POLITECNICO DI MILANO

Availability of Systems

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Introduction

Introduction: reliability and availability

- **Reliability** and **availability**: important performance parameters of a system, with respect to its ability to fulfill the required mission in a given period of time

- Two different system types:
  - Systems which must satisfy a specified mission within an assigned period of time: **reliability** quantifies the ability to achieve the desired objective without failures
  - Systems maintained: **availability** quantifies the ability to fulfill the assigned mission at **any** specific moment of the life time

Availability definition (1)

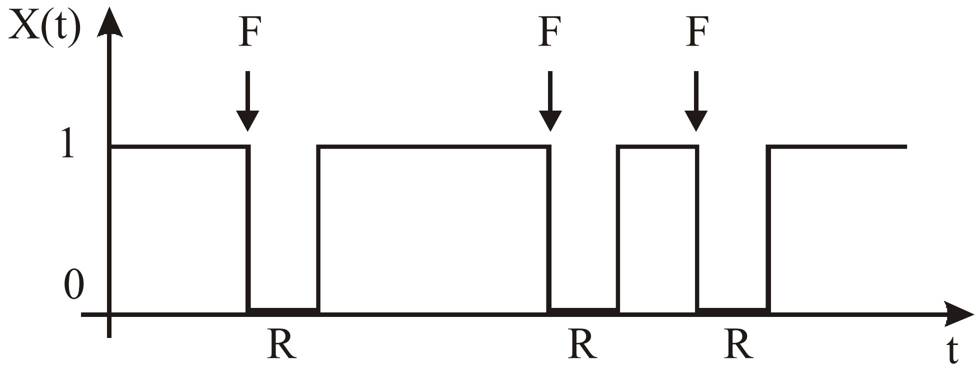
Availability is a concept that applies to situations in which failures are repaired.

Availability is a measure of the degree to which an item is in an operable state when called upon to perform.

Availability is expressed as a probability.

Availability definition (2)

- *X(t)* = indicator variable such that:
  - *X(t)* = 1, system is operating at time *t*
  - *X(t)* = 0, system is failed at time *t*



F = Failed
R = under Repair

- Instantaneous **availability** *p(t)* and **unavailability** *q(t)*

$$p(t) = P[X(t) = 1] = E[X(t)] \qquad q(t) = P[X(t) = 0] = 1 - p(t)$$

Contributions to Unavailability

- **Unrevealed failure**

  A stand-by component fails unnoticed. The system goes on without noticing the component failure until a test on the component is made or the component is demanded to function

- **Testing / preventive maintenance**

  A component is removed from the system because it has to be tested or must undergo preventive maintenance

- **Repair**

  A component is unavailable because under repair

Average availability descriptors

- How to compare different maintenance strategies?

- We need to define quantities for an **average** description of its probabilistic behavior:
  - Components under corrective maintenance (stochastic repair time):

$$p = \lim_{t \to \infty} p(t)$$

  - Components under periodic maintenance:

$$p_T = \frac{1}{T}\int_0^T p(t)dt = \frac{\overline{UPtime}}{T} = \frac{\overline{T_U}}{T}$$

Average time the system is functioning (UP) within T_M

Availability of an unattended component (no repair allowed)

Availability of an unattended component (no repair allowed)

- The probability *q(t)* that at time *t* the component is not functioning is **equal** to the probability that it failed before *t*

$$q(t) \equiv F(t)$$

$$p(t) = 1 - q(t) \equiv R(t)$$

where *F(t)* is the cumulative failure probability and *R(t)* is the reliability

Availability of a continuously monitored component

Availability of a continuously monitored component (1)

- **Objective**:
  - Computation of the availability *p(t)*
- **Hypotheses**:
  - N = number of identical components at time $t = 0$
  - Restoration starts immediately after the component failure
  - Probability density function of the random time duration T_R of the repair process = $\boldsymbol{g(t)}$

Balance equation between time t and time $t+\Delta t$

Availability of a continuously monitored component (2)

$$\underbrace{N \cdot p(t+\Delta t)}_{(1)} = \underbrace{N \cdot p(t)}_{(2)} - \underbrace{N \cdot p(t) \cdot \lambda \cdot \Delta t}_{(3)} + \int_0^t \underbrace{N \cdot p(\tau) \cdot \lambda \cdot \Delta\tau \cdot g(t-\tau) \cdot \Delta t}_{(4)}$$

1) Number of items UP at time $t+\Delta t$
2) Number of items UP at time t
3) Number of items failing during the interval Δt
4) Number of items that had failed in (τ, $\tau+\Delta\tau$) and whose restoration terminates in (t, $t+\Delta t$);

Availability of a continuously monitored component (3)

- The integral-differential form of the balance

$$\frac{dp(t)}{dt} = -\lambda \cdot p(t) + \int_0^t \lambda \cdot p(\tau) \cdot g(t-\tau) \cdot d\tau \qquad p(0) = 1$$

- The solution can be obtained introducing the Laplace transforms

$$f(x) \quad \rightarrow \quad L[f(x)] = \widetilde{f}(s) = \int_0^\infty e^{-s \cdot x} f(x) dx$$

$$\frac{df(x)}{dx} \quad \rightarrow \quad L\left[\frac{df(x)}{dx}\right] = s \cdot \widetilde{f}(s) - f(0)$$

Availability of a continuously monitored component (4)

- Applying the Laplace transform we obtain:

$$s \cdot \tilde{p}(s) - 1 = -\lambda \cdot \tilde{p}(s) + \lambda \cdot \tilde{p}(s) \cdot \tilde{g}(s)$$

which yields:

$$\tilde{p}(s) = \frac{1}{s + \lambda \cdot \left(1 - \tilde{g}(s)\right)}$$

- **Inverse** Laplace transform $\rightarrow$ *p(t)*
- Limiting availability:

$$p_{\infty} = \lim_{t \to \infty} p(t) = \lim_{s \to 0} \left[s \cdot \tilde{p}(s) \right] = \lim_{s \to 0} \left[\frac{s}{s + \lambda \cdot \left(1 - \tilde{g}(s)\right)} \right]$$

Availability of a continuously monitored component (5)

- As $s \to 0$, the first order approximation of $\tilde{g}(s)$ is the following:

$$\tilde{g}(s) = \int_0^\infty e^{-s\tau} g(\tau)d\tau = \int_0^\infty (1 - s\cdot\tau + \ldots) g(\tau)d\tau \cong 1 - s\cdot\int_0^\infty \tau g(\tau)d\tau = 1 - s\cdot\bar{\tau}_R$$

$$\bar{\tau}_R = E_g\left[T_R\right]$$

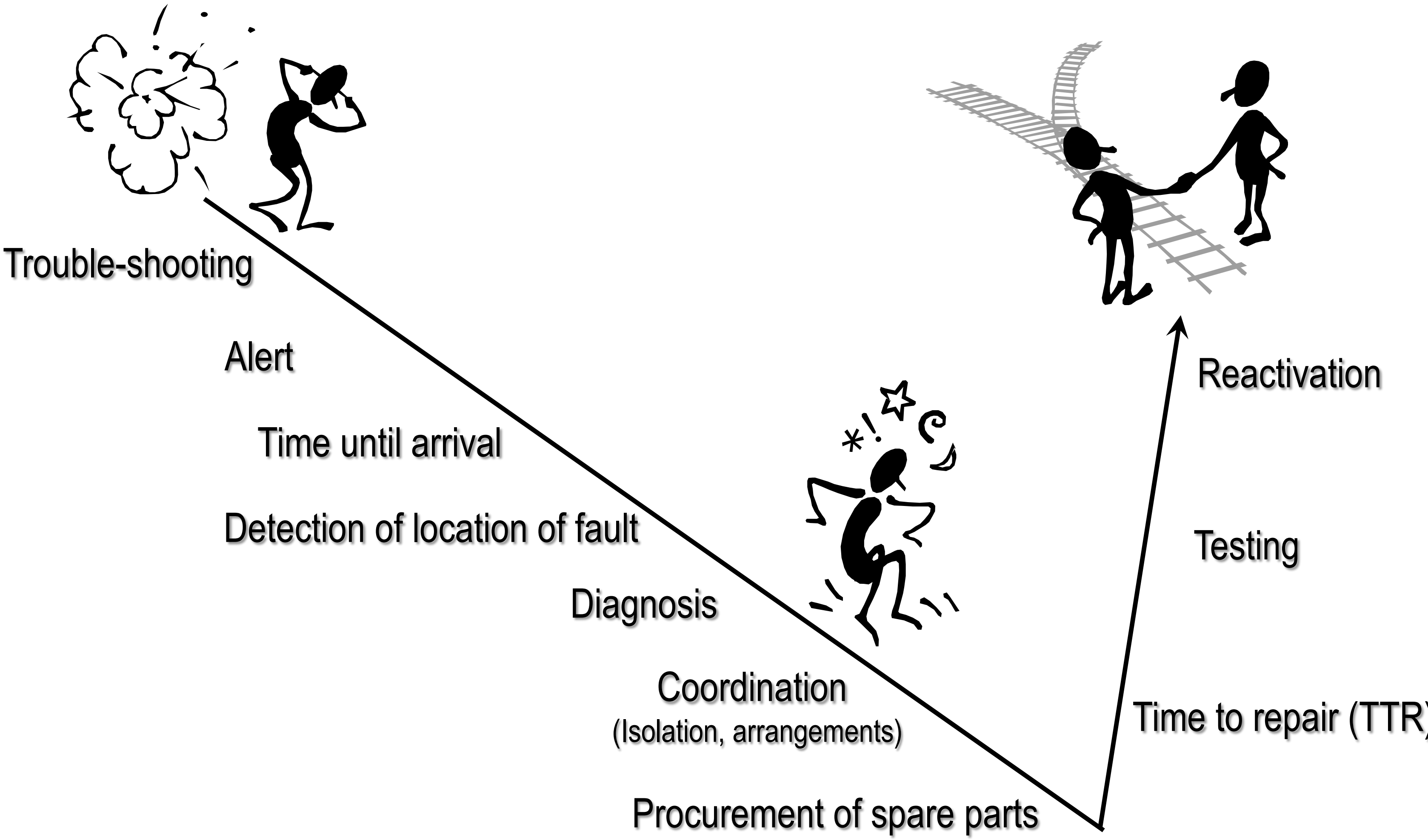
MTTF ⟶ MTBF

$$p_\infty = \lim_{s\to 0}\frac{s}{s+\lambda\cdot s\cdot\bar{\tau}_R} = \frac{1}{1+\lambda\cdot\bar{\tau}_R} = \frac{1/\lambda}{1/\lambda+\bar{\tau}_R} = \frac{MTTF}{MTTF+MTTR} = \frac{\text{average time the component is UP}}{\text{average time of a failure/repair "cycle"}}$$

General result !

Availability of a continuously monitored component (5)

Interruption of Operation (Down Time, DT)



Availability of a component under periodic maintenance (1)

- Safety systems are generally in standby until accident and their components must be periodically tested
- The instantaneous unavailability is a periodic function of time

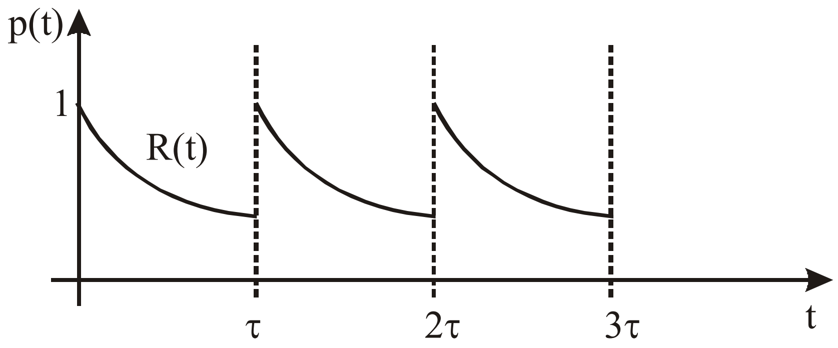
- The performance indicator used is the average unavailability (which is not a probability !)

$$q_T = \frac{1}{T} \cdot \int_0^T q(t)dt = \frac{\overline{DOWNtime}}{T} = \frac{\overline{T_D}}{T}$$

Availability of a component under periodic maintenance (2)

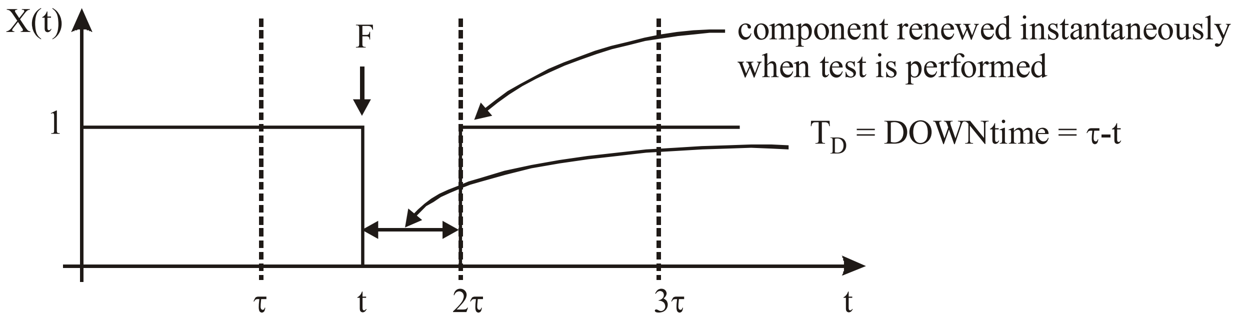
- Suppose the unavailability is due to unrevealed random failures, e.g. with constant rate λ
- Assume also instantaneous and perfect testing and maintenance procedures

- The instantaneous availability within a period τ coincides with the reliability



Availability of a component under periodic maintenance (3)

- **Objective**: computation of the average unavailability



- The average unavailability within one period τ is:

$$q_\tau = \frac{mean \;\; DOWNtime}{\tau} = \frac{\overline{T}_D}{\tau}$$

$$\overline{T}_D = \int_0^\tau (\tau - t) f_T(t) dt = \int_0^\tau (\tau - t) dF_T =$$

$$= (\tau - t) F_T(t) \Big|_0^\tau + \int_0^\tau F_T(t) dt = \int_0^\tau F_T(t) dt$$

Availability of a component under periodic maintenance (4)

- The average unavailability and availability are then:

$$q_\tau = \frac{\overline{T}_D}{\tau} = \frac{\int_0^\tau F_T(t)dt}{\tau} \qquad p_\tau = \frac{\overline{T}_U}{\tau} = \frac{\int_0^\tau R(t)dt}{\tau}$$

For different systems, we can compute p_τ e q_τ by first computing their failure probability distribution $F_T(t)$ and reliability $R(t)$ and then applying the above expressions

Availability of a component under periodic maintenance (5)

- Assume a finite repair time τ_R

- The average unavailability and availability over the complete maintenance cycle period $\tau + \tau_R$ are:

$$\bar{q}_{\tau+\tau_R} = \frac{\tau_R + \int_0^{\tau} F_T(t)dt}{\tau + \tau_R} \approx [\tau_R << \tau] \approx \frac{\tau_R + \int_0^{\tau} F_T(t)dt}{\tau}$$

$$\bar{p}_{\tau+\tau_R} = \frac{\int_0^{\tau} R(t)dt}{\tau + \tau_R} \approx [\tau_R << \tau] \approx \frac{\int_0^{\tau} R(t)dt}{\tau}$$

EXAMPLES

Availability of a continuously monitored component

- Find instantaneous and limiting availability for an exponential component whose restoration probability density is

$$g(t) = \mu \cdot e^{-\mu \cdot t}$$

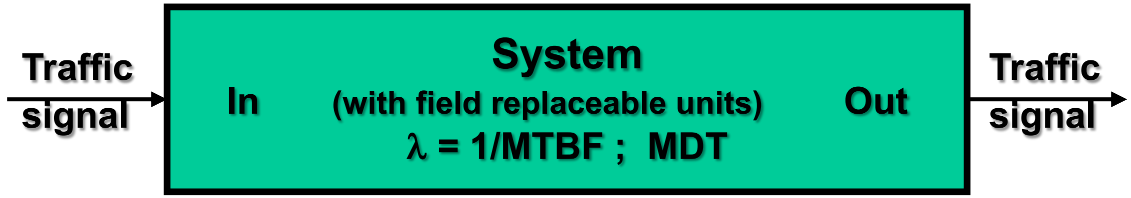
- The Laplace transform of the restoration density is

$$\tilde{g}(s) = L[g(t)] = \frac{\mu}{s+\mu}$$

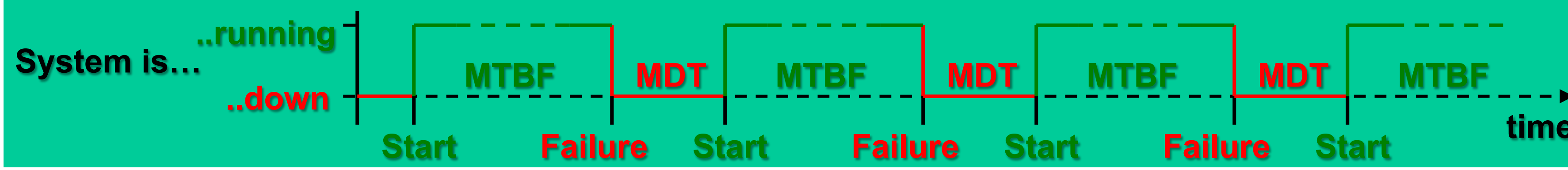
$$\tilde{p}(s) = \frac{1}{s + \lambda \cdot \frac{s}{s+\mu}} = \frac{s+\mu}{s \cdot (s+\mu+\lambda)} \longrightarrow \begin{cases} p(t) = L^{-1}[\tilde{p}(s)] = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} \cdot e^{-(\mu+\lambda)\cdot t} \\ p_\infty = \frac{\mu}{\mu+\lambda} \end{cases}$$

Availability of a continuously monitored system

Traffic signalling by a system with field replaceable units :



A field replaceable unit follows on average he following schedule:



Availability A:

$$A = \frac{MTBF}{MTBF + MDT}$$

Unavailability N (=q) :

$$N = 1 - A = \frac{MDT}{MTBF + MDT}$$

MTBF: Mean Time between Failure

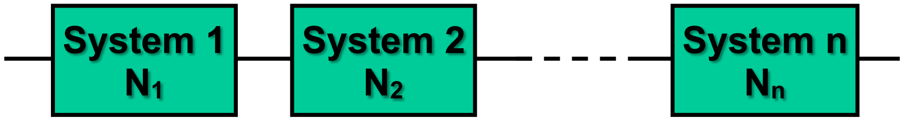
MDT: Mean Down Time (˜MTTR)

MDT-figures includes travelling-, administrative-, fault detection- + active repair times (MTTR), assuming 24h/d readiness of maintenance staff and availability of sufficient spares.

Repair or replace of	MDT (i˜MTTR)
Unit	4 h
Subrack	6 … 8 h

Availability of a continuously monitored system

Unavailability of Series Systems N_s (=q_s)



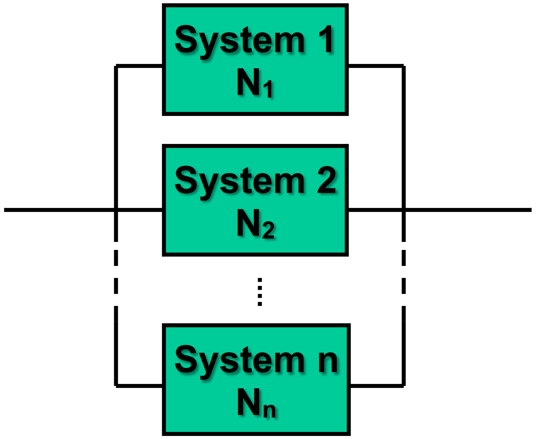
$$N_s \approx N_1 + N_2 + \cdots + N_n = \sum_{i=1}^{n} N_i$$

Example: System 1 = System 2
MTBF = 10yrs = 87 600h; MDT = 4h :

$$N_{1/2} = \frac{MDT}{MTBF + MDT} = \frac{4h}{87\,600h + 4h} \approx 4{,}6 \cdot 10^{-5}$$

$$N_{s\_tot} = N_1 + N_2 = 2 \cdot N_{1/2} = 9{,}2 \cdot 10^{-5} \approx 10^{-4}$$

Unavailability of Parallel Systems N_p (=q_p)



$$N_p = N_1 \bullet N_2 \bullet \cdots \bullet N_n = \prod_{i=1}^{n} N_i$$

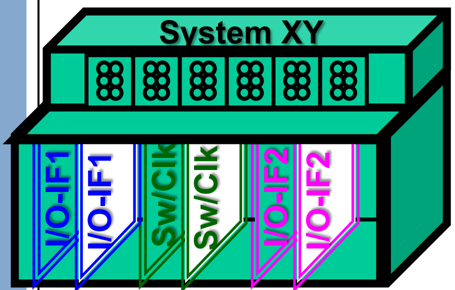
Example: System 1 = System 2
MTBF = 5yrs = 43 800h; MDT = 5h :

$$N_{1/2} = \frac{MDT}{MTBF + MDT} = \frac{5h}{43\,800h + 4h} \approx 1{,}14 \cdot 10^{-4}$$

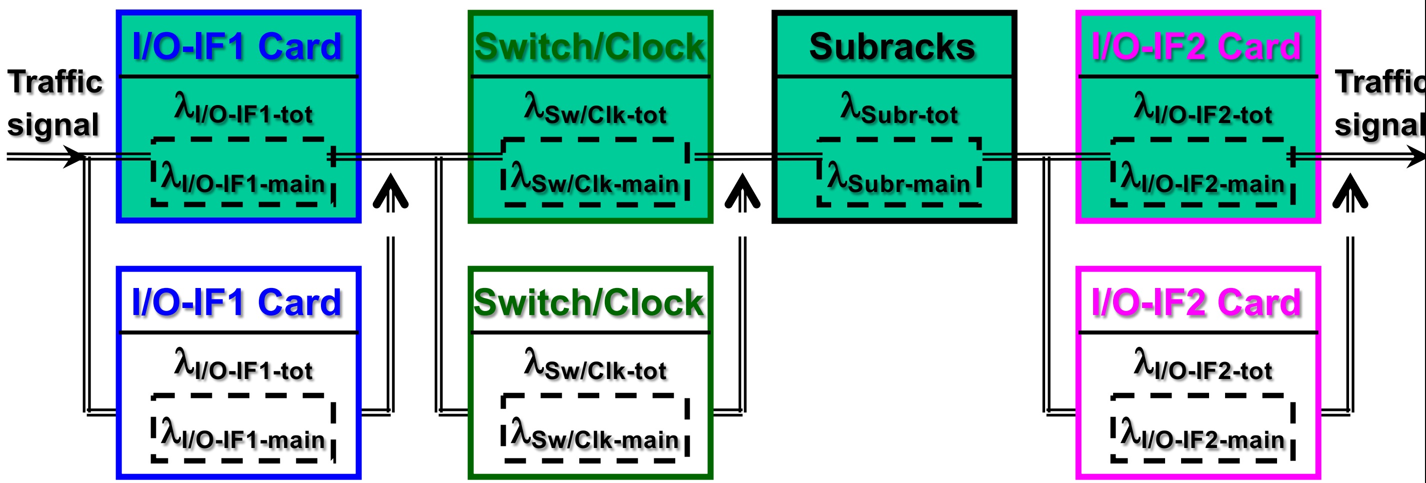
$$N_{tot} = N_1 \cdot N_2 = N_{1/2}^2 = 1{,}3 \cdot 10^{-8}$$

Availability of a continuously monitored system

System XY with protected field replaceable units :



Traffic signal through systems XY :



Unit In Operation

Unit In Stand-by Mode

Unavailability of a system with unprotected units only:

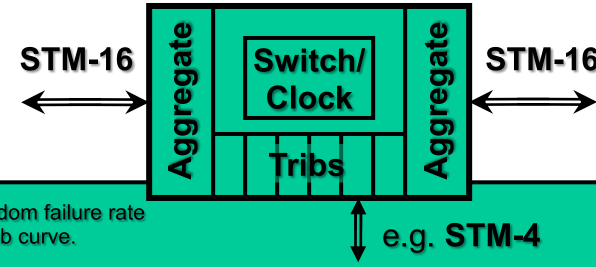
$$N_{unprot} = N_{I/O\text{-}IF1} + N_{Sw/Clk} + N_{Subr} + N_{I/O\text{-}IF2}$$

Unavailability of a system with possible protected units:

$$N_{prot} = N^2_{I/O\text{-}IF1} + N^2_{Sw/Clk1} + N_{Subr} + N^2_{I/O\text{-}IF2} \approx N_{Subr}$$

Availability of a continuously monitored system

Example: Availability Calculation
for traffic signal of a transport system
with List of Material (LoM) (no current values):



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The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve.
The calculation method is in acc.with IEC 61709!

Item-No	Name	total		Main path				Quantity of items STM-16 and given ports: a) unprotected				b) Trib-+Aggreg-+Switch-prot.			
		λ_{tot} FIT	$MTBF_{tot}$ (year)	λ_{main} FIT	$MTBF_{main}$ (year)	MDT (h)	Non-Availab.	Total	STM1e	STM4	STM16	Total	STM1e	STM4	STM16
SM10-1.11	Subrack	2.334	48,9	540	211,4	4	2,16E-6	1	1	1	1	1	1	1	1
SM10-1.12	SCOH	4.694	24,3	0	infinite	4	0	1				1			
SM10-5.1	CLU	2.771	41,2	2.024	56,4	4	8,10E-6	1	1	1	1	2	1+1	1+1	1+1
SM10-8.1	SWITCH FABRIC VC-4	2.980	38,3	2.302	49,6	4	9,21E-6	1	1	1	1	2	1+1	1+1	1+1
SM10-20.1	STM-16 BOARD	5.133	22,2	3,242	35,2	4	1,30E-5	1	1	1	2	2	1+1	1+1	1+1
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	500	228,3	4	2,00E-6	4	1	1	2	8	1+1	1+1	1+1
SM10-23.1	STM-4 BOARD	2.931	38,9	1.685	67,7	4	6,74E-6	1		1		2		1+1	
SM10-23.11	STM-4 MODULE L-4.2/3	500	228,3	500	228,3	4	2,00E-6	2		1		4		1+1	
SM10-27.1	STM-1 CARD, 8 X STM-1e	3.489	32,7	2.288	49,9	4	9,15E-6	1	1			2	1+1		
SM10-6.1	SWITCH FABRIC VC-12	4.866	23,5	4.014	28,4	4	1,61E-5	1				2			
	Failure rates in FIT (1FIT=1Failure/10^9h)				Total			37.565				66.102			
	MTBF in years				Total			3,04				1,73			
	Failure rates in FIT (1FIT=1Failure/10^9h)				Main path				10.896	10.793	12.350		540	540	540
	MTBF in years				Main path				10,48	10,58	9,24		211,4	211,4	211,4
	Non-availability				Main path				4,36E-5	4,32E-5	4,94E-5		2,16E-6	2,16E-6	2,16E-6
	Failure intensity in failures per year				Main path				22,91	22,69	25,96		1,14	1,14	1,14
	Non-availability in min/year				Main path				0,095	0,095	0,108		0,005	0,005	0,005
	Availability in %				Main path				99,996	99,996	99,995		99,9998	99,9998	99,9998

Availability/Reliability Table

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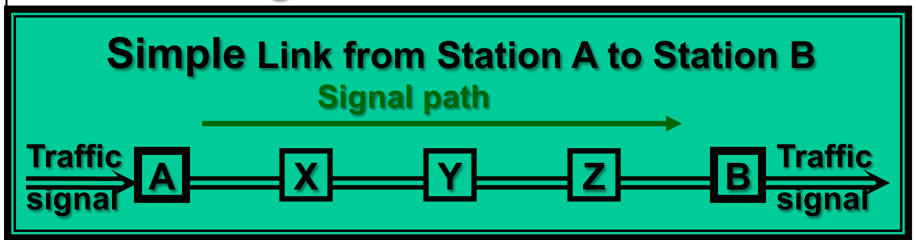
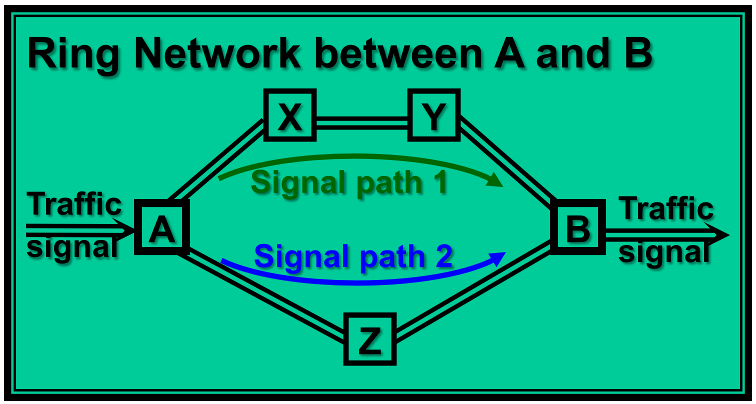
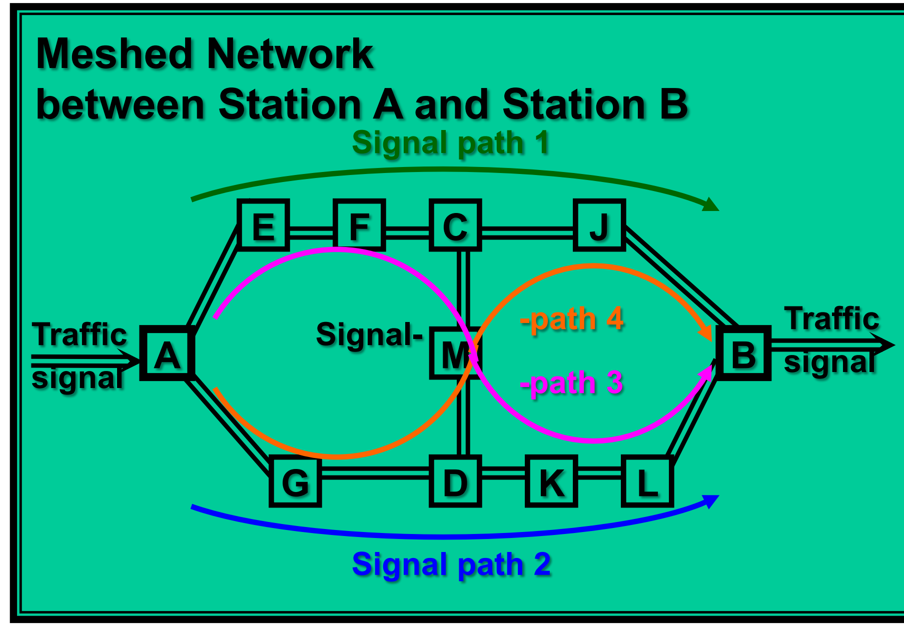
Availability of a continuously monitored network system

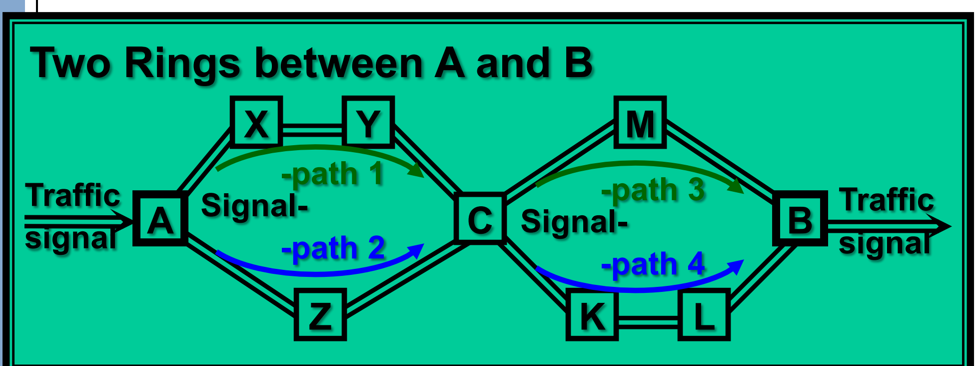
Traffic signal transmission from station A to station B for several types of Networks



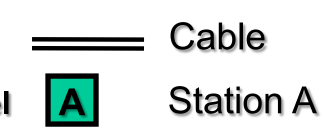
Which signal paths are possible to come from A to B?





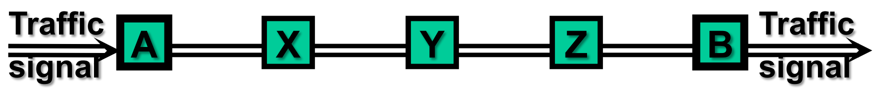


Traffic signal can be e.g.
PDH/SDH-signal and/or **fibre channel**



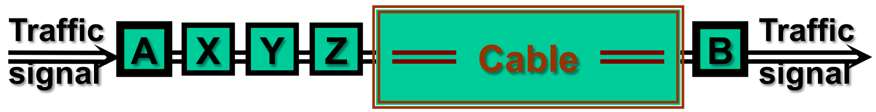
Availability of a continuously monitored network system

Link from Station A to Station B



The stations contain the transport system only

What about cables between the stations?



The cable is a very important function block in the consideration of system availability

Table: Failure rates of several cable types (approx. values)

Types of (duct-)cable	Failure/100km/year	Failure rate / km	MTBF * km	MDT x) (incl. MTTR,etc.)
Terrestrial cable City	10 … 25	11.500 … 28.600 FIT/km	4 … 10 yr*km	12 … 14 h
Terrestrial cable Country	0,1 … 0,6	115 … 700 FIT/km	160 … 1.000 yr*km	12 … 14 h
Sea cable in depth of < 1.000 km	0,2 … 0,4	230 … 450 FIT/km	250 … 500 yr*km	10 … 30 d
Sea cable in depth of > 1.000 km	0,1… 0,2	115 … 230 FIT/km	500 … 230 yr*km	10 … 30 d

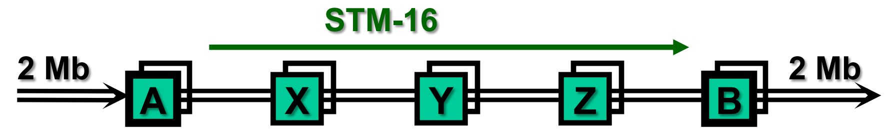
x) fault detection- + active repair times (MTTR), assuming 24h/d readiness of maintenance staff

(Duct-) Cable   A Station A

Availability of a continuously monitored network system

Example: Availability of a simple link:

(no current values)



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The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve.
The calculation method is in acc.with IEC 61709!

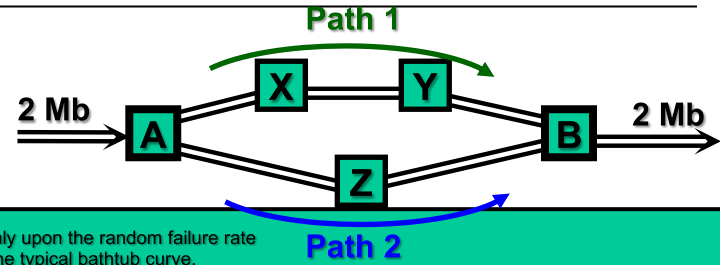
		Main transmission path (Main Path)				Quantity of items for following items: STM-16 Link from A to B	
Item-No	Name	λ_{main} FIT	$MTBF_{main}$ (year)	MDT (h)	Non-Availab.	with unprotected parts	with protected parts
SM10-1.11	Subrack	540	211,4	4	2,16E-6	5	5
SM10-1.12	SCOH	0	infinite	4	0		
SM10-5.1	CLU	2.024	56,4	4	8,10E-6	5	1+1
SM10-8.1	SWITCH FABRIC VC-4	2.302	49,6	4	9,21E-6	5	1+1
SM10-20.1	STM-16 BOARD	3,242	35,2	4	1,30E-5	5	1+1
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	4	2,00E-6	8	1+1
SM10-6.1	SWITCH FABRIC VC-12	4.014	28,4	4	1,61E-5	5	1+1
SM10-25.2	IF2M CARD 63*E1 120ohm)	1.894	161,9	4	2,82E-6	2	1:n
SM10-25.12	LSU CARD 63*E1 120ohm)	705	228,3	4	6,74E-6	2	2
0,3 Failure intensity of fibre (failures(100km/year):							
	Fibre per km	342	333,3	12	4,11E-6	200 km	200 km
	Failure rates in FIT (1FIT=1Failure/10^9h)		Main path			138.301	72.603
	MTBF in years		Main path			0,83	1,57
	Non-availability of unprotected parts					1,10E-3	8,38E-4
	Non-availability of paths 1 & 2						
	Non-availability of 1+1 protected parts						
	Total Non-availability					1,10E-3	8,38E-4
	Total Non-availability in min/year					578,76	440,64
	Total Availability in %					99,8899	99,9162

Computation rules: The signal flow through equipment is marked by the figures in the relevant columns.
Remark: The results for the main transmission path are related to one bi-directional signal/channel.

Availability/Reliability Table **Page 1 - 1** ICN CN S M EP2/Eberlin/12.05.2004/ 2004_A&Btel_MTBF-Spares.xls

Availability of a continuously monitored network system

Example: Availability of a Ring network:
(no current values)



SIEMENS

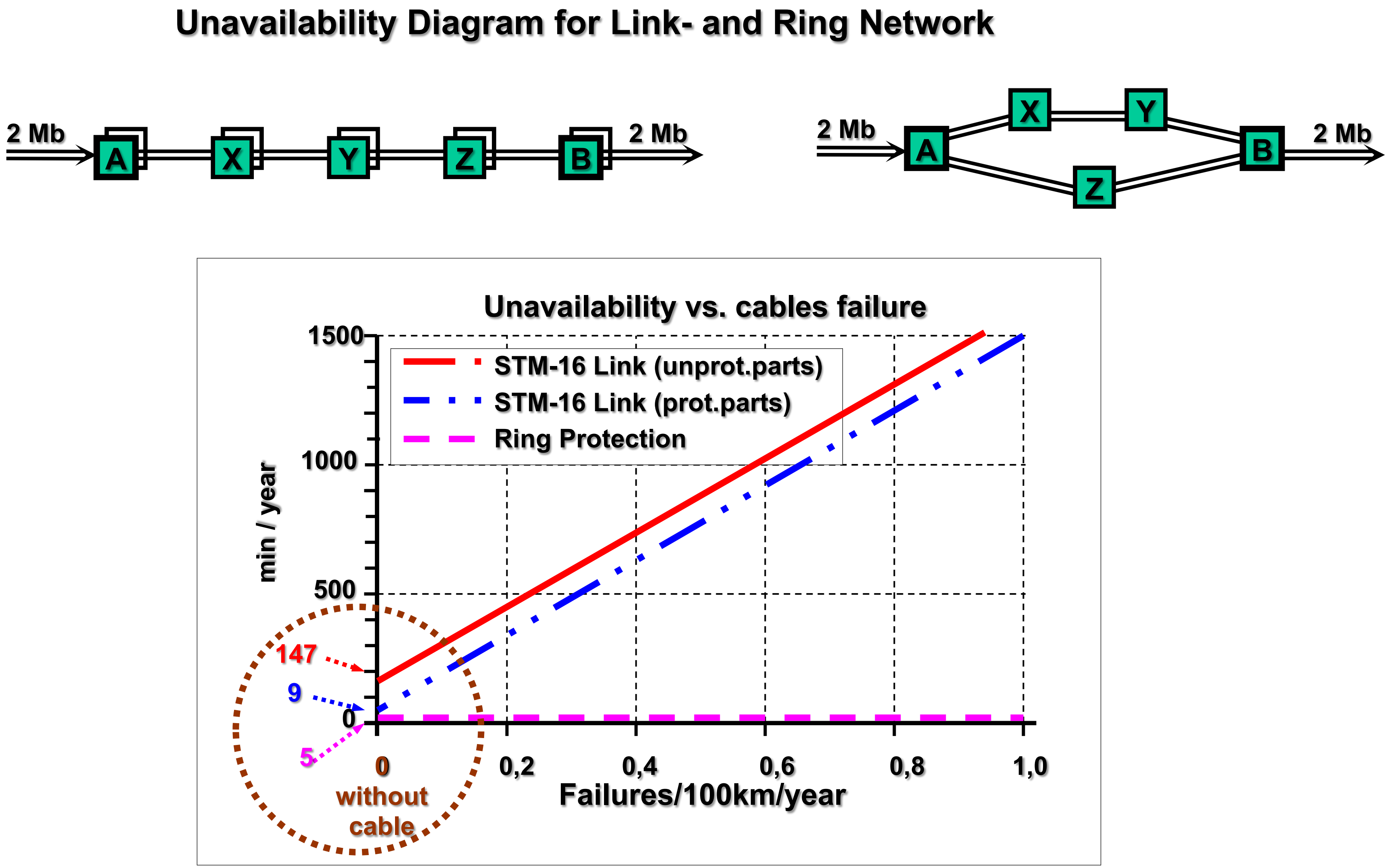
The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve.
The calculation method is in acc.with IEC 61709!

		Main transmission path (Main Path)				Quantity of items for following items: Ring-Protection from A to B		
Item-No	**Name**	**λ_{main} FIT**	**$MTBF_{main}$** (year)	**MDT** (h)	**Non-Availab.**	**unprotected parts**	**STM-16 Ring: Path 1**	**STM-16 Ring: Path 2**
SM10-1.11	Subrack	540	211,4	4	2,16E-6	2	2	1
SM10-1.12	SCOH	0	infinite	4	0			
SM10-5.1	CLU	2.024	56,4	4	8,10E-6	1+1	1+1	1+1
SM10-8.1	SWITCH FABRIC VC-4	2.302	49,6	4	9,21E-6	1+1	1+1	1+1
SM10-20.1	STM-16 BOARD	3,242	35,2	4	1,30E-5		4	3
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	4	2,00E-6		6	4
SM10-6.1	SWITCH FABRIC VC-12	4.014	28,4	4	1,61E-5	1+1		
SM10-25.2	IF2M CARD 63*E1 120ohm)	1.894	161,9	4	2,82E-6	1+n		
SM10-25.12	LSU CARD 63*E1 120ohm)	705	228,3	4	6,74E-6	2		
0,3 Failure intensity of fibre (failures(100km/year):								
	Fibre per km	342	333,3	12	4,11E-6	0 km	150 km	100 km
Failure rates in FIT (1FIT=1Failure/10^9h)		**Main path**				2.490	68.418	45.513
MTBF in years		**Main path**				45,85	1,67	2,45
Non-availability of unprotected parts						9,96E-6		
Non-availability of paths 1 & 2							6,85E-4	4,60E-4
Non-availability of 1+1 protected parts							3,15E-7	
Total Non-availability						1,03E-5		
Total Non-availability in min/year						5,40		
Total Availability in %						99,9990		

Computation rules: The signal flow through equipment is marked by the figures in the relevant columns.
Remark: The results for the main transmission path are related to one bi-directional signal/channel.

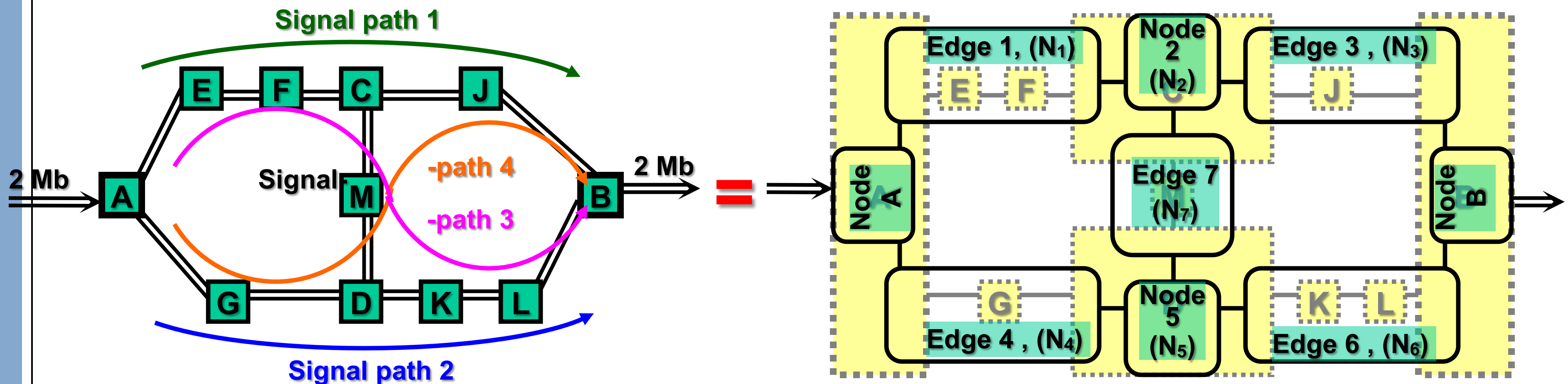
Availability/Reliability Table **Page 1 - 1** ICN CN S M EP2/Eberlin/12.05.2004/ 2004_A&Btel_MTBF-Spares.xls

Availability of a continuously monitored network system



Meshed Network Structure

Unavailability Calculation for a meshed network:



Possible paths from A to B

Path 1: Edge1+Node2+Edge3

Path 2: Edge4+Node5+Edge6

Path 3: Edge1+Node2+Edge7+Node5+Edge6

Path 4: Edge4+Node5+Edge7+Node2+Edge3

Theorem of total probability (addition rule):

$$\text{Prob}\{a \cup b\} = \text{Prob}\{a\} + \text{Prob}\{b\} - \text{Prob}\{a \cap b\}$$

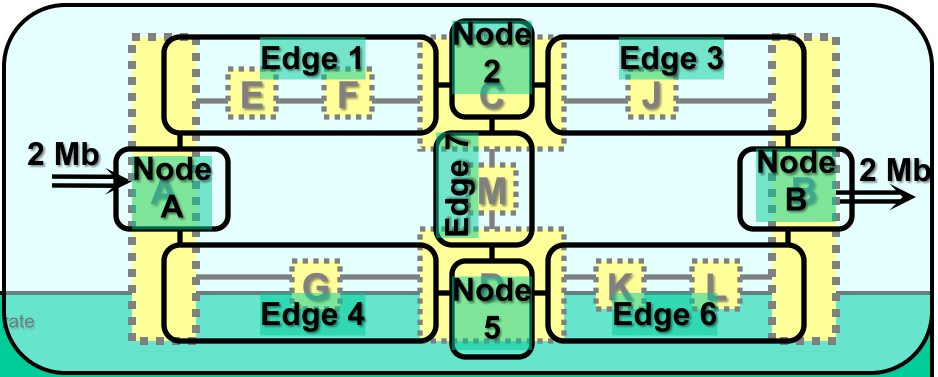
Unavailability N_{tot} for this meshed network, for $N_i << 1$:

$$\begin{aligned} N_{tot} \approx N_{A+B} &+ N_1\left[N_4(1-N_2) + N_5(1-N_2-N_3-N_4) + (N_6 \cdot N_7)\right] \\ &+ N_2\left[N_4(1-N_6) + N_5(1-N_3-N_4-N_6) + N_6(1-N_3)\right] \\ &+ N_3\left[N_5 + N_6 + N_4 \cdot N_7 - N_5 \cdot N_6\right] \end{aligned}$$

Availability of a continuously monitored network system

Example: Availability for a meshed network

(no current values)



SIEMENS

The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve.
The calculation method is in acc.with IEC 61709!

Item-No	Name	λ_{main} FIT	$MTBF_{main}$ (year)	MDT (h)	Non-Availab	Unprotect Parts A,B	Edge 1 A,E,F,C	Node 2 C	Edge 3 C,J,B	Edge 4 A,G,D	Node 5 D	Edge 6 D,K,L,B	Edge 7 C,M,D
	Main transmission path (Main Path)					**Quantity of items for following items:**							
SM10-1.21	Subrack DC	268	466,0	4	1,07E-6	2	2	1	1	1	1	2	2
SM10-5.1	CLU	2.024	56,4	4	8,10E-6	1+1	1+1	1+1	1+1	1+1	1+1	1+1	1+1
SM10-8.1	SWITCH FABRIC VC-4	2.302	49,6	4	9,21E-6	1+1	1+1	1+1	1+1	1+1	1+1	1+1	1+1
SM10-20.1	STM-16 BOARD	3,242	35,2	4	1,30E-5		4		3	3		4	3
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	4	2,00E-6		6		4	4		6	4
SM10-6.1	SWITCH FABRIC VC-12	4.014	28,4	4	1,61E-5	1+1	1+1	1+1	1+1	1+1	1+1	1+1	1+1
SM10-25.2	IF2M CARD 63*E1 120ohm)	1.894	161,9	4	2,82E-6	1+1							
SM10-25.12	LSU CARD 63*E1 120ohm)	705	228,3	4	6,74E-6	2							
0,3 Failure intensity of fibre (failures(100km/year):													
	Fibre per km	342	333,3	12	4,11E-6		249 km		161 km	174 km		231 km	157 km
Non-availability of unprotected parts at Node A and Node B						7,78E-6							
Non-availability of each Edges/Nodes 1 … 7							1,09E-3	1,07E-6	7,10E-4	7,63E-4	1,07E-6	1,02E-3	6,93E-4
Non-availability over all Edges/Nodes 1 … 7										1,55E-6		w/o Edge 7	
Total Non-availability related to 2Mb end to end										9,341E-6		1,094E-5	
Total Non-availability in min/year related to 2Mb end to end										4,909		5,750	
Total Availability in % related to 2Mb end to end										99,999066		99,998906	

Computation rules: The signal flow through equipment is marked by the figures in the relevant columns.
Remark: The results for the main transmission path are related to one bi-directional signal/channel.

Availability/Reliability Table **Page 1 - 1**

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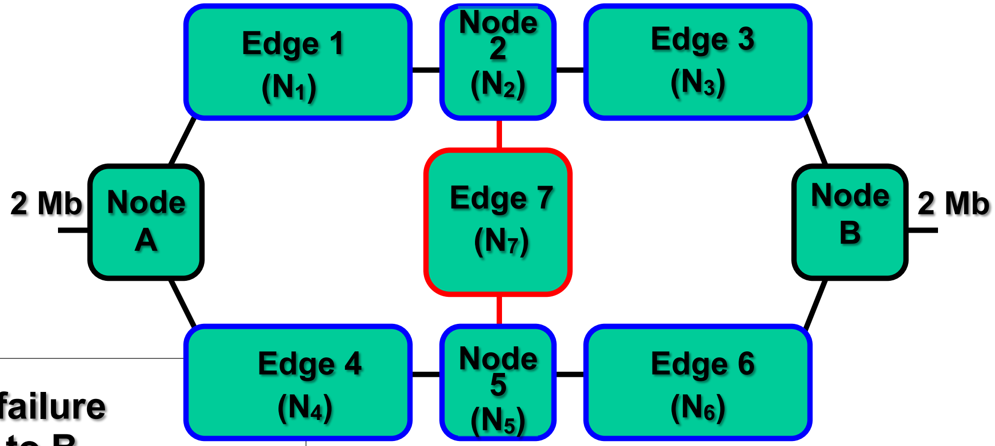
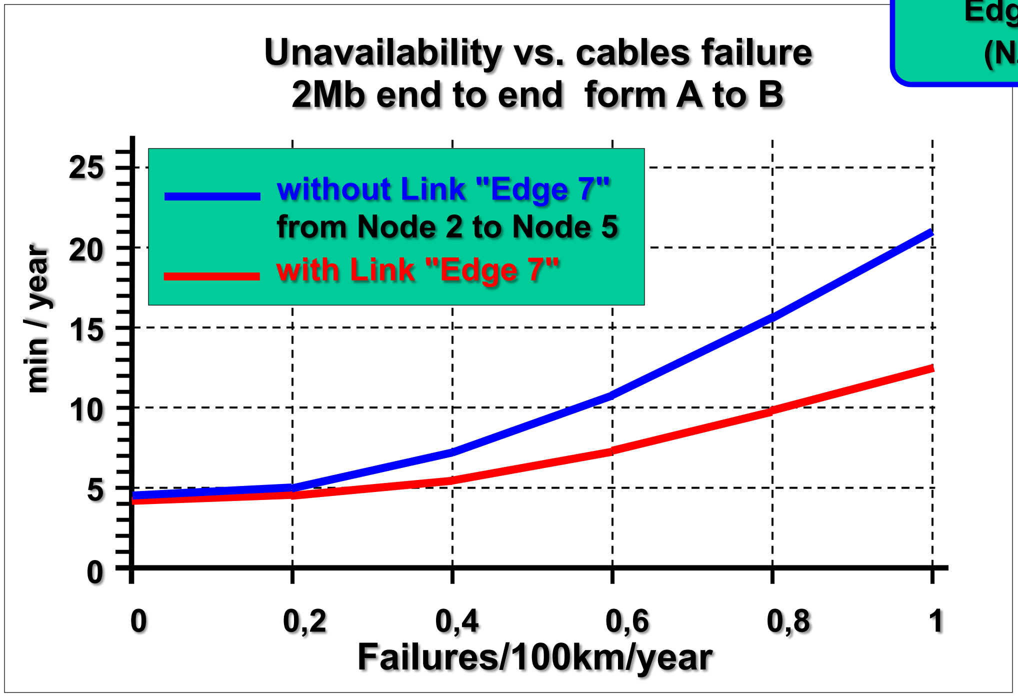
Meshed Network Non-availability Diagram

Example: Unavailability Diagram for a meshed Network





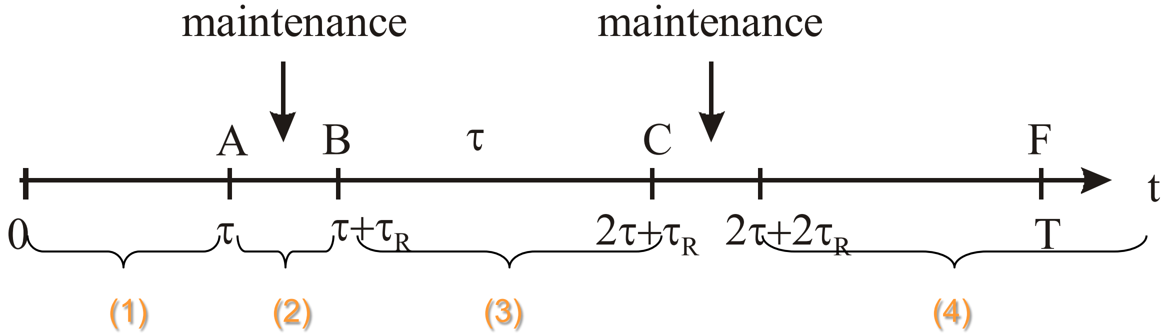
Availability of a component under periodic maintenance

- **Objective**: computation of the average unavailability over the lifetime [0, T_M]

$$\overline{q_{0T_M}} = \frac{\overline{T_D}}{T_M}$$

- **Hypoteses**:
  - The component is initially working: $q(0) = 0$; $p(0) = 1$
  - Failure causes:
    1. random failure at any time $T \sim F_T(t)$
    2. on-line switching failure on demand $\sim Q_0$
    3. maintenance disables the component $\sim \gamma_0$ (human error during inspection, testing or repair)

Component under periodic maintenance: a more realistic case



1) The probability of finding the component DOWN at the generic time *t* is due either to the fact that it was demanded to start but failed or to the fact that it failed unrevealed randomly before *t*. The average DOWNtime is:

$$q_{0A}(t) = Q_0 + (1 - Q_0) F_T(t)$$

$$\overline{T}_{D(0A)} = \int_0^{\tau} q_{0A}(t)dt = \int_0^{\tau} \left[Q_0 + (1-Q_0)\cdot F_T(t)\right]dt = Q_0 \cdot \tau + (1-Q_0)\cdot \int_0^{\tau} F_T(t)dt$$

Component under periodic maintenance: a more realistic case

2) During the maintenance period the component remains disconnected and the average DOWNtime is the whole maintenance time:

$$\overline{T}_{D(AB)} = \tau_R$$

3) The component can be found failed because, by error, it remained disabled from the previous maintenance or because it failed on demand or randomly before *t*. The average DOWNtime is:

$$q_{BC}(t) = \gamma_0 + (1-\gamma_0)\cdot\left[Q_0 + (1-Q_0)\cdot F_T(t)\right]$$

$$\overline{T}_{D(BC)} = \int_0^{\tau} q_{BC}(t)dt = \gamma_0\cdot\tau + (1-\gamma_0)\cdot\left[Q_0\cdot\tau + (1-Q_0)\cdot\int_0^{\tau} F_T(t)dt\right]$$

Component under periodic maintenance: a more realistic case

4) The normal maintenance cycle is repeated throughout the component lifetime T_M. The number of repetitions, i.e. the number of AB-BC maintenance cycles, is:

$$k = \frac{T_M}{\tau + \tau_R}$$

- The total expected DOWNtime is:

$$\overline{T}_D = Q_0\tau + (1-Q_0)\cdot\int_0^{\tau} F_T(t)dt + \frac{T_M}{\tau+\tau_R}\cdot\left\{\tau_R + \gamma_0\tau + (1-\gamma_0)\cdot\left[Q_0\cdot\tau + (1-Q_0)\cdot\int_0^{\tau} F_T(t)dt\right]\right\}$$

$$\overline{q}_{T_M} = \frac{\overline{T}_{D(0T_M)}}{T_M} = \frac{Q_0\tau}{T_M} + \frac{1-Q_0}{T_M}\cdot\int_0^{\tau} F_T(t)dt + \frac{1}{\tau+\tau_R}\cdot\left\{\tau_R + \gamma_0\tau + (1-\gamma_0)\cdot\left[Q_0\cdot\tau + (1-Q_0)\cdot\int_0^{\tau} F_T(t)dt\right]\right\}$$

Component under periodic maintenance: a more realistic case

- Q_0 and $F_T(t)$ are generally small, and since typically $\tau_R << \tau$ and $\tau << T_M$, the average unavailability can be simplified to:

$$\overline{q}_{T_M} \cong \frac{\tau_R}{\tau} + \gamma_0 + (1-\gamma_0)\cdot\left[Q_0 + \frac{1-Q_0}{\tau}\cdot\int_0^{\tau} F_T(t)dt\right]$$

- Consider an exponential component with small, constant failure rate $\lambda \Rightarrow F_T(t) = 1 - e^{-\lambda \cdot t} \cong \lambda \cdot t$

- Since typically $\gamma_0 << 1$, $Q_0 << 1$, the average unavailability reads:

$$\overline{q}_{0T_M} \cong \frac{\tau_R}{\tau} + \gamma_0 + Q_0 + \frac{1}{2}\cdot\lambda\cdot\tau$$

Maintenance

Error after test

Switching failure on demand

Random, unrevealed failures between tests