=j, X(t)=i] = P[X(t+dt)=j \mid X(t)=i] \cdot P[X(t)=i] = p_{ij}(dt)P_i(t)$$

- **Frequency of departure from state *i* to state *j***:

$$v_{ij}^{dep}(t) = \lim_{dt\to 0}\frac{P[X(t+dt)=j, X(t)=i]}{dt} = \lim_{dt\to 0}\frac{p_{ij}(dt)P_i(t)}{dt} = \alpha_{ij}P_i(t)$$

(at steady state) $= \boxed{v_{ij}^{dep} = \alpha_{ij}\cdot\Pi_i}$

- **Total frequency of <u>departure</u> from state *i* to any other state *j***:

$$v_i^{dep}(t) = \sum_{\substack{j=0\\ j\neq i}}^{N} v_{ij}^{dep}(t) = \sum_{\substack{j=0\\ j\neq i}}^{N}\alpha_{ij}\cdot P_i(t) = P_i(t)\sum_{\substack{j=0\\ j\neq i}}^{N}\alpha_{ij} = -\alpha_{ii}\cdot P_i(t)$$

(at steady state) $\boxed{v_i^{dep} = -\alpha_{ii}\cdot\Pi_i}$

Frequency of arrival to a state

- **In analogy**, considering the **arrivals** to state i from any state k:

$$v_i^{arr}(t) = \sum_{\substack{k=0 \\ k \neq i}}^{N} \alpha_{ki} \cdot P_k(t)$$

$$v_i^{arr} = \sum_{\substack{k=0 \\ k \neq i}}^{N} \alpha_{ki} \cdot \Pi_k \quad \text{(at steady state)}$$

$$\underline{\Pi} \cdot \underline{\underline{A}} = 0 \quad \Rightarrow \quad \sum_{k=0}^{N} \alpha_{ki} \cdot \Pi_k = 0 \quad (i = 0, 1, 2, \ldots, N)$$

$$-\alpha_{ii} \cdot \Pi_i = \sum_{\substack{k=0 \\ k \neq i}}^{N} \alpha_{ki} \cdot \Pi_k \quad (i = 0, 1, 2, \ldots, N)$$

AT STEADY STATE:

frequency of departures from state *i* = frequency of arrivals to state *i*

System Failure Intensity

- **SYSTEM FAILURE INTENSITY W_f:**
  - **Rate** at which **system failures** occur
  - **Expected number** of system **failures** per **unit of time**
  - **Rate** of **exiting a success state** to go into one of **fault**

- **SYSTEM FAILURE INTENSITY W_f:**
  - **Rate** at which **system failures** occur
  - **Expected number** of system **failures** per **unit of time**
  - **Rate** of **exiting a success state** to go into one of **fault**

$$W_f(t) = \sum_{i \in S} P_i(t) \cdot \lambda_{i \to F}$$

S = set of success states of the system

F = set of failure states of the system

$P_i(t)$ = probability of the system being in the functioning state i at time t

$\lambda_{i \to F}$ = conditional (transition) probability of leaving success state i towards failure states

- **SYSTEM REPAIR INTENSIT W_r:**
  - **Rate** at which **system repairs** occur
  - **Expected number** of system **repairs** per **unit of time**
  - **Rate** of **exiting a failed state** to go into one of **success**

$$W_r(t) = \sum_{j \in F} P_j(t) \cdot \mu_{j \to S}$$

S = set of success states of the system

F = set of failure states of the system

$P_j(t)$ = probability of the system being in the failure state j at time t

$\mu_{j \to S}$ = conditional (transition) probability of leaving failure state j towards success states

Example 5: one component/one repairman

Consider a system made by one component which can be in two states: working (‘0’) or Failed (‘1’). Assume constant failure rate λ and constant repair rate μ and that the component is working at $t = 0$: $\underline{C} = [1 \quad 0]$

- You are required to find the failure and repair intensities

Sojourn Time in a state (1)

- **Time of occupance of state *i* (sojourn time), $\boldsymbol{T_i}$** = time spent in a state i
- Is the time (t) that the system has already been in state i influencing the time (s) the system will remain in state i?

$$\mathrm{P}(T_i > t + s | T_i > t) = P(X(t+u) = i, 0 \le u \le s |\, X(\tau) = i, 0 \le \tau \le t) =$$

$$= \mathrm{P}(X(t+u) = i, 0 \le u \le s | X(t) = i) \text{ (by Markov property)}$$

$$= \mathrm{P}(X(u) = i, 0 \le u \le s | X(0) = i) \text{ (by homogeneity)}$$

$= P(T_i > s)$ **Memoryless Property**

- The only distribution satisfying the memoryless property is the **Exponential distribution**

$$T_i \sim Exp$$

Sojourn Time in a state (2)

- System departure **rate from state *i*** (at steady state): $-\alpha_{ii}$

$$T_i \sim Exp(-\alpha_{ii})$$

- **Expected sojourn time $\boldsymbol{l_i}$**: average time of occupancy of state i

$$l_i = \mathbb{E}\{T_i\} = \frac{1}{-\alpha_{ii}}$$

Sojourn Time in a state (3)

- Total frequency of departure at steady state: $\boxed{\nu_i^{dep} = -\alpha_{ii} \cdot \Pi_i}$
- Average time of occupancy of state: $\boxed{l_i = \frac{1}{-\alpha_{ii}}}$

$$\nu_i^{dep} = -\alpha_{ii} \cdot \Pi_i = \frac{\Pi_i}{l_i}$$

$$\boxed{v_i^{dep} = v_i^{arr}}$$

$$\Pi_i = \nu_i^{arr} \cdot l_i$$

The **mean** proportion of time Π_i that the system spends in state *i* is equal to the **total frequency of <u>arrivals</u> to state *i*** multiplied by the mean duration of one visit in state *i*

System Availability

- **System instantaneous availability** at time t

  = **sum** of the **probabilities** of being in a **success** state at time t

$$p(t) = \sum_{i \in S} P_i(t) = 1 - q(t) = 1 - \sum_{j \in F} P_j(t)$$

In the Laplace domain

$$\tilde{p}(s) = \sum_{i \in S} \tilde{P}_i(s) = \frac{1}{s} - \sum_{j \in F} \tilde{P}_j(s)$$

S = set of success states of the system

F = set of failure states of the system

- **TWO CASES:**

1) **Non-Reparaible Systems**
   → No repairs allowed

2) **Reparaible Systems**
   → Repairs allowed

- No repairs allowed ⇒ **Reliability = Availability** $R(t) \equiv p(t) = 1 - q(t)$

- In the Laplace Domain: $\tilde{R}(s) = \sum_{i \in S} \tilde{P}_i(s) = \frac{1}{s} - \sum_{j \in F} \tilde{P}_j(s)$

- **Mean Time to Failure (MTTF):**

$$MTTF = \int_0^\infty R(t)dt = \left[\int_0^\infty R(t)e^{-st}dt \right]_{s=0} = \tilde{R}(0) = \sum_{i \in S} \tilde{P}_i(0) = \left[\frac{1}{s} - \sum_{j \in F} \tilde{P}_j(s) \right]_{s=0}$$

- **TWO CASES:**

1) **Non-reparaible systems**
   → No repairs allowed

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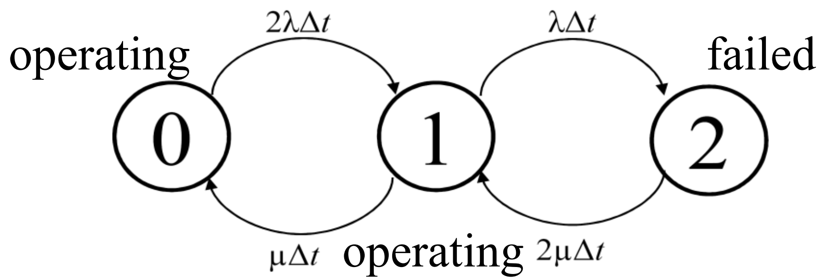
Parallel System of two identical components



$S = \{0,1\}$

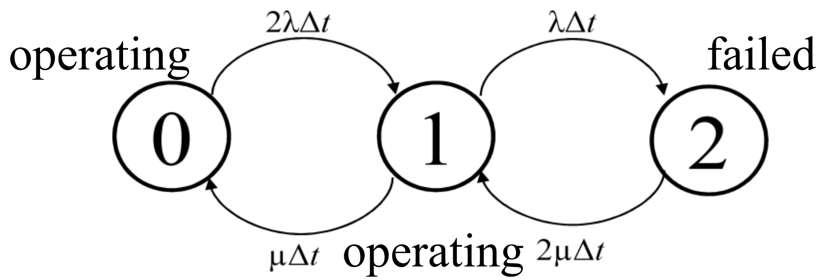
$F = \{2\}$

$$R(t) = P(T > t) = P\{X(\tau) = 0 \ \ or \ X(\tau) = 1, \forall \tau \in [0, t)\} = P_0^*(t) + P_1^*(t)$$

Can I trasnsform the Markov Diagram in such a way that $R(t) = P_0^*(t) + P_1^*(t)$?

System Reliability: Reparaible Systems (1)

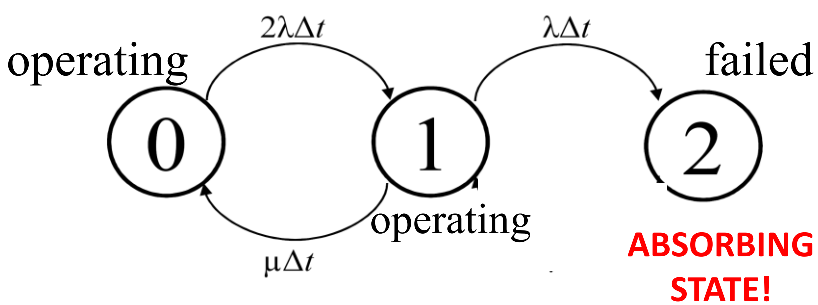
Parallel System of two identical components



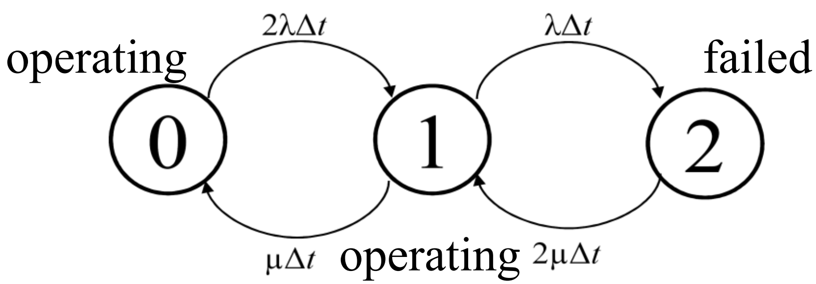
$$S = \{0,1\}$$
$$F = \{2\}$$

$$R(t) = P(T > t) = P\{X(\tau) = 0 \ \ or \ X(\tau) = 1, \forall\tau \in [0, t)\} = P_0^*(t) + P_1^*(t)$$

Can I trasnsform the Markov Diagram in such a way that $R(t) = P_0^*(t) + P_1^*(t)$?



1. Transform the failed states $j \in F$ into absorbing states (the system cannot be repaired → it is not possible to escape from a failed state)**. i.e., exclude all the failed states $j \in F$ from the transition rate matrix**



$$S = \{0,1\}$$
$$F = \{2\}$$

$$\underline{\underline{A}} = \left(\begin{array}{cc|c} -2\lambda & 2\lambda & 0 \\ \mu & -(\mu+\lambda) & \lambda \\ \hline 0 & 2\mu & -2\mu \end{array}\right) \qquad \underline{\underline{A}}^* = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\mu+\lambda) & \lambda \\ 0 & 0 & 0 \end{bmatrix} \qquad \underline{\underline{A}}^* = \begin{bmatrix} -2\lambda & 2\lambda \\ \mu & -(\mu+\lambda) \end{bmatrix}$$

The new matrix $\underline{\underline{A}}^*$ contains the transition rates for transitions **only among the success states** $i \in S$

(the "reduced" system is virtually functioning continuously with no interruptions)

2. Solve the **reduced problem** of $\underline{\underline{A}}^*$ for the probabilities $P_i^*(t),\ i \in S$ of being in these **(transient) safe states**

$$\frac{d\underline{P}^*(t)}{dt} = \underline{P}^*(t) \cdot \underline{\underline{A}}^*$$

Reliability

$$R(t) = \sum_{i \in S} P_i^*(t)$$

Mean Time To Failure (MTTF)

$$MTTF = \int_0^\infty R(t)\,dt = \sum_{i \in S} \tilde{P}_i^*(0) = \tilde{R}(0)$$

NOTICE: in the reduced problem we have only transient states $\Rightarrow \Pi_i^* = P_i^*(\infty) = 0$

Example 6

Consider a system made by 2 identical components in parallel. Each component can be in two states: working or failed. Assume constant failure rate λ, that 2 repairmen are available and that the single component repair rate is constant and equal to μ. You are required to:

- find the system reliability
- find the system MTTF

Example 7

Consider a system made by 2 identical components in series. Each component can be in two states: working or failed. Assume constant failure rate λ, that 2 repairmen are available and that the single component repair rate is constant and equal to μ. You are required to:

- find the system reliability
- find the system MTTF