POLITECNICO DI MILANO

Markov Chain

Exercise lesson

Politecnico di Milano
Dipartimento di Energia

Exercise 1

A nuclear steam supply system has two turbo-generator units; unit 1 operates and unit 2 is in standby whenever both are good. The units have a constant MTTF of λ_i^{-1}, i=1 and 2, during active operation while during standby unit 2 has a MTTF of $(\lambda_2^*)^{-1}$. The repair of a unit is assumed to begin instantaneously after it fails, but its duration is random so that the instantaneous repair rates will be μ_1 and μ_2, respectively. The repairs can be done on only one unit at a time and any unit under repair will remain so until the task is completed.

1. Draw the system diagram.
2. Write the Markov equations.

Exercise 2

An alarm system is subject to both unrevealed (u) and revealed (r) faults each of which have time to occurrence which are exponentially distributed with mean values of 200 h and 100 h, respectively. If a revealed failure occurs, then the complete system is restored to the time-zero condition by a repair process which has exponentially distributed times to completion with a mean value of 10 h. If an unrevealed fault occurs, then it remains in existence until a revealed fault occurs when it is repaired along with the revealed fault.

1. What is the asymptotic unavailability of the alarm system?
2. What is the asymptotic failure intensity?
3. What is the mean number of system failures in a total time of 1000 h?

Exercise 3

Two identical pumps are working in parallel logic. During normal operation both pumps are functioning. When one pump fails, the other has to do the whole job alone, with a higher load. The pumps are assumed to have exponentially distributed failure times:
$\lambda_h = 1.5 * 10^{-4}\ h^{-1}$ when the pumps are bearing half load
$\lambda_f = 3.5 * 10^{-4}\ h^{-1}$ when the pumps are bearing the full load

Both pumps may fail at the same time due to some external stresses. The failure rate with respect to this common cause failure has been estimated to be $\lambda_c = 3.0 * 10^{-5}\ h^{-1}$. This type of external stresses affects the system irrespective of how many units are working.

Repair is initiated as soon as one of the pumps fails. The mean time to repair a pump, μ^{-1} , is15 hours. When both pumps are in the failed state, the whole system has to be shut down. In this case, the system will not be put into operation again until both pumps have been repaired. The mean downtime, ${\mu_b}^{-1}$, when both pumps are failed, has been estimated to be 25 hours.

1. Establish a state-space diagram for the system.
2. Write down the state equation in matrix format.
3. Compute the system MTTF
4. Determine the steady states probabilities.
5. Determine the percentage of time when:
    1. Both pumps are functioning
    2. Only one of the pumps is functioning
    3. Both pumps are in the failed state
6. Determine the mean number of pump repairs that are needed during a period of 5 years.
7. How many times we may expect to have a total pump failure (i.e. both pumps in a failed state at the same time) during a period of 5 years?