POLITECNICO DI MILANO

Basic notions of probability theory

- Discrete Random Variables

EXERCISES

Flipped class

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GROUP ASSIGNMENTS

Group	1	2	3	4	5	6	7	8	9	10	11	12	13
Exercise 2 (Version)	V1	V2	V3	V1	V2	V3	V1	V2	V3	V1	V2	V3	V1
Exercise 6 (Version)	V1	V1	V1	V2	V2	V2	V1	V1	V1	V2	V2	V2	V1

All groups must attempt to solve all exercises. Exercises with more than one version are assigned in the table above

Exercise 0

In an energy production plant there are two turbines: 'turbine T_1' and 'turbine T_2'. Each turbine can be in three different states:

'0'= operating;

'1'= degraded;

'2'= failed.

Consider the following events:

A = {'turbine T_1' is failed}

B = {both 'turbine T_1' and 'turbine T_2' are degraded}

C = {'turbine T_2' is not degraded}

Questions:

a) Define the sample space Ω of the state of the turbines and represent it graphycally.

b) Represent events A, B and C in the sample space Ω.

c) Find and represent $A \cap C$, $A \cup B$, $(A \cap C) \cup B$.

Exercise 1

Consider a car headlight system composed of two taillights, $light_a$ and $light_b$.

The probability that $light_a$ is failed is 0.045, the probability that $light_b$ is failed is 0.055 and the probability that $light_a$ or $light_b$ are failed is 0.070.

Questions:

a) What is the probability that both lights are failed?

b) Are the two events $\{light_a$ *is failed.*$\}$ and $\{light_b$ *is failed.*$\}$ statistically independent? If not, try to justify it from an engineeristic point of view.

c) What is the probability that $light_a$ is also failed given that $light_b$ is failed?

d) What is the probability that $light_b$ is also failed given that $light_a$ is failed?

e) What is the probability that $light_a$ is not failed given that $light_b$ is not failed?

Exercise 2 version 1

A motor operated valve opens and closes intermittently on demand to control the coolant level in an industrial process. An auxiliary battery pack is used to provide power for approximately the 0.4% of the time when there are plant power outages. The demand failure probability of the valve is $4 \cdot 10^{-5}$ when operated from the plant power and $8 \cdot 10^{-4}$ when operated from the battery pack. You are requested to:

- find the demand failure probability assuming that the number of demands is independent of the power source. Is the increase due to the battery pack operation significant?

Exercise 2 version 2

A transport company has three routes within a county. 70% of its buses cover the first route, 20% the second and 10% the third. It is known that the daily probabilities of a breakdown in each route are 3%, 4% and 2%, respectively. You are required to estimate the probability that a generic bus of the company has a breakdown any day.

Exercise 2 version 3

An operating system can work either in user mode or in kernel mode.

It is known that:

- 70% of the operations that are required to perform a given complex task are conducted in user mode while the remaining operations are conducted in kernel mode;
- Any operation failure causes the task fail.

Assuming that:

- The probability that the operating system fails in performing a complex task is $7.3 \cdot 10^{-4}$
- the probability of operation failure in user mode is $2 \cdot 10^{-4}$.

You are required to estimate the probability of operation failure in kernel mode.

Exercise 3

In an energy production plant, there are 5 pumps in parallel sharing a common load. Let X indicates the number of operating pumps. According to some historical data, we assume that the following equation properly describes the distribution of the number of working pumps:

$$f_X(i) = P(X = i) = \begin{cases} A, & i = 0, 1, 2, \quad \mathrm{A} \in \mathbb{R} \\ 3A, & i = 3, 4, 5, \quad \mathrm{A} \in \mathbb{R} \end{cases}$$

You are required to:

a) Compute *A*

b) What is the probability that there are more than 2 operating pumps?

c) What is the mean and standard deviation of X?

Exercise 4 – Part 1

In an electric system, there are 5 identical LED's operating independently. The probability that a LED is operative after 1 year of operation is 70%. You are required to:

a) List the possible combinations of states with 0, 1 and 3 LED's that are operating after 1 year.

b) Compute the probability that there will be only 3 LED's that are operating after 1 year.

Exercise 4 - Part 2

Let X be a random variable whose values represent the number of LED's operative after 1 year of operation. Using the information of Part 1, you are required to:

a) plot the probability mass function (PMF) as well as the cumulative distribution function (CDF) of X.

b) Compute the following quantities:

- Mean of X
- Variance of X
- Standard deviation of X

Exercise 5

There are 30,000 traffic lights in Chile. They are all based on the same technology, and they have a single bulb. The probability that the bulb of a traffic light fails during one month of service is p=0.003.

- At the beginning of March 2024, all traffic lights are properly working. Also, the traffic department has in stock only 90 bulbs to replace failed bulbs of traffic lights in all the country. What is the probability that the traffic department will not be able to replace a traffic light during the period between the 1^{st} and 31^{st} of March?
- How many traffic lights bulbs should the traffic department buy to decrease the probability of having a shortage of traffic lights in the same period of time to 0.001?

Exercise 6 version 1

The occurrences of earthquakes in region A may be modelled by a Poisson process with constant rate of occurrence υ. Let $p(k;\ t,\ \upsilon)$ denote the probability of k earthquakes occurrences in t years.

If the mean occurrence rate of earthquakes in a certain region A is once every 5 years, you are required to find:

- the probability of no earthquake in a 10 years period;
- the probability of 1 earthquake in a 10 years period;
- the probability of more than 2 earthquakes in a 10 years period.

Exercise 6 version 2

The occurrences of a fire in region A may be modelled by a Poisson process with constant rate of occurrence υ. Let $p(k;\ t,\ \upsilon)$ denote the probability of k fire occurrences in t month.

If the mean occurrence rate of a fire in a certain region A is once every 3 months, you are required to find:

- the probability of no fire in one year (12 months) period
- the probability of 1 fire in one year (12 months) period
- the probability of more than 2 fires in one year (months) period.

Exercise 7

The mean occurrence rate of extremely cold air events in a region is once every 2 years.

In case of extremely cold air events, the probability that the local power supply system is seriously damaged is 0.03. You are required to compute:

- the probability that the power supply system is not seriously damaged over a 10-year period.