POLITECNICO DI MILANO

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Centre de recherche sur les Risques et les Crises

Fault tree analysis

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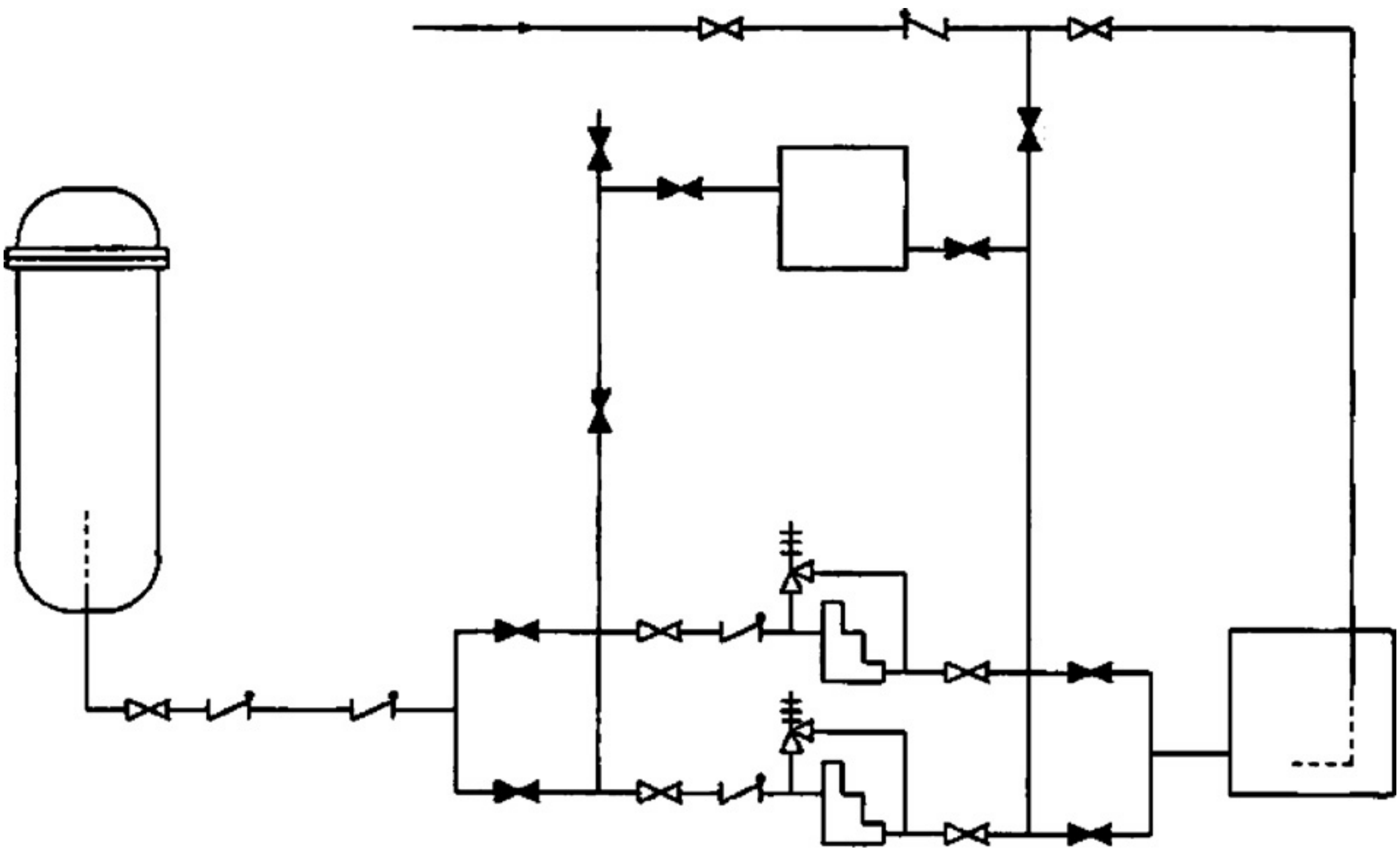
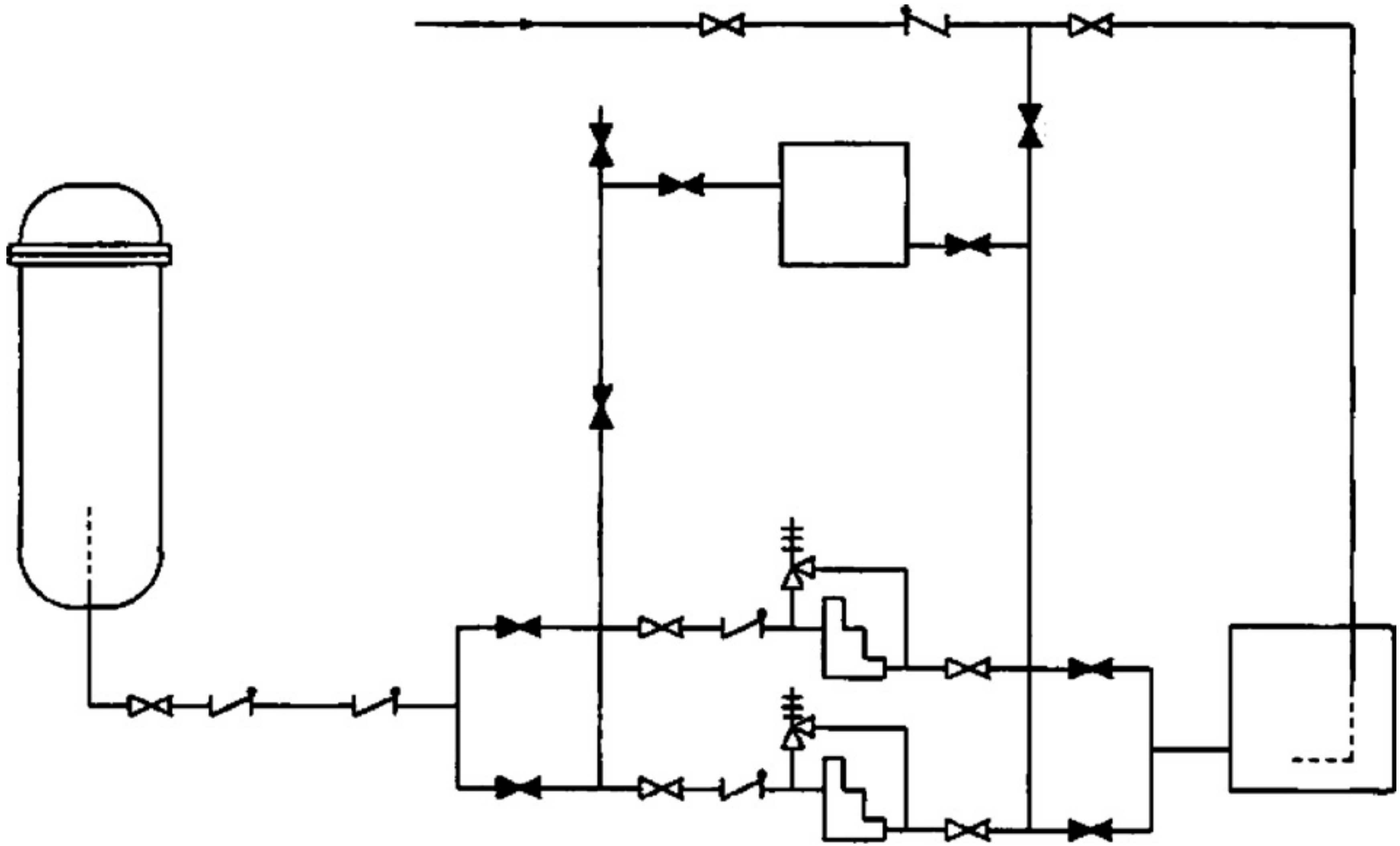


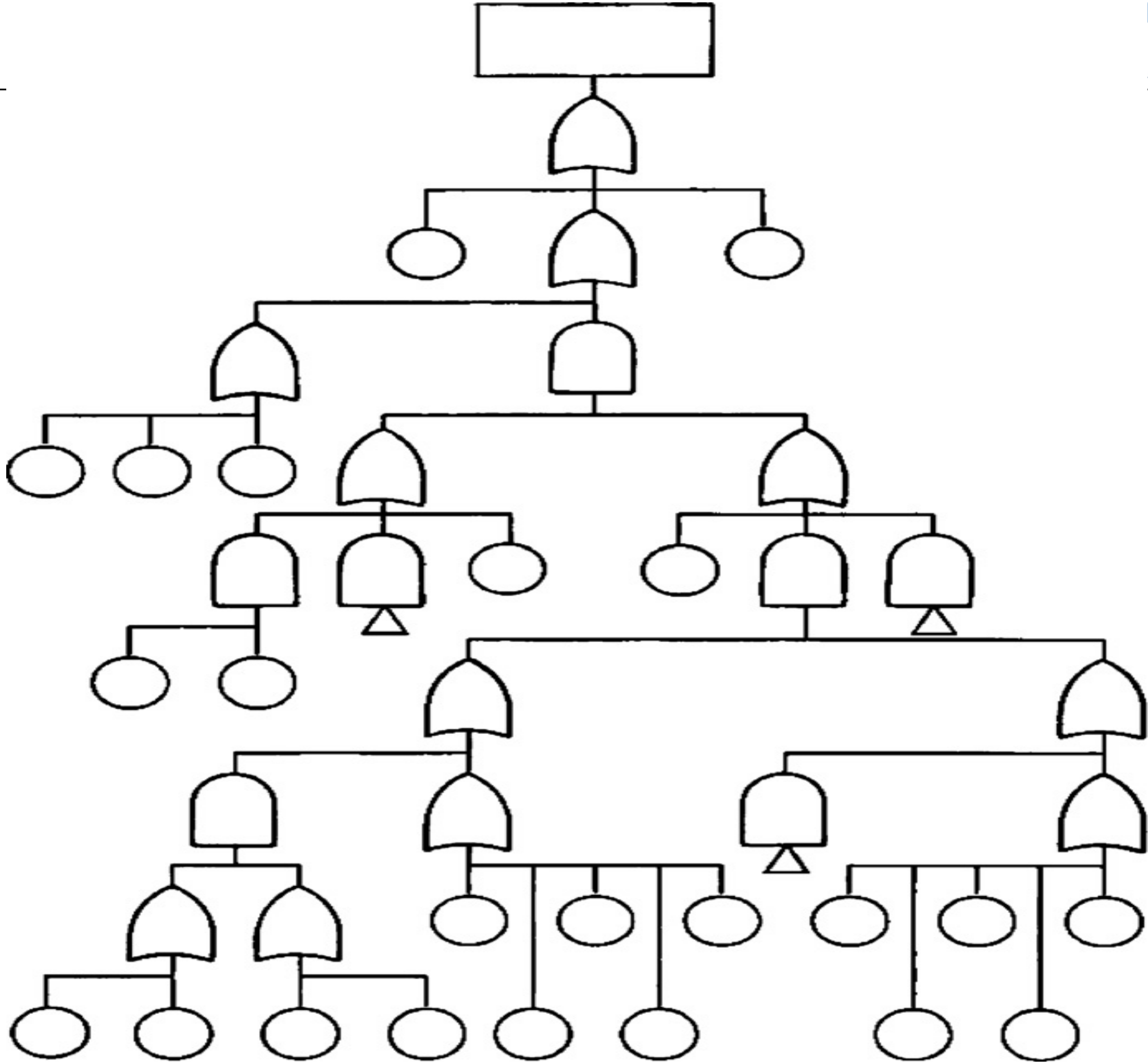
Fault Tree Analysis (FTA)

- Systematic and quantitative
- Deductive

AIM:

1. Decompose the system failure in elementary failure events of constituent components
2. Computation of system failure probability, from component failure probabilities



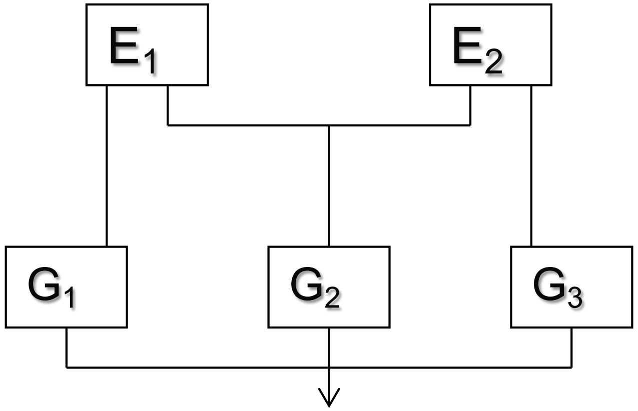


FT construction

FT construction: Procedure steps

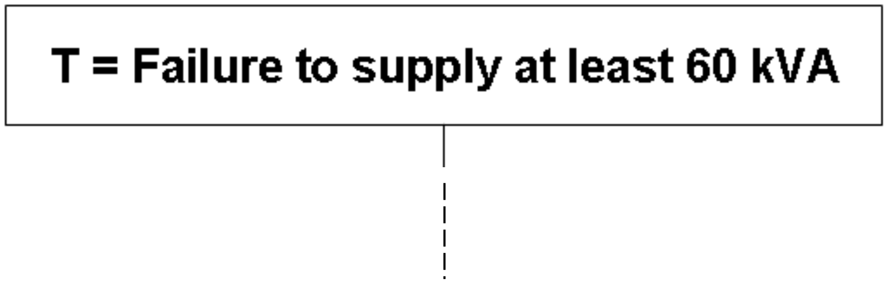
1. **Define top event (system failure)**

Electric power generation system



E1, E2 = engines

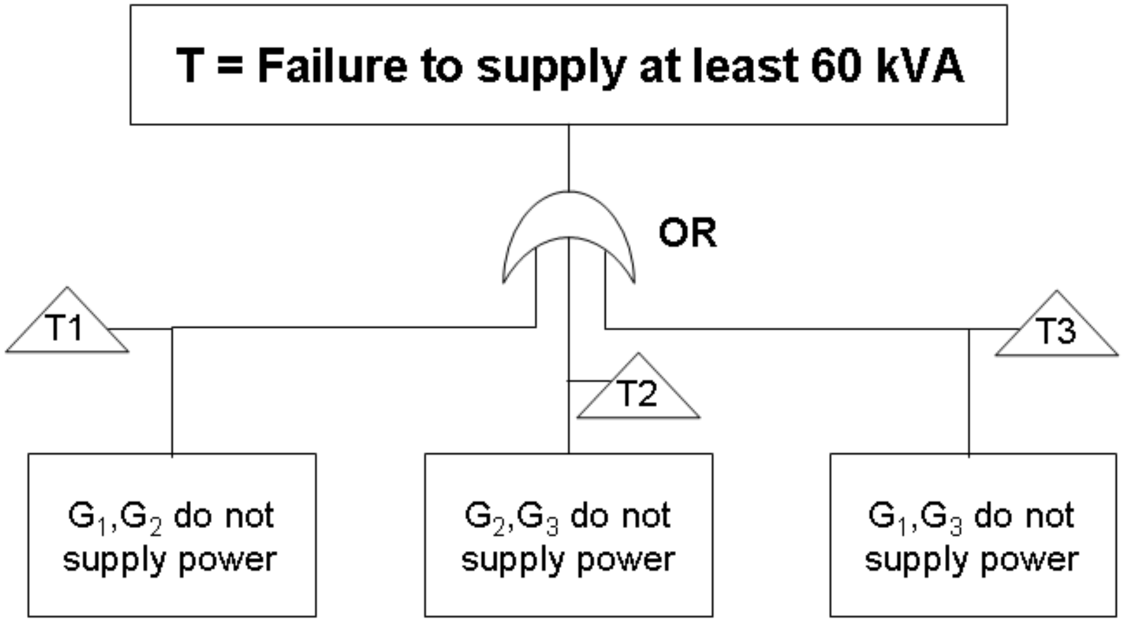
G1, G2, G3 = generators, each one is rated at 30 KVA

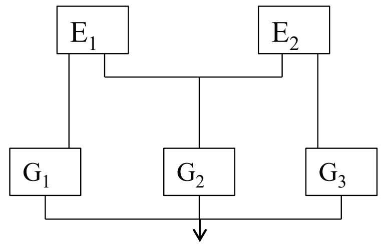


FT construction: Procedure steps

1. **Define top event (system failure)**
2. **Decompose top event by identifying subevents which can cause it.**

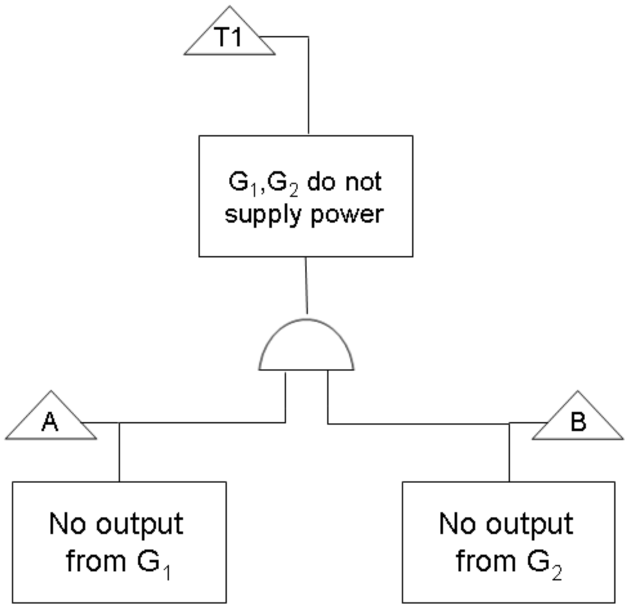
At least two out of the three generators do not work

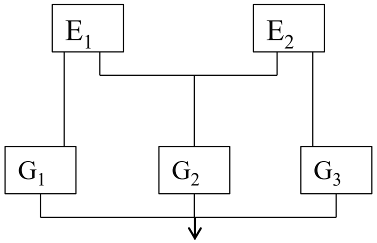




FT construction: Procedure steps

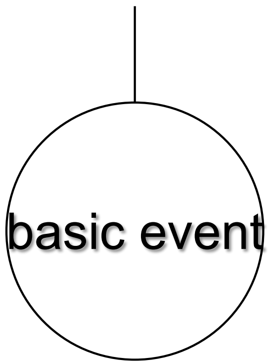
1. Define top event (system failure)
2. Decompose top event by identifying subevents which can cause it
3. Decompose each subevent in more elementary subevents which can cause it





FT construction: Procedure steps

1. **Define top event (system failure)**
2. **Decompose top event by identifying subevents which can cause it**
3. **Decompose each subevent in more elementary subevents which can cause it**
4. **Stop decomposition when subevent probability data are available (resolution limit): subevent = basic or primary event**



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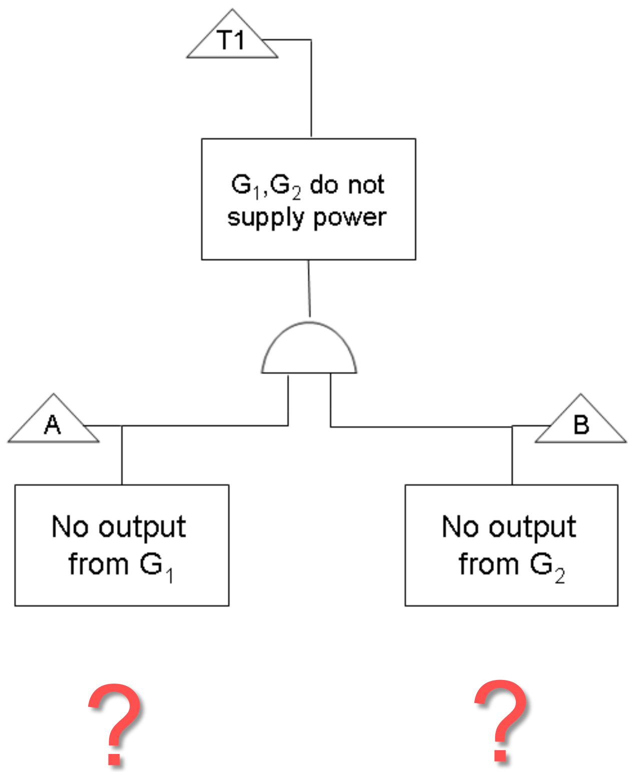
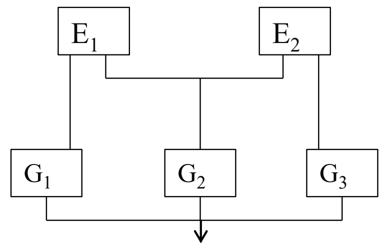
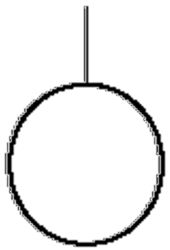
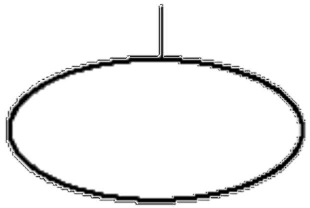
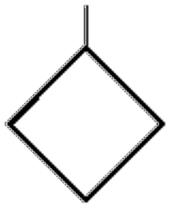
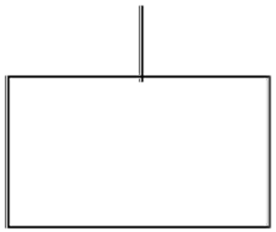
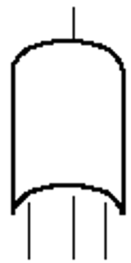
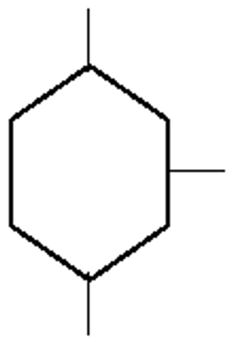




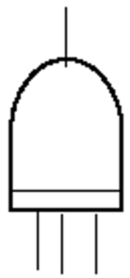
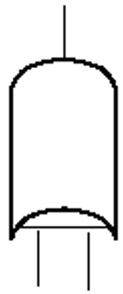
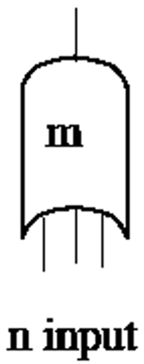
FT event symbols

Symbol	Description	Symbol	Description
	Basic event with sufficient data		Condition event used with inhibit gate
	Undeveloped event		Transfer symbol
	Event represented by a gate		

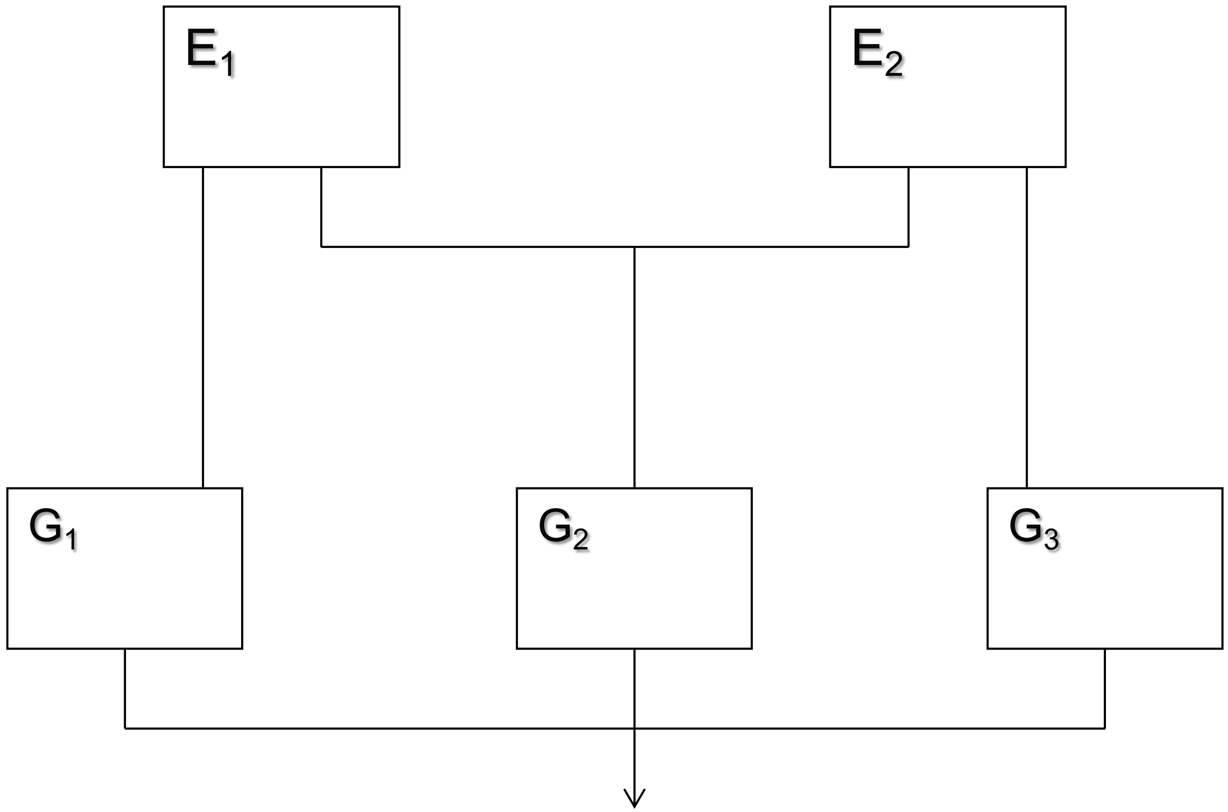
FT gate symbols

	AND gate	Output event occurs if all input events occur simultaneously.
	OR gate	Output event occurs if any one of the input events occurs.
	Inhibit gate	Input produces output when conditional event occurs.

FT gate symbols

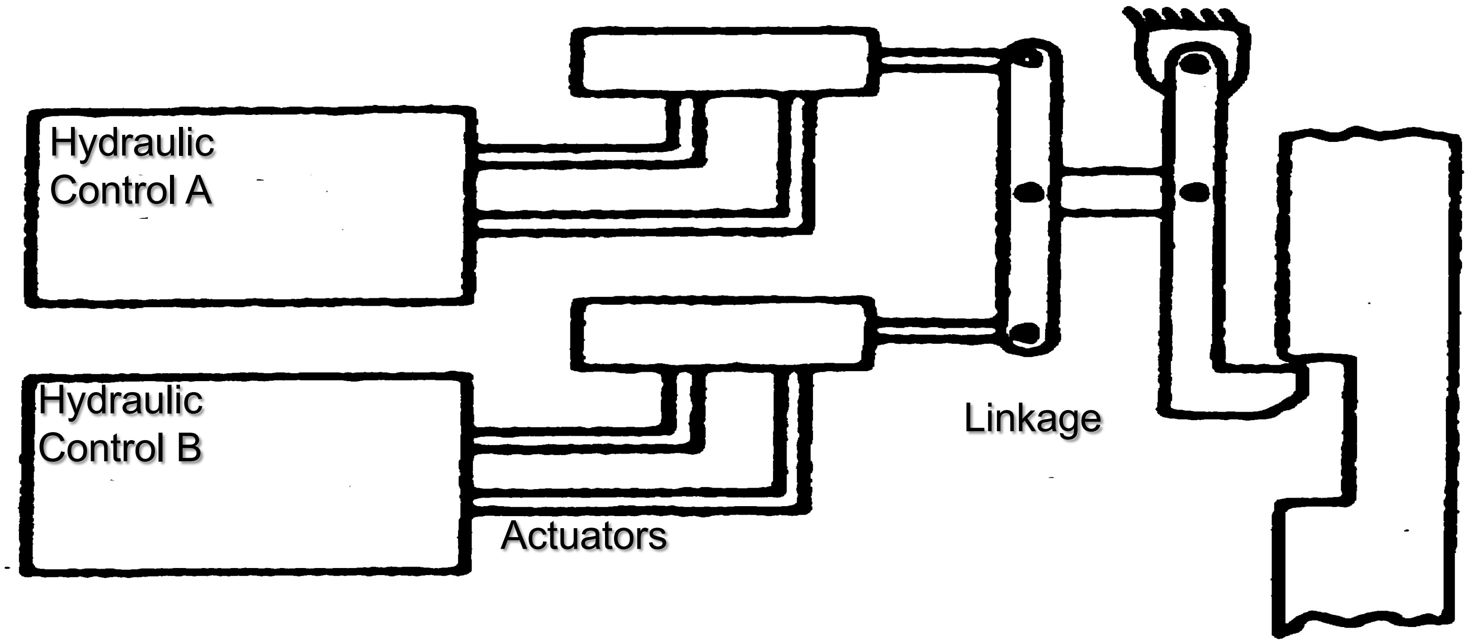
	Priority AND Gate	Output event occurs if all input events occur in the order from left to right
	Exclusive OR Gate	Output event occurs if one, but not both, of the input events occur.
	m out of n gate (volume or sample gate)	Output event occurs if m out of n input events occur.

FT Example 1: Electric Power Generation System

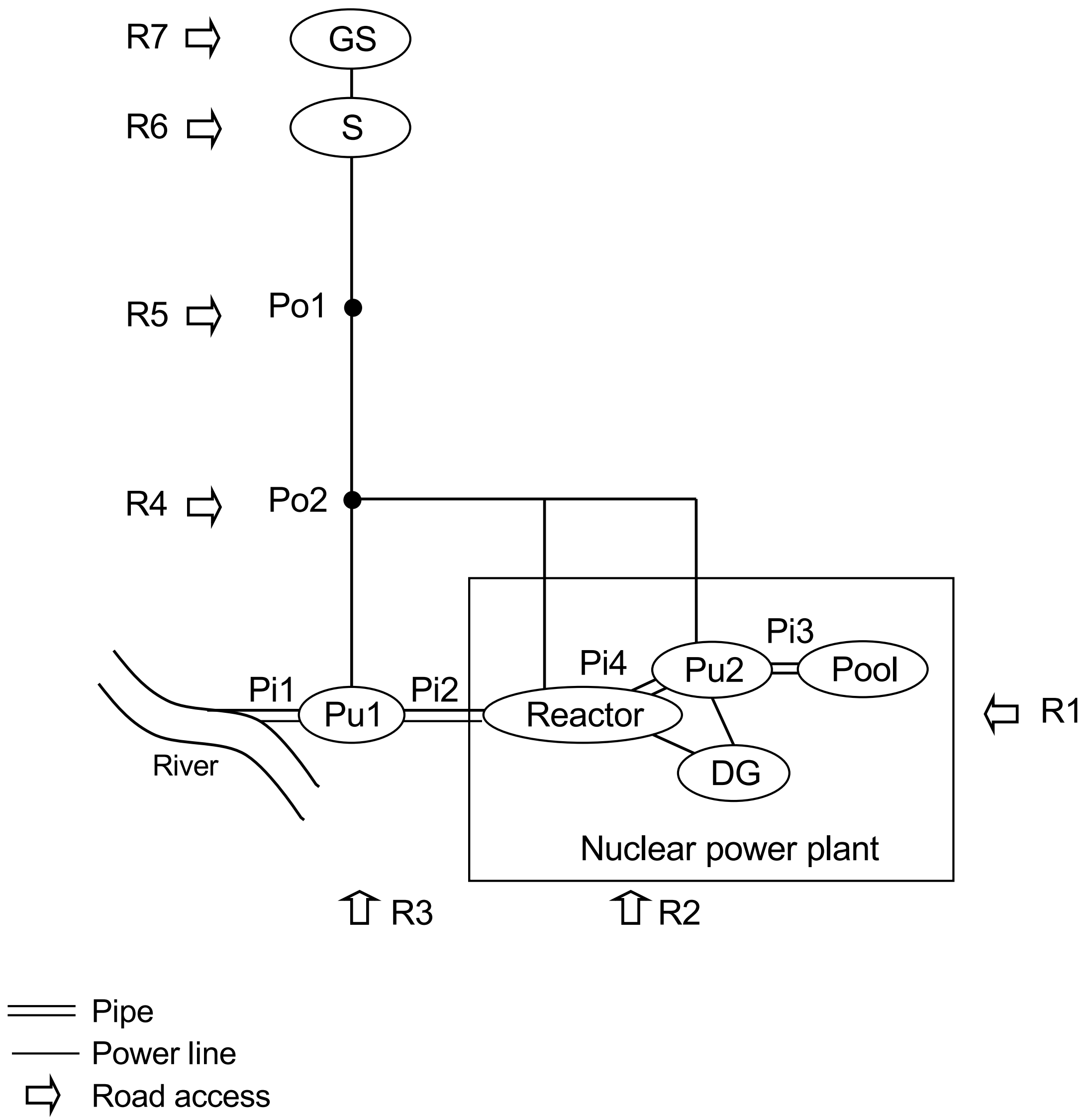


Fault tree?

FT Example 2: The Shutdown System



FT Example 3: The System of Systems



Internal emergency devices:

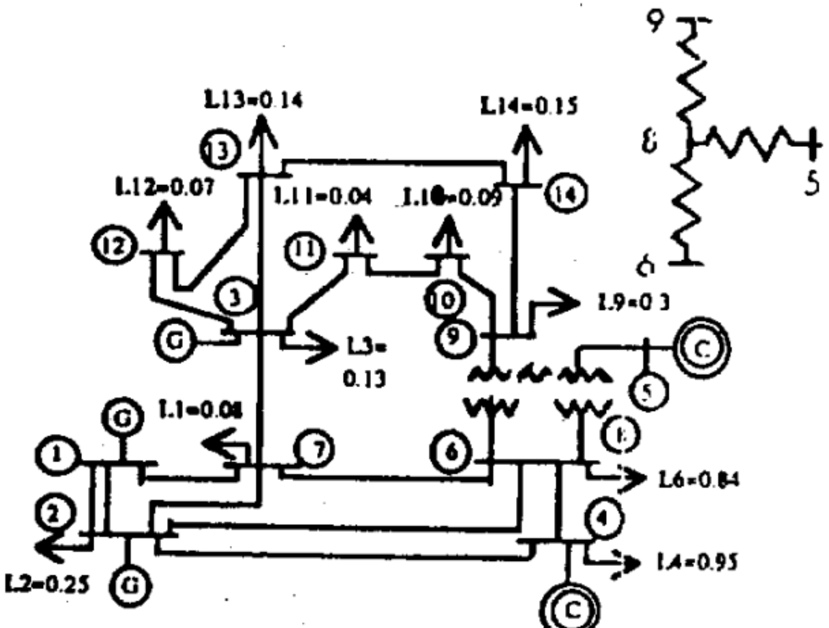
- Power system
  - Diesel Generator (DG)
- Water system
  - Pipe (Pi)
  - Pump (Pu)
  - Pool

Interdependent CIs:

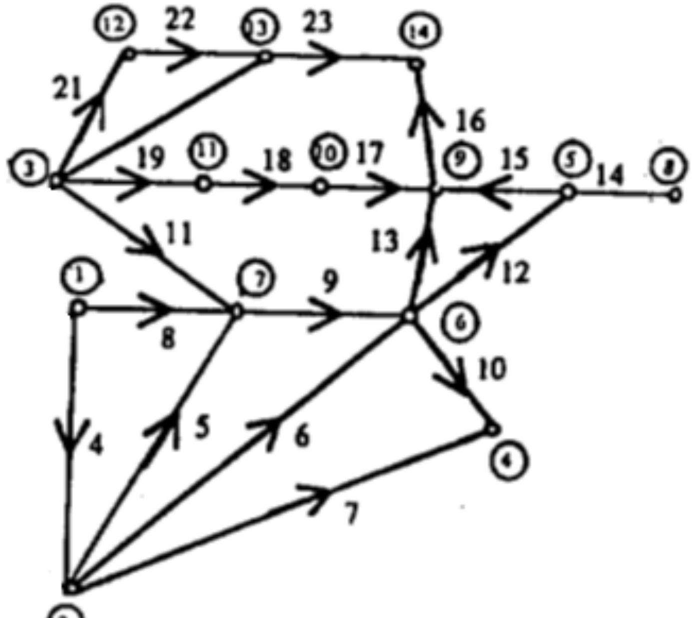
- Power system
  - Generation Station (GS)
  - Substation (S)
  - Pole (Po)
- Water system
  - Pipe (Pi)
  - Pump (Pu)
  - River
- Road transportation system
  - Road access (R)

FT Example 4: IEEE14 Bus Power Distribution System

Generators (G1, G2 , G3)
Loads (2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14)
Power delivery paths: lines (L) and buses (B).



(a)



(b)

FT Example 4: IEEE14 Bus Power Distribution System

Draw a Fault Tree (FT) for the top event “failure to supply power to bus 2” (Load2)

FT Example 4: IEEE14 Bus Power Distribution System

Draw a Fault Tree (FT) for the top event “failure to supply power to bus 4” (Load4)

FT qualitative analysis

FT qualitative analysis

Introducing:

X_i : binomial indicator variable of i-th component state (basic event)

$$X_i = \begin{cases} 1 & \text{failure event } \underline{\text{true}} \\ 0 & \text{failure event } \underline{\text{false}} \end{cases}$$

- **FT = set of boolean algebraic equations (one for each gate) => structure (switching) function *Φ*:**

$$X_T = \Phi(X_1, X_2, \ldots, X_n)$$

Fundamental Products

- **FT = set of boolean algebraic equations (one for each gate) => structure (switching) function Φ:**

$$X_T = \Phi(X_1, X_2, \ldots, X_n)$$

An important theorem states that a structure function can be written uniquely as the union of the fundamental products which correspond to the combinations of the variables which render the function true. This is called the canonical expansion or disjunctive normal form of Φ.

Fundamental Laws

1) Commutative Law:
   (a) $XY = YX$
   (b) $X + Y = Y + X$

2) Associative Law
   (a) $X(YZ) = (XY)Z$
   (b) $X + (Y + Z) = (X + Y) + Z$

3) Idempotent Law
   (a) $XX = X$
   (b) $X + X = X$

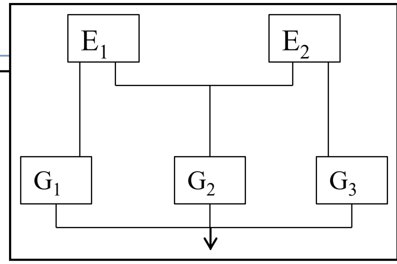
4) Absorption Law
   (a) $X(X + Y) = X$
   (b) $X + XY = X$

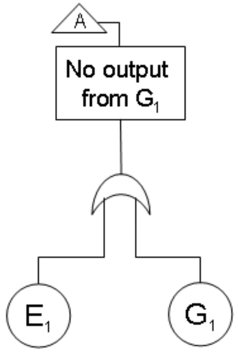
5) Distributive Law
   (a) $X(Y + Z) = XY + XZ$
   (b) $(X + Y)(X + Z) = X + YZ$

6) Complementation*
   (a) $X\overline{X} = \varnothing$
   (b) $X + \overline{X} = \Omega$
   (c) $\overline{\overline{X}} = X$

7) Unnamed relationships but frequently useful
   (a) $X + \overline{X}Y = X + Y$
   (b) $\overline{X}(X + Y) = \overline{X}\,\overline{Y}$

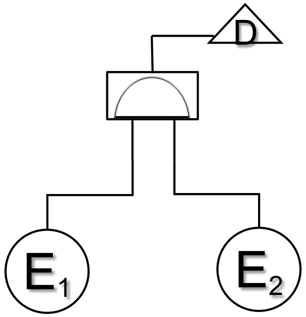
Structure function: Example 1





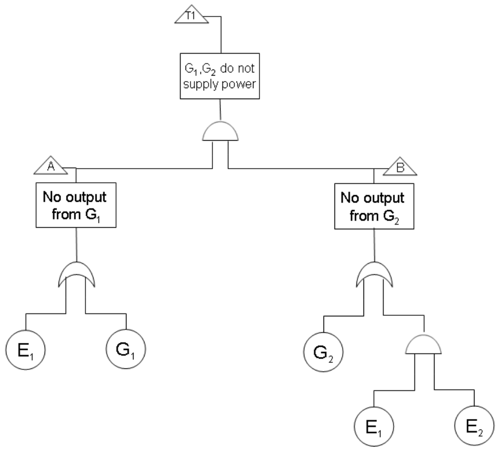
OR gate

$$\mathbf{X_A = X_{E_1} + X_{G_1} - X_{E_1} X_{G_1}} =$$
$$= 1 - (1 - \mathbf{X_{E_1}})(1 - \mathbf{X_{G_1}})$$



AND gate

$$\mathbf{X_D = X_{E_1} X_{E_2}}$$



$$X_{T_1} = \Phi(X_{E_1}, X_{E_2}, X_{G_1}, X_{G2})$$

Coherent structure functions

A physical system would be quite unusual (or perhaps poorly designed) if improving the performance of a component (that is, replacing a failed component by a functioning one) caused the system to change from the success to the failed state.

Thus, we restrict consideration to structure functions that are monotonically increasing in each input variable. These structure functions do not contain complemented variables; they are called *coherent* and can always be expressed as the union of fundamental products.

The main properties of a coherent structure function are:

1. $\Phi(\underline{1}) = 1$ if all the components are in their success state, the system is successful;
2. $\Phi(\underline{0}) = 0$ if all the components are failed, the system is failed;
3. $\Phi(\underline{X}) \geq \Phi(\underline{Y})$ for $\underline{X} \geq \underline{Y}$

FT qualitative analysis

Coherent structure functions can be expressed in reduced expressions in terms of minimal path or cut sets. A path set is a set $\underline{X}$ such that $\Phi(\underline{X})=1$; a cut set is a set $\underline{X}$ such that $\Phi(\underline{X})=0$. Physically, a path (cut) set is a set of components whose functioning (failure) ensures the functioning (failure) of the system.

- **Reduce Φ in terms of minimal cut sets (mcs)**

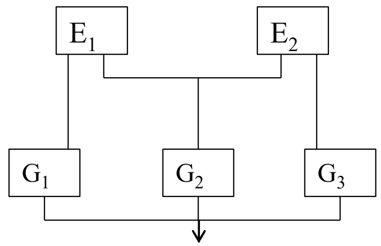
- **cut sets =** logic combinations of primary events which render true the top event
- **minimal cut sets =** cut sets such that if one of the events is not verified, the top event is not verified

FT qualitative analysis

- FT = set of boolean algebraic equations (one for each gate) => structure (switching) function Φ:

$$X_T = \Phi(X_1, X_2, \ldots, X_n)$$

- Boolean algebra to solve FT equations



$$X_{T_1} = X_A X_B =$$

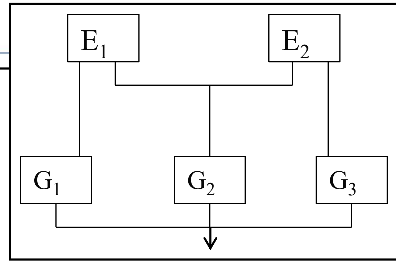
$$= (X_{E_1} + X_{G_1} - X_{E_1}X_{G_1})(X_{G_2} + X_{E_1}X_{E_2} - X_{E_1}X_{E_2}X_{G_2}) =$$

$$= X_{E_1}X_{G_2} + X_{E_1}X_{E_2} - X_{E_1}X_{E_2}X_{G_2} + X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_1} +$$

$$- X_{E_1}X_{E_2}X_{G_1}X_{G_2} - X_{E_1}X_{G_1}X_{G_2} - X_{E_1}X_{E_2}X_{G_1} + X_{E_1}X_{E_2}X_{G_1}X_{G_2} =$$

$$= X_{E_1}X_{G_2} + X_{E_1}X_{E_2} + X_{G_1}X_{G_2} - X_{E_1}X_{E_2}X_{G_2} - X_{E_1}X_{G_1}X_{G_2}$$

FT Example 1: mcs



$$X_{T_1} = X_{E_1}X_{G_2} + X_{E_1}X_{E_2} + X_{G_1}X_{G_2} - X_{E_1}X_{E_2}X_{G_2} - X_{E_1}X_{G_1}X_{G_2}$$

$$= 1 - [1 - X_{E_1}X_{G_2} - X_{E_1}X_{E_2} - X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_2} + X_{E_1}X_{G_1}X_{G_2}] =$$

$$= 1 - [1 - X_{E_1}X_{G_2} - X_{E_1}X_{E_2} - X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_2} + X_{E_1}X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_1}X_{G_2} - X_{E_1}X_{E_2}X_{G_1}X_{G_2}] =$$

$$= 1 - [1 - X_{E_1}X_{E_2} - X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_1}X_{G_2} - X_{E_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_2} + X_{E_1}X_{G_1}X_{G_2} - X_{E_1}X_{E_2}X_{G_1}X_{G_2}] =$$

$$= 1 - [1 - X_{E_1}X_{E_2} - X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_1}X_{G_2} - X_{E_1}X_{G_2}(1 - X_{E_1}X_{E_2} - X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_1}X_{G_2})] =$$

$$= 1 - [(1 - X_{E_1}X_{G_2})(1 - X_{E_1}X_{E_2} - X_{G_1}X_{G_2} + X_{E_1}X_{E_2}X_{G_1}X_{G_2})] =$$

$$= 1 - [(1 - X_{E_1}X_{G_2})(1 - X_{E_1}X_{E_2})(1 - X_{G_1}X_{G_2})]$$

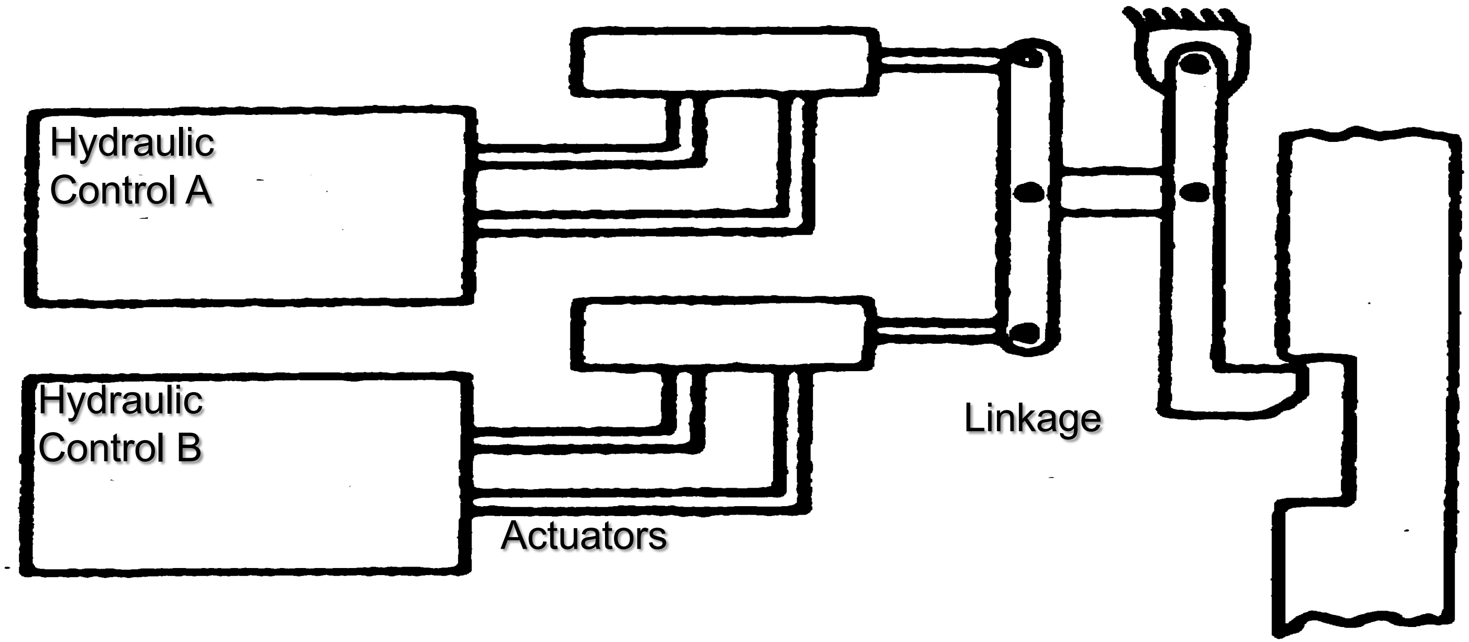
3 minimal cut sets:

$$M_1 = \{E_1 G_2\}$$

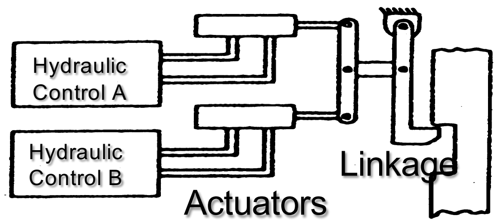
$$M_2 = \{E_1 E_2\}$$

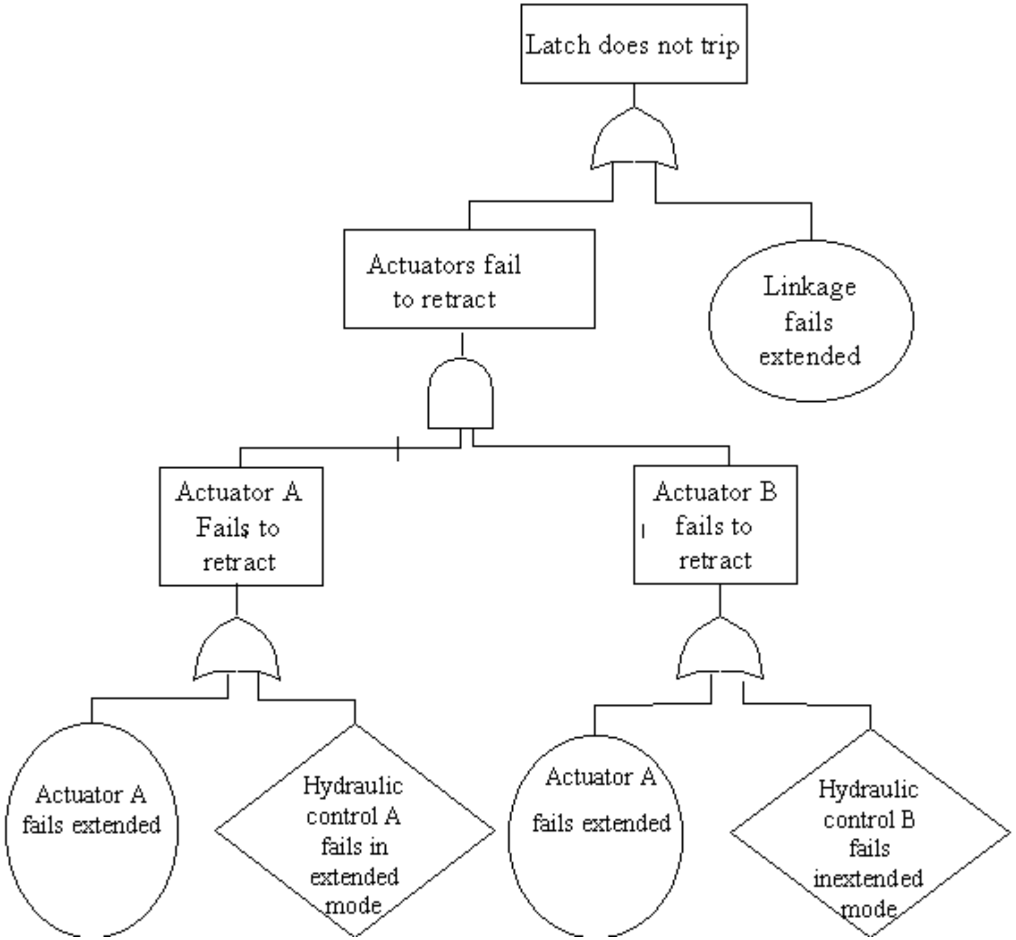
$$M_3 = \{G_1 G_2\}$$

FT Example 2: The Shutdown System



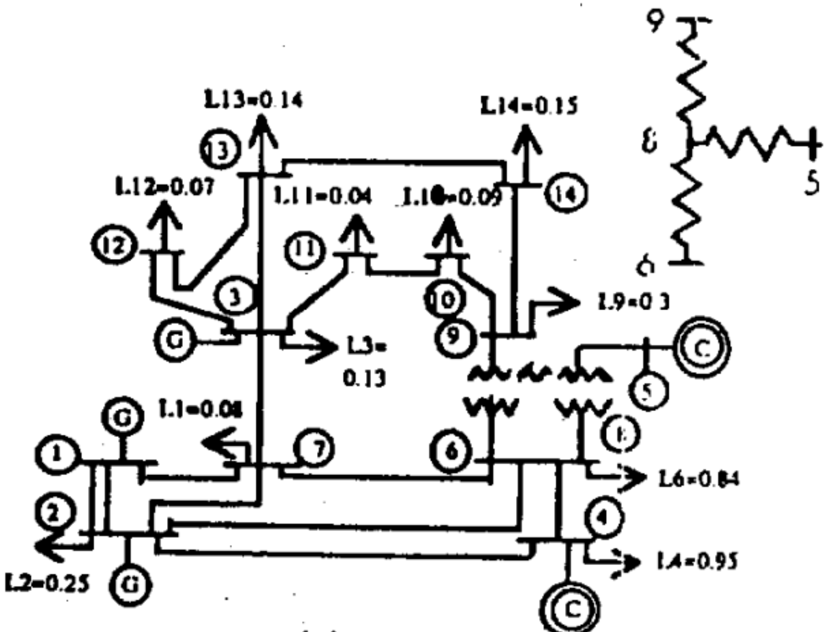
FT Example 2: The Fault Tree



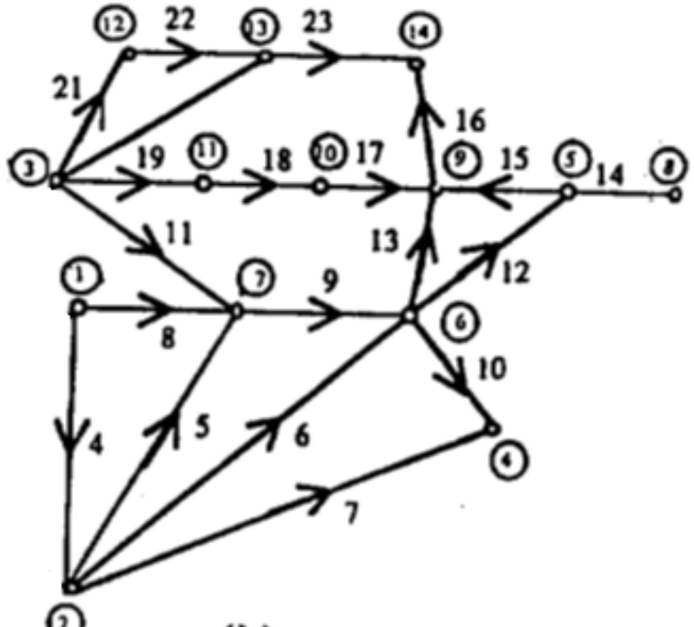


FT Example 4: IEEE14 Bus Power Distribution System

Generators (G1, G2 , G3)
Loads (2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14)
Power delivery paths: lines (L) and buses (B).



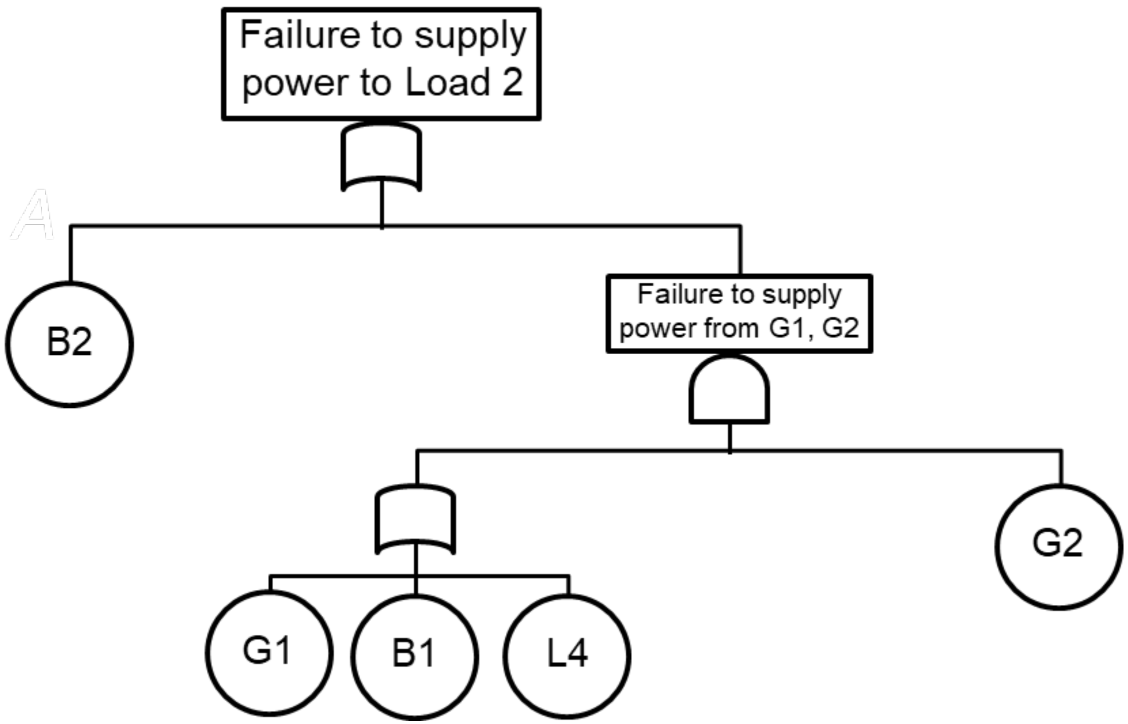
(a)



(b)

FT Example 4: IEEE14 Bus Power Distribution System

Find the Mcs for the top event "failure to supply power to bus 2" (Load2)



FT qualitative analysis: results

1. mcs identify the component basic failure events which contribute to system failure
2. qualitative component criticality: those components appearing in low order mcs or in many mcs are most critical

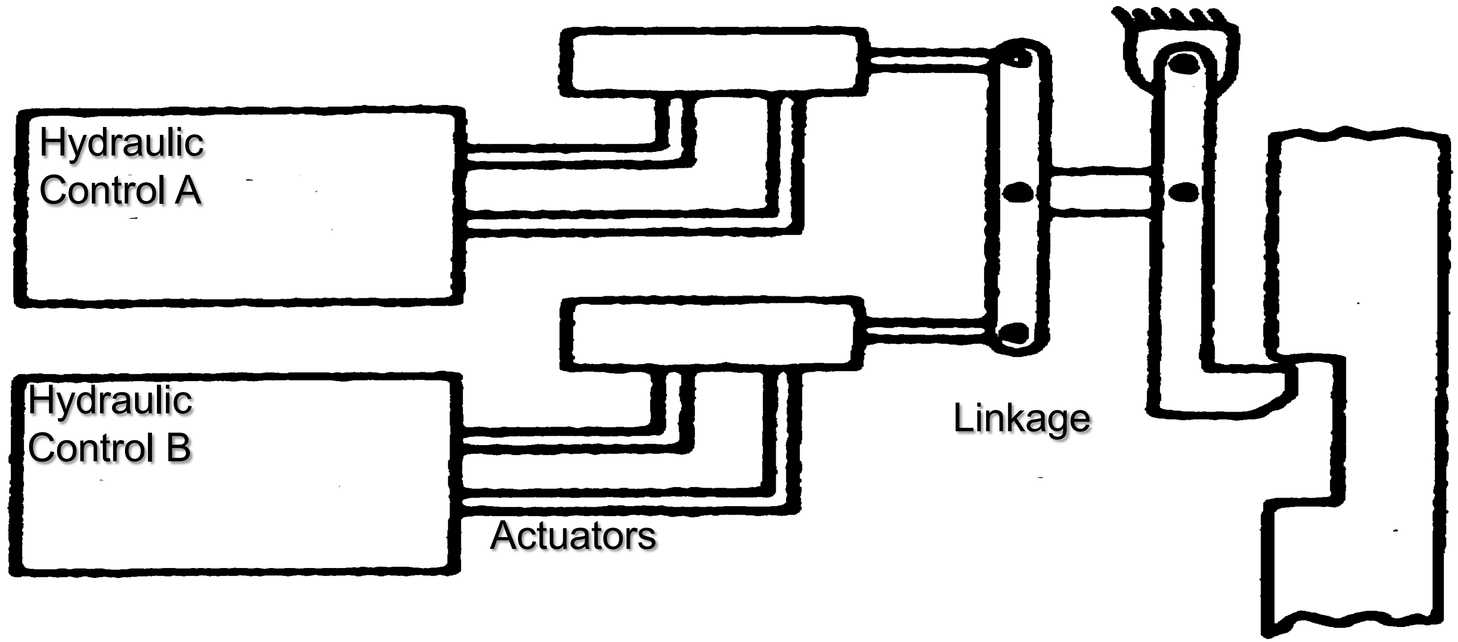
FT quantitative analysis

FT quantitative analysis

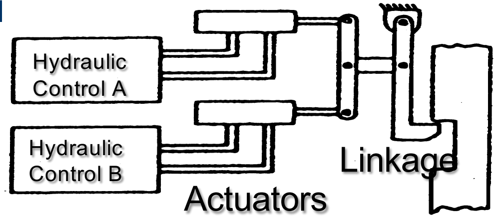
Compute system failure probability from primary events probabilities by:

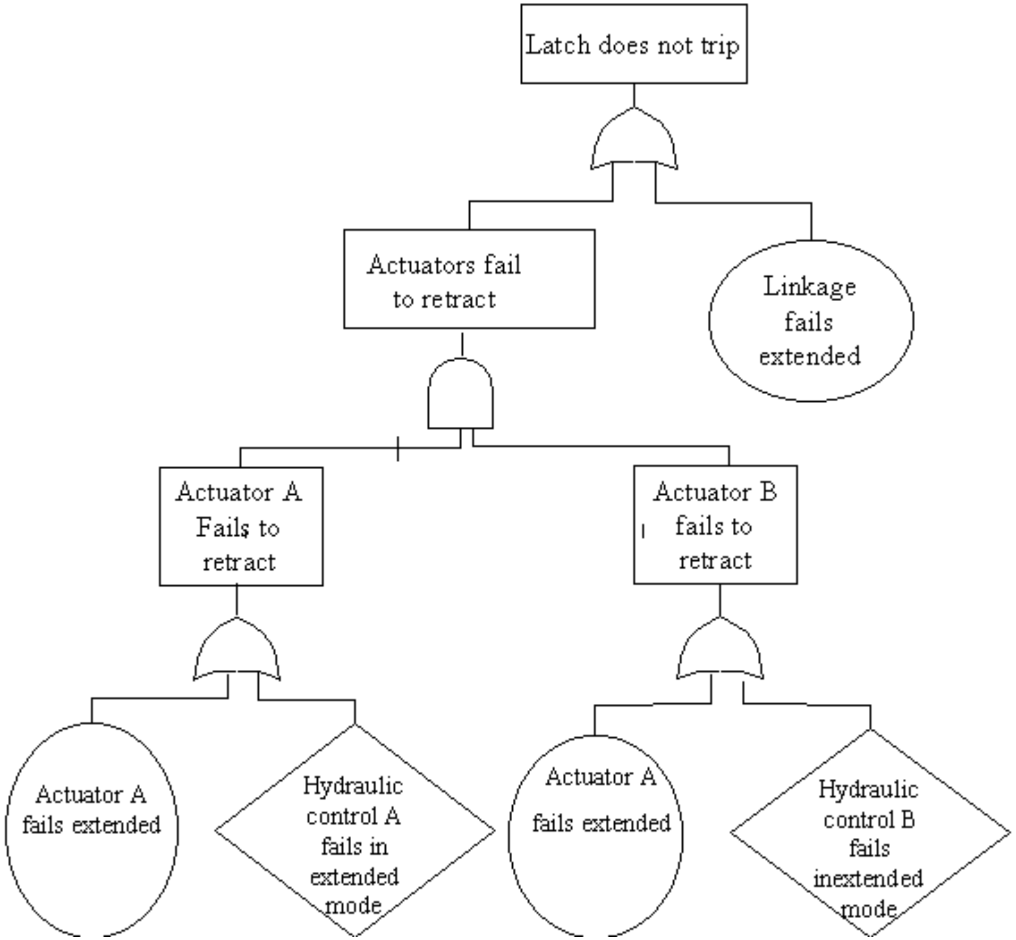
1. **using the laws of probability theory at the fault tree gates**

FT Example 2: The Shutdown System



FT Example 2: The Fault Tree

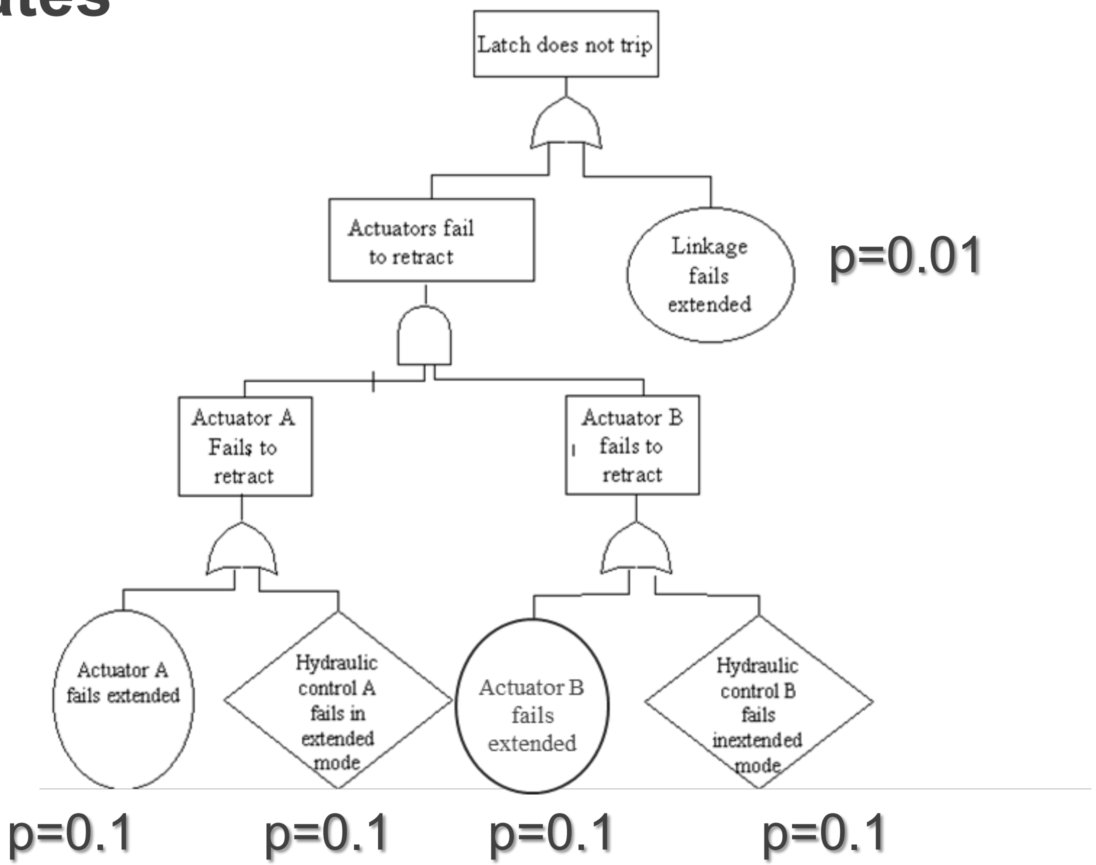


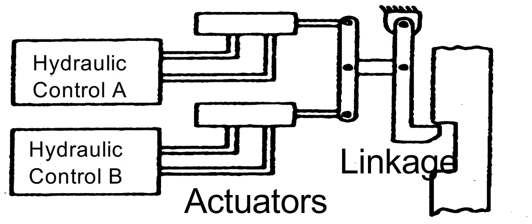


FT quantitative analysis-Example 2

Compute system failure probability from primary events probabilities by:

1. **using the laws of probability theory at the fault tree gates**





FT quantitative analysis

Compute system failure probability from primary events probabilities by:

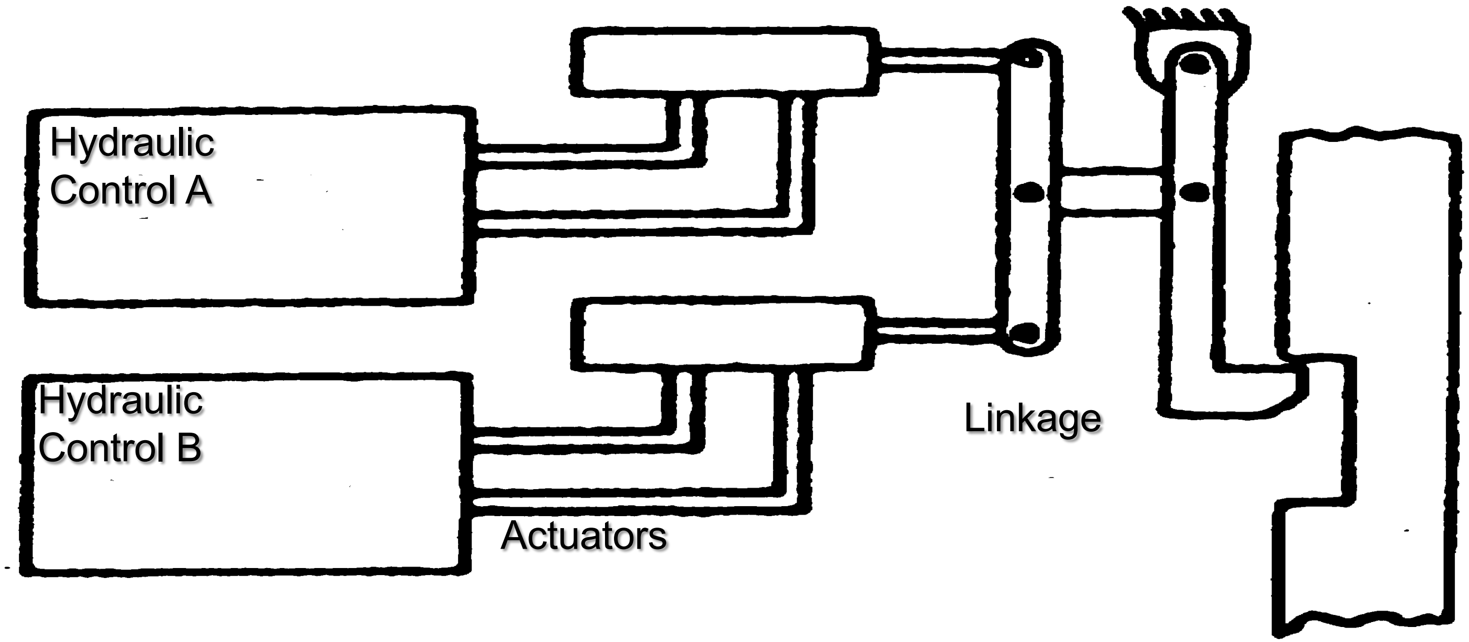
1. using the laws of probability theory at the fault tree gate
2. using the mcs found from the qualitative analysis

$$P[\Phi(\underline{X})=1]=\sum_{j=1}^{mcs}P[M_j]-\sum_{i=1}^{mcs-1}\sum_{j=i+1}^{mcs}P[M_iM_j]+\cdots+(-1)^{mcs+1}P[\prod_{j=1}^{mcs}M_j]$$

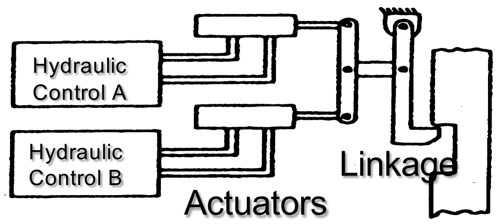
It can be shown that:

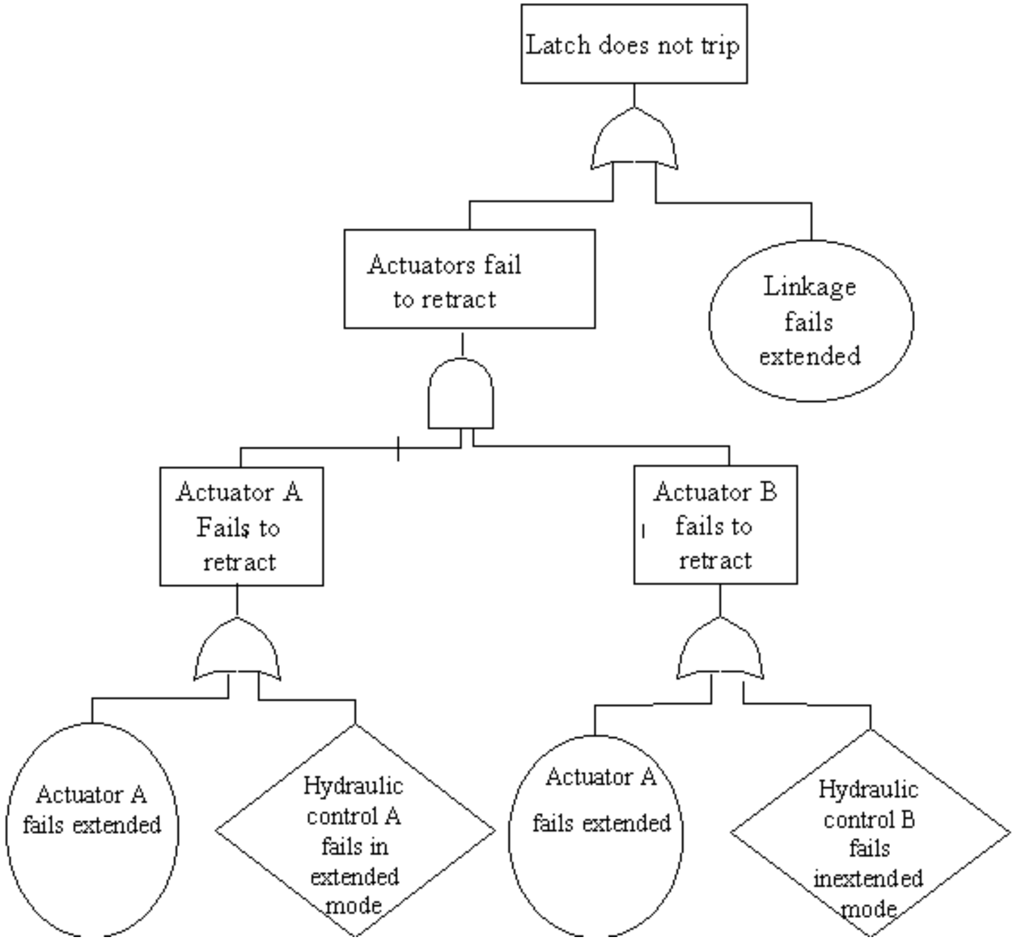
$$\sum_{j=1}^{mcs}P[M_j]-\sum_{i=1}^{mcs-1}\sum_{j=i+1}^{mcs}P[M_iM_j]\leq P[\Phi(\underline{X})=1]\leq\sum_{j=1}^{mcs}P[M_j]$$

FT Example 2: The Shutdown System

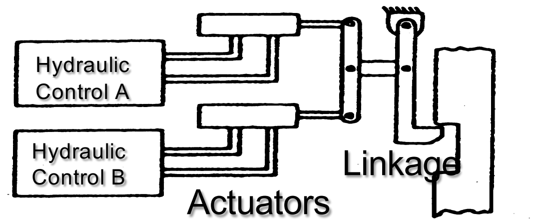


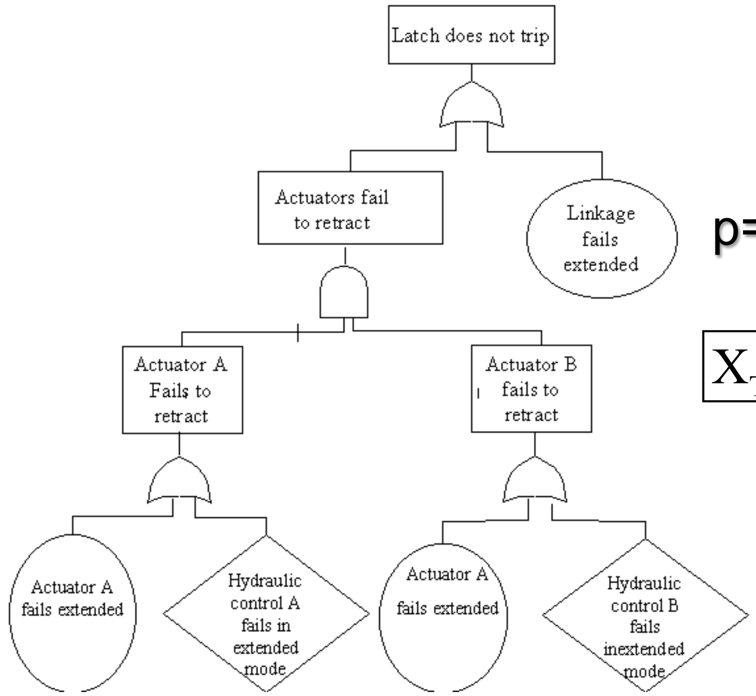
FT Example 2: The Fault Tree





FT Example 2: mcs





p=0.01

p=0.1 p=0.1 p=0.1 p=0.1

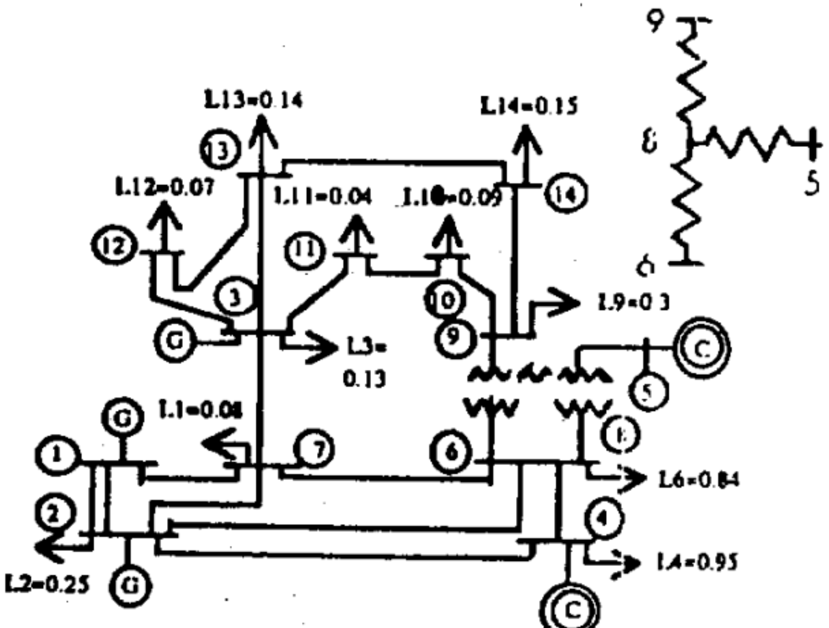
$$X_T = 1-(1-X_L)(1-(X_A + X_{HA} - X_A X_{HA})(X_B + X_{HB} - X_B X_{HB}))$$

5 minimal cut sets:

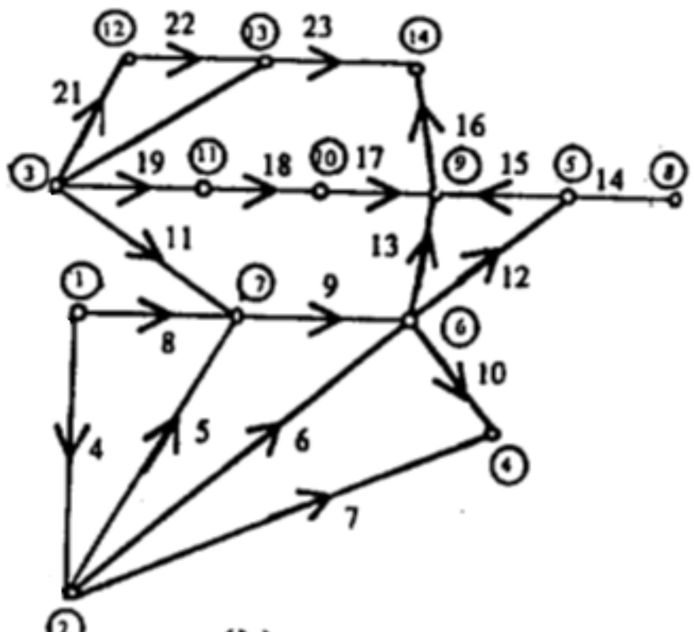
- $M_1 = X_L$
- $M_2 = X_A X_B$
- $M_3 = X_A X_{HB}$
- $M_4 = X_{HA} X_B$
- $M_5 = X_{HA} X_{HB}$

FT Example 4: IEEE14 Bus Power Distribution System

Generators (G1, G2 , G3)
Loads (2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14)
Power delivery paths: lines (L) and buses (B).

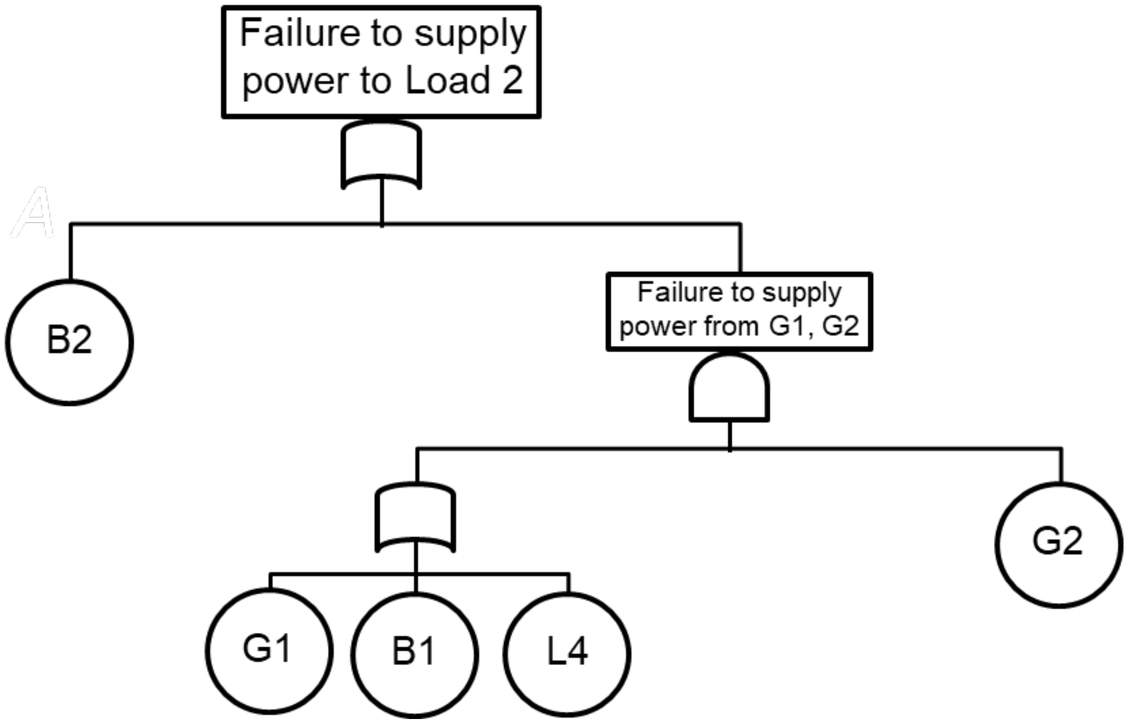


(a)


(b)

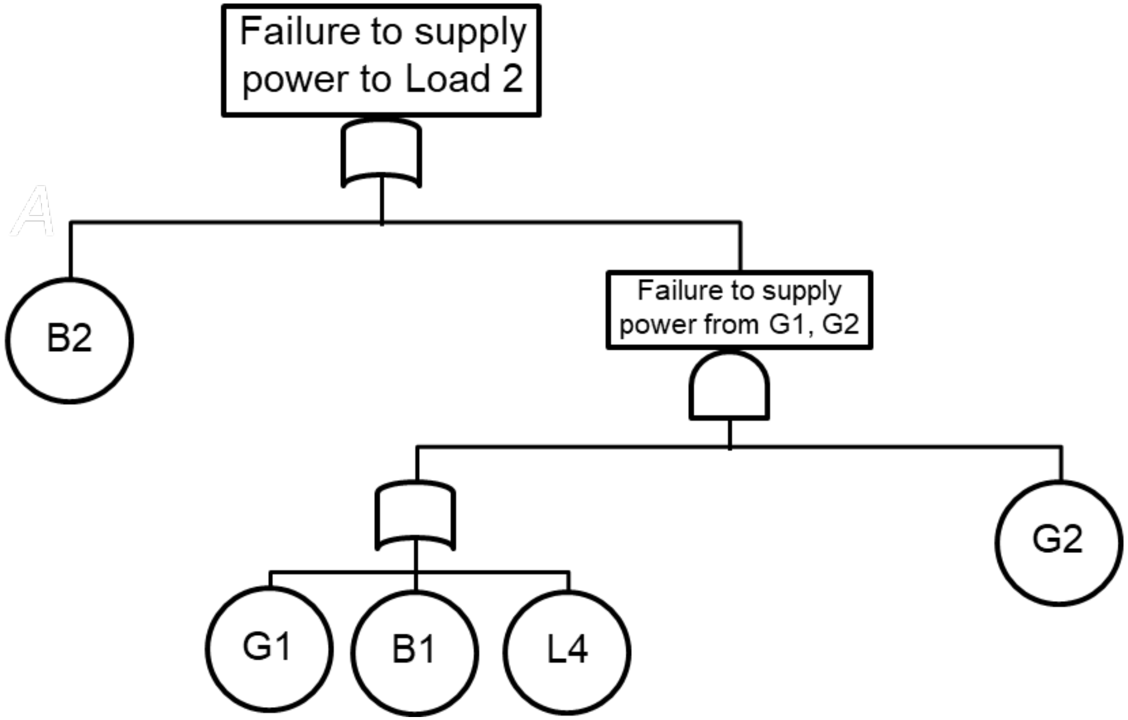
FT Example 4: IEEE14 Bus Power Distribution System

Find the Mcs for the top event "failure to supply power to bus 2" (Load2)



FT Example 4: IEEE14 Bus Power Distribution System

Find the Mcs for the top event "failure to supply power to bus 2" (Load2)



$M_1 = \{B_2\}$

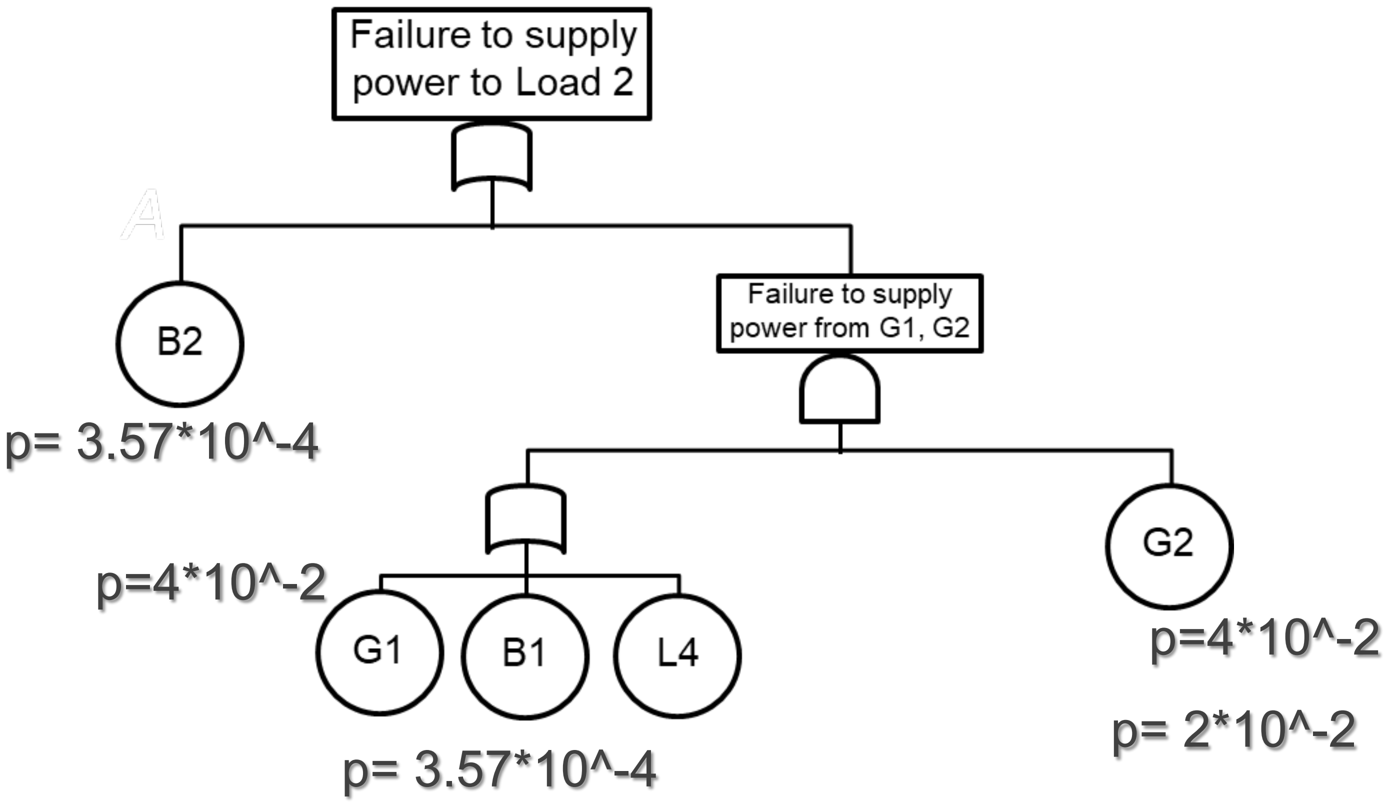
$M_2 = \{G_1, G_2\}$

$M_3 = \{B_1, G_2\}$

$M_4 = \{L_4, G_2\}$

FT Example 4: IEEE14 Bus Power Distribution System

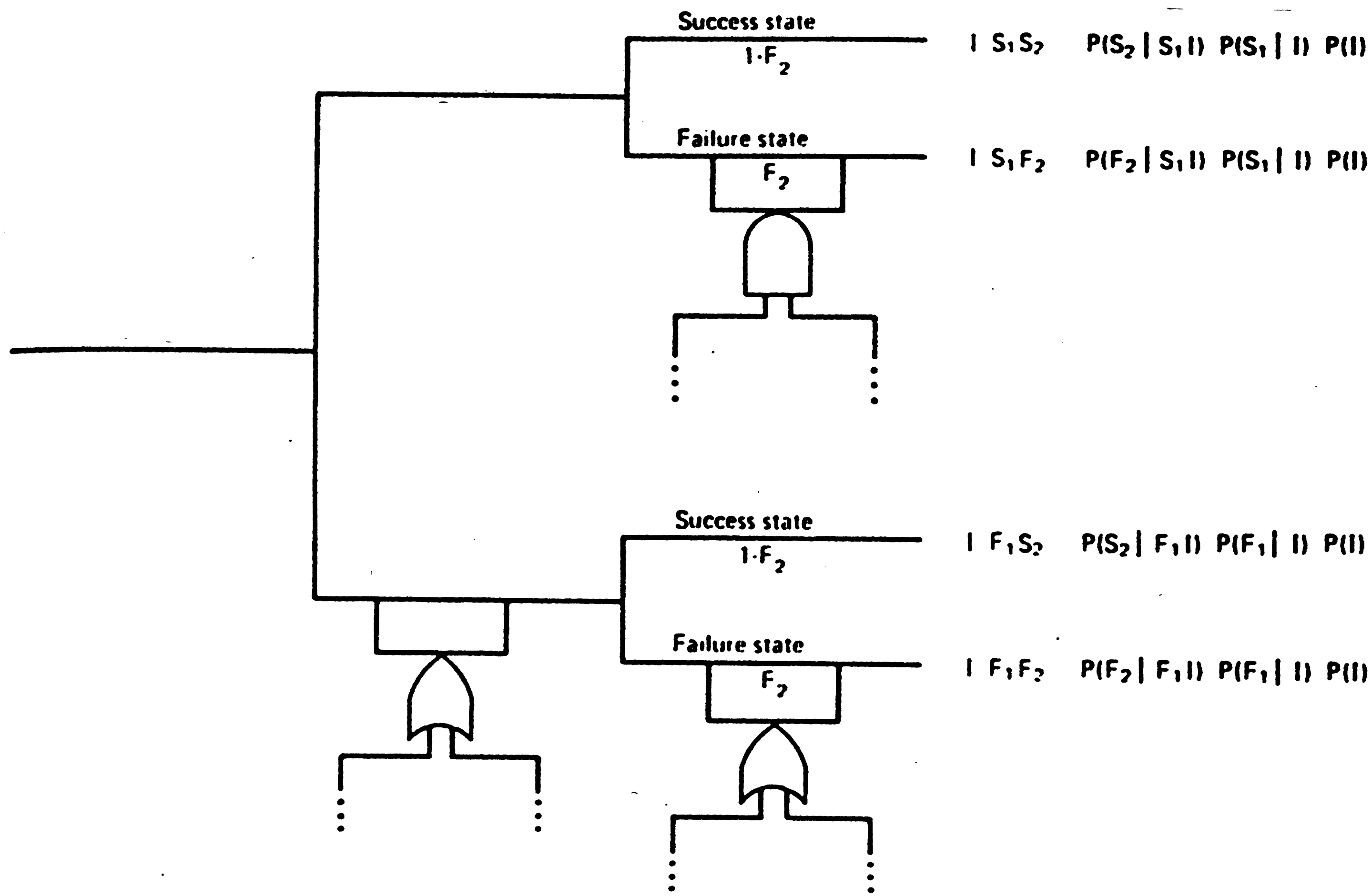
Find the Mcs for the top event “failure to supply power to bus 2” (Load2)



FT: comments

1. Straightforward modelization via few, simple logic operators;
2. Focus on one top event of interest at a time;
3. Providing a graphical communication tool whose analysis is transparent;
4. Providing an insight into system behaviour;
5. Minimal cut sets are a synthetic result which identifies the critical components.

ETA+FTA



Success state
1-F2
Failure state
F2
I S1S2 P(S2 | S1 I) P(S1 | I) P(I)
I S1F2 P(F2 | S1 I) P(S1 | I) P(I)
Success state
1-F2
Failure state
F2
I F1S2 P(S2 | F1 I) P(F1 | I) P(I)
I F1F2 P(F2 | F1 I) P(F1 | I) P(I)