







Decision analysis for resilience (Game theory, Adversarial Risk Analysis, Reinforcement Learning,...)

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Concept: what is system resilience?



 Resilience concerns the whole evolution dynamics of a system to disruptions

Concept: difference with risk



Concept: difference with risk





Resilience quantification

Resilience metrics

• Classification: depend on the applications of interest, calculation methods, available data, etc.





Resilience metric 1 (Henry et al. 2012)



- $\rightarrow P(t)$ system performance function
- \rightarrow t_o: the time when the external disruptive event occurs
- \rightarrow t_d : the time when system performance reach its lowest level

Measures the ratio between the recovered performance up to time t and the maximal loss of system performance due to a disruptive event.



Resilience metric 1 (Henry et al. 2012)



- Deterministic, dynamic
- P(t) could be sometimes high than $P(t_o)$, therefore R(t) could >1
- Consider the lowest level of system performance, while not embrace the failure process, $t < t_d$, in the failure phase



- $\rightarrow P^{N}(t)$: normalized system performance function
- $\rightarrow P_l^N$: loss of normalized system performance after a disruption
- \rightarrow t_o: the instant when P(t) reaches its minimum
- \rightarrow t_e: the instant when the system performance returns to original level
- $\rightarrow t^*$: a strict upper bound for restoration time t_e
- $\rightarrow T = t_e t_o$ and $T^* = t^* t_o$.



- Measures the ratio between the <u>area between the actual (simplified)</u> system performance curve and the desidered function (=1) and the area below the desired function (=1) and the time axis
- Deterministic, static
- Considers both the performance loss and the length of recovery

Resilience metric 3 (Bruneau 2003; Bruneau and Reinhorn 2007; Cimellaro 2010 from MCEER)



- $\rightarrow t_{0E}$: the time when P(t) reaches its minimum
- \rightarrow *T_{RE}*: the total recovery time
- The area between the actual system functionality and the time axis, normalized by the recovery time

Resilience metric 3 (Bruneau 2003; Bruneau and Reinhorn 2007; Cimellaro 2010 from MCEER)



- One of the fundamental metrics
- Deterministic, static
- Give an more accurate evaluation of the resilience level, by considering the true evolution of system performance

Resilience metric 4 (Chang & Shinozuka 2004)



- \rightarrow *P*_o: the initial system performance loss after a disruption, i.e., the largest loss during the disruptive event
- \rightarrow P*: the maximum acceptable loss of performance
- $\rightarrow t_e$: the time when the performance returns to its original level
- \rightarrow t^{*}: the maximum acceptable system recovery time

Resilience metric 4 (Chang & Shinozuka 2004)



- Probabilistic metric, static
- Take into account both the loss of performance and the length of recovery, but not detailed restoration curve.
- Consider uncertainties within the process



• The use of resilience metrics depends on the **applications** and **available** data, e.g., the types of systems, types of disruptive events.

Metrics	Value	Data required in system performance (SP) dimension	Data required in time dimension	Feature/advanta ge in the application
Metric 1 $\boldsymbol{R}(t) = \frac{P(t) - P(t_d)}{P(t_o) - P(t_d)}$	Function of time	P(t): SP function	t_o and t_d	It is actually a normalized SP function
Metric 2 $\boldsymbol{R} = 1 - \frac{P_l^N T}{2T^*}$	Single value ∈ [0,1]	The lowest value of the normalized SP.	$t_o, t_e, \text{ and } t^*$	Provide a rough estimation based on relatively less information
Metric 3 $\boldsymbol{R} = \int_{t_0}^{t_e} \frac{P^N(t)}{T_{RE}} dt$	Single value ∈ [0,1]	$P^{N}(t)$: the normalized SP function of time	t_o and t_e	Including more information of SP
Metric 4 $\mathbf{R} = \Pr(P_o < P^* \cap t_e < t^*)$	Single value Probability	SP value at a critical time instant	t_e and t^*	Taking into account the uncertainty of the event or system



A network with nodes and directed links

- Each node has two states: operation vs. failure
- The direction of links indicates the functional dependency
- A node operates when all the nodes it depends on are functioning well



- System performance *P*(*t*) is defined as the proportion of operational nodes at each time *t*.
- At $t = t_o$, a disruption occurs and leads to the failure of certain nodes, and the failure propagates to other nodes.
- It takes $\Delta t_r = 1$ for a node to shift from operation (failure) to failure (operation) when its dependent nodes are failed (repaired)
- Nodes having redundancy (yellow circle) are able to sustain disruptions



Failure scenario

- \rightarrow At $t_o = 1$, a disruptive event strikes the system, node 1 is failed at t=2
- \rightarrow At $t_r = 8$, the failed node (node 1) is fixed;
- → The maximum acceptable system recovery instant $t^* = 12$.





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System performance curve calculation





System performance curve calculation





Resilience metric 1 (Henry)

$$\boldsymbol{R}(t) = \frac{P(t) - P\left(t_{d}\right)}{P\left(t_{o}\right) - P\left(t_{d}\right)}, t \ge t_{d}$$

- The data required are:
 - $\rightarrow P(t)$
 - $\rightarrow t_o = 0$
 - $\rightarrow t_d = 4$
 - $\rightarrow P(t_o) = P(t=0) = 1$
 - $\rightarrow P(t_d) = P(t=0) = 0.5$

• Essentially, it is a normalized system performance function for $t \ge t_d$



Resilience metric 1 (Henry)

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The data required are:



• Essentially, it is a normalized system performance function for $t \ge t_d$



Resilience metric 2 (Zoebel)

$$\boldsymbol{R} = \frac{T^* - P_l^N \cdot T/2}{T^*} = 1 - \frac{P_l^N T}{2T^*}$$

- The data required are:
 - $\begin{array}{l} \Rightarrow \ P_l^N = 0.5 \\ \Rightarrow \ t_o = \\ \Rightarrow \ t_e = 10 \\ \Rightarrow \ t^* = 12 \\ \Rightarrow \ T = t_e t_o \\ \Rightarrow \ T^* = t^* t_o \end{array}$



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- A single value in the range of [0, 1]
- Only require field data about P_l^N and T, a rough estimation



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- A single value in the range of [0, 1]
- Only require field data about P_l^N and T, a rough estimation (more conservative \rightarrow considers the failure process)



Resilience metric 3 (Bruneau)

$$\mathbf{R} = \int_{t_{0E}}^{t_{0E}+T_{RE}} \frac{P^{N}(t)}{T_{RE}} dt$$

- The data required are:
 - $\begin{array}{l} \rightarrow \ P^{N}(t) \\ \rightarrow \ t_{oE} = 4 \\ \rightarrow \ t_{e} = 10 \end{array}$
 - $\rightarrow T_{RE} = t_e t_{oE}$
- A single value in the range of [0, 1]
- Require data of system performance $P^N(t)$ for the whole recovery process



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Resilience metric 3 (Cimellaro)

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- The data required are:
 - $\begin{array}{l} \rightarrow \ P^{N}(t) \\ \rightarrow \ t_{oE} = 1 \\ \rightarrow \ t_{e} = 10 \\ \searrow \ T \ -t \end{array}$
 - $\rightarrow T_{RE} = t_e t_{oE}$
- More realistic → considers the failure process



Resilience metric 3 (Cimellaro)

$$\mathbf{R} = \int_{t_{0E}}^{t_{0E}+T_{RE}} \frac{P^{N}(t)}{T_{RE}} dt$$

- The data required are:
- More realistic → considers the failure process



Resilience metric 4 (Chang)

$$\boldsymbol{R} = \Pr(P_o < P^* \text{ and } t_e < t^*)$$

- The data required are:
 - → **Consider uncertainty:** the recovery time Δt_r of failed node is a discrete random variable



- \rightarrow The maximum acceptable performance loss $P^*=0.5$
- \rightarrow The maximum acceptable recovery time $t^* = 12$



Resilience metric 4 (Chang)

$$\boldsymbol{R} = \Pr(P_o < P^* \text{ and } t_e < t^*)$$

- Monte Carlo simulation for the recovery process
- Example of system performance curves



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$$\boldsymbol{R} = \Pr(P_o < P^* \text{ and } t_e < t^*)$$

- Monte Carlo simulation for the recovery process
- Example of system performance curves



• we only need to check if the time that the system is recovered to 100% is earlier than t=12.



- Objective: quantitatively evaluate the resilience of a system against specific hazards
- Methods:
 - **Statistical** methods based on historical events data if available
 - **Simulation**-based computational methods if hazards are predictable and calculable, e.g., can be modeled via probabilistic approaches
 - Worst-case analysis methods for deep uncertain hazards
 - \rightarrow Nondeliberate hazards, e.g., extreme disasters
 - \rightarrow Climate Change
 - \rightarrow Deliberate threats (e.g., vandalism, sabotage, and terrorism)

"WORST-CASE" DISRUPTION MODEL

- Hypothetical intelligent adversary
- Disruption: loss of sets of system components
- Worst-case adversary behavior: use limited capability to inflict maximum damage

SYSTEM RESPONSE MODEL

- Model the function of a system of interest
- Operator makes decisions about system activities after disruption
- What we want (objectives) vs. What is feasible (constraints)

Attacker-Defender Interdiction Model

 $\min_{\boldsymbol{x} \in \boldsymbol{X}} \max_{\boldsymbol{y} \in \boldsymbol{Y}(\boldsymbol{x})} P(\boldsymbol{y})$

\rightarrow X: limited capability of the attacker

- \rightarrow **Y**(**x**): defender's feasible operation space, in a function of **x**
- $\rightarrow P(\mathbf{y})$: What we want, system performance function
- \rightarrow Min or max depending on the definition of P(y)



Attacker-Defender Interdiction Model

 $\min_{\boldsymbol{x} \in \boldsymbol{X}} \max_{\boldsymbol{y} \in \boldsymbol{Y}(\boldsymbol{x})} P(\boldsymbol{y})$

- A Stackelberg (leader-follower) game
- Solution is given by its Stackelberg Equilibrium
- Theorem: in a finite game with 2 players, i.e., X and Y(x) are finite, there is always a Stackelberg equilibrium

A game-theory worst-case assessment framework

An example: fuel supply network



- White \rightarrow demand =1 barrel of fuel
- Black \rightarrow suppliers = 10 barrels
- Links → bidirectional, capacity = 15, transmission cost = 1€/barrel
- Penalty of unsatisfied demand: 10€/barrel
- We care about the total cost



$\min_{y \in Y} Cost(y)$

- A baseline flow pattern corresponding the minimum cost flow solution
- Total cost = 25€

A game-theory worst-case assessment framework

An example: fuel supply network

$\max_{\pmb{x}\in \pmb{X}}\min_{\pmb{y}\in \pmb{Y}(\pmb{x})}Cost(\pmb{y})$



Total cost = **30**€

Total cost = **33**€

- $x \rightarrow \text{only single link break is allowed}$
- Recourse actions **y**: re-dispatching network flow
- Incurs additional operating cost, but does not prevent fuel from being delivered to each demand

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• Worst-case single disruption: **[10, 13]** → increase **8**€ cost

A game-theory worst-case assessment framework

How does the worst-case assessment help?

- Avoid the two biggest gotchas:
 - \rightarrow "We didn't know that X would cause Y..."
 - \rightarrow "We never thought that could happen..."
- We don't have to guess at scenarios
 - \rightarrow (or try to assess the intent of bad guys)

Resilience-oriented decision making





Resilience strategies

- To enhance resilience, resilience strategies include:
 - \rightarrow Enhance the resilience awareness
 - \rightarrow Share information
 - \rightarrow Make integrated decision makings
 - ightarrow Train staff and managers
 - → Harden system components
 - \rightarrow Adjust system topology
 - \rightarrow Control system demand level
 - → Deploy backup systems (redundancy)
 - → Optimize repair sequence
 - Resilience strategies are system specific
 - Generally, in the time domain: pre-event strategies vs. postevent strategies, e.g., hardening vs. repair crew scheduling

Resilience-oriented decision making

Optimal pre-event planning for resilience enhancement against threats

 The system operators make decisions about the pre-event resilience strategies in order to reconcile the objectives (enhance resilience) with its constraints in an intelligent and efficient manner.



Resilience-oriented decision making

Defender-Attacker-Defender models $\max_{w \in W} \min_{x \in X(w)} \max_{y \in Y(w,x)} P(y)$

System defender: determine the pre-event resilience strategies, *w*, pursuing maximal system performance

Disruptive agent: maximize the damage considering his ability X(w)

System operator: mitigate the negative effect of the disruptive attack by recourse actions $y \in Y(w, x)$



The Advanced Lead-cooled Fast Reactor European Demonstrator (ALFRED)



ALFRED subjected to cyber attacks

Objective: to provide a one-sided (i.e., defender) prescriptive support strategy d* for optimizing allocation of resources for the defensive barriers based on a <u>subjective expected utility model</u>.

State of the art: traditional game-theoretical defend-attack modeling

o The players (defender and attacker) rely on shared knowledge;

The defender:

knows own beliefs and preferences;

 $\sqrt{}$ knows those of the attacker (and vice versa);



[J] Wang W., Di Maio F., Zio E. Risk Analysis, (2018)

Approach: Adversarial Risk Analysis (ARA) game-theoretical defend-attack modeling

- o incomplete information,;
 - The defender:

only know own beliefs and preferences;





[J] Wang W., Di Maio F., Zio E. Risk Analysis, (2018) under review.

Approach: Monte Carlo (MC) scheme embedded within the ARA modeling

$\,\circ\,$ Attacker decision for $\pi(a|d),$ as seen by defender:

	nown to Defender <u>ven d</u> , propagate uncertainty among N _m runs, for estimating <u>frequency of occurrence of a</u> eing the optimal $a^{*,m}(d)$, $\pi(a d)$:	π(a d)
INNER	calculate attacker expected utility of each a, for finding the optimal one: $a^{*,m}(d) = \operatorname{argmax}_a \Psi^m_A(a d)$ \circ sample incomplete information (e.g. costs) from subjective distributions	π(a d) for each d
Defend	er decision for d*:	

propagate uncertainty among N_v runs, for estimating frequency of d being the optimal $d^{*,v}$, and finding the optimal allocation d* of resource for defensive barriers by: $d^* = \operatorname{argmax}_d f(d)$

d*

calculate defender expected utility of each d, for finding the optimal one: $d^{*,\nu} = \operatorname{argmax}_{d} \Psi_{D}^{\nu}(d)$

where,

$$\Psi_D^{\nu}(d) = \sum_a \left[\sum_s p_D(s|d,a) \cdot u_D(d,a,s) \right] \cdot \pi(a|d)$$

o sample incomplete information (e.g. costs) from subjective distributions

Results: ARA model



What we have done:

• Optimization of the allocation of defensive barriers against cyber attacks by Adversarial Risk Analysis (ARA);

What we have found (case study):

ARA beats Nash equilibrium in ALFRED;

[J] W. Wang, F. Di Maio, E. Zio. Adversarial Risk Analysis to Allocate Optimal Defense Resources for Protecting Nuclear Power Plants from Cyber Attack. Risk Analysis,

Resilience-oriented decision making

Optimal pre-event planning for resilience enhancement against threats

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Profit-driven-reward Reinforcement Learning

Approach: Deep Reinforcement Learning



Tabular RL





For continuous state space or complex system, it's impossible to list all the combinations





Approach: Imitation Learning (IL)

Imitation Learning (IL) is a type of supervised learning in which an agent learns to perform a task by mimicking the behavior of an expert.

IL can help speed up the convergency when the state space is large and it is hard to find global optimum.



Approach: Proximal Policy Optimization (PPO)

Proximal Policy Optimization (PPO) is a popular actor-critic RL algorithm, which aims at stabilizing the policy optimization by optimizing a surrogate objective function that is a compromise between the current policy and a new candidate policy, and constraining/clipping the gradient updates, in the attempt to monotonically improve the policy.



Application: Cyber-Physical Energy Systems (CPESs)







High level of variability and uncertainty penetrate the grid

Context : CPES Flexible Operation

Flexible operation: The ability of a plant to adjust its power output to match fluctuations in electricity demand, while maintaining safety and efficiency (load-following includes three phases: a power decrease, a low power duration and a power ramp).



Load-following scheme





The Advanced Lead Fast Reactor European Demonstrator (ALFRED)

ALFRED multi-loop control scheme 4 Single-Input-Single-Output control loops



Dynamic Reliability Assessment Framework



[1] Di Maio F, Mascherona R, Zio E. Risk analysis of cyber-physical systems by GTST-MLD[J]. IEEE Systems Journal, 2019, 14(1): 1333-1340.

Dynamic Reliability Assessment Framework



Cyber aging IF models the controller aging under flexible load-following operation



Result: Maintenance timing comparison



- Predictive strategy randomly arranges maintenance activities.
- RL strategies arrange maintenance intervention mostly on 000 and 001 sequence days to satisfy load-following operation as much as possible.

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Z. Hao, F. Di Maio, E. Zio, "A Sequential Decision Problem Formulation and Deep Reinforcement Learning Solution of the Optimization of O&M of Cyber-Physical Energy Systems (CPESs) for Reliable and Safe Power Production and Supply", Reliability Engineering & System Safety, Vol. 235, 109231, 2023

Protection and resilience of critical infrastructures: scientific and technical issues





