Name:____________________

POLIMI ID number:____________________

Reliability, Safety and Risk Analysis: Exam Simulation (29/05/2020)

Note:

1. Make sure to write your name and POLIMI ID number
2. The exam consists of 2 numerical problems on the course topics
3. Exam time is 1 hours and 15 minutes

Problem 1 on the course topics (15 points)

Consider a system made by 1 component with constant failure and repair rates, equal to: $\lambda = 10^{-3} h^{-1}$ and $\mu = 10^{-2} h^{-1}$, respectively. The system mission time is 10^3h. You are required to:

1a. Provide the pseudocode for the Monte Carlo estimation of:

- 1a.1) The reliability at mission time.
- 1a.2) The availability at time 210.
- 1a.3) The average unavailability over the mission time
- 1a.4) The MTTF and the uncertainty of the Monte Carlo estimator.

Please comment the main steps of the pseudocode

1b. Evaluate the quantities in 1a.1), 11a.2) and 1a.3) by performing 3 system life simulations. Assume that a generator of random numbers from a uniform distribution in the range [0,1) provides the numbers reported in Table 1 below. Notice that some random numbers of the list may be not needed for the computation.

Table 1

1.	0.1853	**10.**	0.0351
2.	0.4531	**11.**	0.9992
3.	0.8730	**12.**	0.9997
4.	0.0866	**13.**	0.0428
5.	0.7215	**14.**	0.9996
6.	0.3676	**15.**	0.1997
7.	0.9025	**16.**	0.8581
8.	0.0942	**17.**	0.5782
9.	0.0425	**18.**	0.0843

Problem 2 on the course topics (15 points)

Consider the system made by six components whose block diagrams is reported in Figure 1. The system fails when an input signal is able to go from the input node *I* to the output node *O*. You are required to:

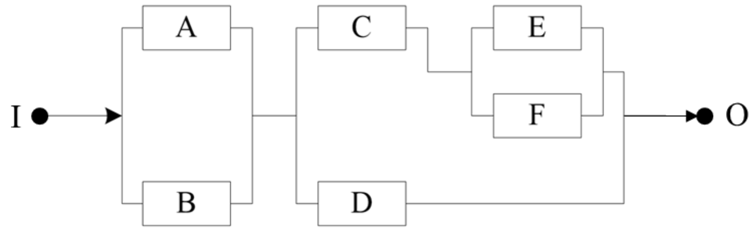

Figure 1: System block diagram

2a. Build the Fault Tree corresponding to the top event: ‘System Failure’.

2b. Assuming that each component has a constant failure rate $\lambda = 10^{-4}$ h^{-1}:

- 2b.1) Find the minimal cut sets;
- 2b.2) Compute the system reliability at time $t = 1000h$;
- 2b.3) Compute the Birnbaum, Fussel-Vesely and RRW importance measures of component D, E and F at t=1000h. Comments the obtained results.

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Problem 1 on the course topics (15 points)

The mean time to failure of a component of a safety system is 1000 days. Testing the component requires $\tau = 6$ hours, whereas the time to repair is negligible. The time T between the end of the previous test and the beginning of the next one is assumed to be 50 days.
You are required to:

1. Plot the evolution of the instantaneous unavailability of the system.
2. Compute the average unavailability of the component.
3. Consider the operation time between tests (T) as a quantity to be optimized by the maintenance engineers. Which is the value of T that minimizes the average unavailability of the component?
4. Repeat 1 and 2) and only consider the first maintenance cycle, assuming that if the component is found failed at the test, the time to repair is 8 hours.

Problem 2 on the course topics (15 points)

Consider a standby system of two identical units. The failure rate of the unit is λ_1 when it is operating and λ_2 when it is in standby. Both units are functioning at time 0 (one is operative, the other one is standby). In case of failure of the operating unit, the maintenance team starts immediately the repair, which is characterized by a constant repair rate, μ_1. The failure of the standby unit is unnoticed until the unit is asked to operate. In the case in which the operating unit had a failure and the standby had previously failed, the maintenance team start repairing both units in parallel. This repair process, which will result in the repair of both units at the same time, is characterized by a constant repair rate $\mu_2 < \mu_1$.
You are required to:

A. Draw the Markov diagram of the system, upon proper definition of the system states.
B. Write the transition matrix and the Kolmogorov equation in the matrix form.
C. Find the MTTF of the system.
D. Modify the Markov diagram, assuming that an external event may occur with rate λ_C; the external event has a probability p_1 of causing the failure of the operating unit and p_2 of the standby unit.

Reliability, Safety and Risk Analysis: Exam Simulation Solutions (29/05/2020)

Problem 1

1a.

```
Set N, lambda, mu, Tm,
Initialize rel,unav,TTF
for n=1:N                                   % repeat for N Monte Carlo lifes
        s=1
        t=0
        while t<Tm
                if s=1                      % If the component is working the failure rate, lambda, is used,
                                            otherwise the repair rate, mu, is used
                        x=lambda
                else
                        x=mu

                T = -ln(rand)/x             % The transition time is sampled
                t'=t
                t=t+T
                if t<Tm
                        if s==1
                                rel(n)=0        % If a failure happens the reliability of the story is set to 0
                                TTF=[TTF,T]     % If a failure happens the Time To Failure is recorded
                        else
                                unav(n,t':t)=1  % If a repairment happens the unavailability vector is updated
                                DT=[DT, t-t'];  % If a repairment happens the Downtime is recorded
                if t>Tm & s==0
                        unav(n,t':Tm)=1
                        DT=[DT, Tm-t'];
                s=1-s

Rel_Tm=mean(rel)                            % Compute the Reliability at Tm
Av=(mean(1-unav))
Av_210=Av(210)                              % Compute the Availability at t = 210
MDT=sum(DT)/(N*Tm)                          % Compute the Average Unavailability over Tm
MTTF=mean(TTF)                              % Compute the MTTF
std= sqrt(var(TTF)/length(TTF))             % Compute the estimator standard deviation
```

1b.

Life 1:

$$T = \frac{-ln(R)}{lambda}$$

R=0.1853, T=1658, t=1658

t>Tm

Rel(1)=1

Av(1,210)=1
DT=[]
TTF=[1658]

Life 2:

$$T = \frac{-ln(R)}{lambda}$$

R=0.4531, T=791, t=791
t<Tm
Rel(2)=0
Av(2,210)=1
DT=[]
TTF=[1658, 791]

$$T = \frac{-ln(R)}{mu}$$

R=0.8730, T=14, t=805
t<Tm
Rel(2)=0
Av(2,210)=1
DT=[14]
TTF=[1658, 791]

$$T = \frac{-ln(R)}{lambda}$$

R=0.0866, T=2446, t=3251
t>Tm
Rel(2)=0
Av(2,210)=1
DT=[14]
TTF=[1658, 791, 2446]

Life 3:

$$T = \frac{-ln(R)}{lambda}$$

R=0.7215, T=326, t=326
t<Tm
Rel(3)=0
Av(3,210)=1
DT=[14]
TTF=[1658, 791, 2446, 326]

$$T = \frac{-ln(R)}{mu}$$
R=0.3676, T=100, t=426
t<Tm
Rel(3)=0
Av(3,210)=1
DT=[14, 100]
TTF=[1658, 791, 2446, 326]

$$T = \frac{-ln(R)}{lambda}$$
R=0.9025, T=102, t=528
t<Tm
Rel(3)=0
Av(3,210)=1
DT=[14, 100]
TTF=[1658, 791, 2446, 326, 102]

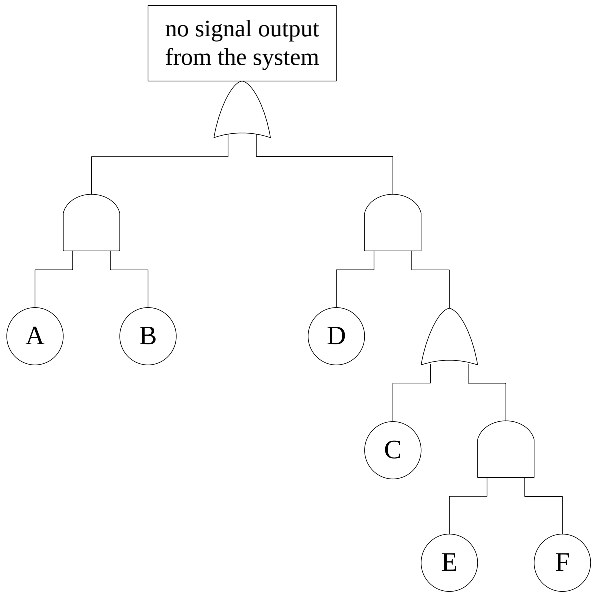
$$T = \frac{-ln(R)}{mu}$$
R=0.0942, T=236, t=764
t<Tm
Rel(3)=0
Av(3,210)=1
DT=[14, 100, 236]
TTF=[1658, 791, 2446, 326, 102]

$$T = \frac{-ln(R)}{lambda}$$
R=0.0425, T=3158, t=3922
t>Tm
Rel(3)=0
Av(3,210)=1
DT=[14, 100, 236]
TTF=[1658, 791, 2446, 326, 102, 3158]

Reliability(Tm)=0,333
Avaliability(210)=1
MDT=0.1167
MTTF=1419 ± 499

Problem 2

2a.



2b.

2b.1) minimal cut sets (MCS): $\{A, B\}, \{C, D\}, \{D, E, F\}$.

2b.2) Component reliability at time 1000h: $R_1(1000) = e^{-\lambda t} = 0.9048$

Component failure probability at time 1000h: $F_1(1000) = 1 - R(1000) = 0.0952$

Probability of the Minimal Cut Sets:

$$MCS_1 = \{A, B\}, MCS_2 = \{C, D\}, MCS_3 = \{D, E, F\}$$

$P(\{A, B\}) = 0.0952^2 = 0.0091$, $P(\{C, D\}) = 0.0952^2 = 0.0091$, $P(\{D, E, F\}) = 0.0952^3 = 8.628 \times 10^{-4}$

System reliability:

$$R_{sys}(1000) = 1 - F_{sys}(1000) \approx 1 - \sum_{i=1}^{n} P(MCS_i) = 0.9810$$

2b.3)

Since component E and F have same failure probability and they appear and only appear in the same Minimal Cut Set $\{D, E, F\}$, the component E and F should have the same importance measure.

- Importance measures of component D:

Birnbaum:

For situation '+', the new MCS is: $\{A, B\}, \{C\}, \{E, F\}$;

for situation '-', the new MCS is: $\{A, B\}$;

$$I_D^B = F^+(1000) - F^-(1000) = P(\{A, B\} \cup \{C\} \cup \{E, F\}) - P(\{A, B\})$$

$$= (0.0952^2 + 0.0952 * 1 + 0.0952^2 * 1) - (0.0952^2) = 0.1043$$

Fussel-Vesely:

$$I_D^{FV} = \frac{P(\{\text{C,D}\} \cup \{\text{D,E,F}\})}{P(\bigcup_i MCS_i)} = \frac{[(0.0952)^2 + (0.0952)^3]}{0.0190} = \frac{0.0099}{0.0190} = 0.5224$$

RRW:

$$I_D^{RRW} = F(1000)/F^-(1000) = \frac{0.0190}{0.0952^2} = \frac{0.0190}{0.0091} = 2.0879$$

- The importance measure for component E

Birnbaum:
For situation '+', the new MCS is: $\{A,B\},\{C,D\},\{D,F\}$;
for situation '-', the new MCS is: $\{A,B\},\{C,D\}$;

$$I_E^B = F^+(1000) - F^-(1000) = P(\{A,B\} \cup \{C,D\} \cup \{D,F\}) - P(\{A,B\} \cup \{C,D\}) =$$

$$(0.0952^2 + 0.0952^2 + 0.0952^2) - (0.0952^2 + 0.0952^2) = 0.0091$$

Fussel-Vesely:

$$I_E^{FV} = \frac{P(\{\text{D,E,F}\})}{P(\cup_i MCS_i)} = \frac{[(0.0952)^3]}{0.0190} = \frac{8.628 \cdot 10^{-4}}{0.0190} = 0.0454$$

RRW:

$$I_E^{RRW} = \frac{F(1000)}{F^-(1000)} = \frac{(0.0952^2 + 0.0952^2 + 0.0952^3)}{(0.0952^2 + 0.0952^2)} = 1.0476$$

- The importance measure for component F,

Birnbaum:
For situation '+', the new MCS is: $\{A,B\},\{C,D\},\{D,E\}$;
for situation '-', the new MCS is: $\{A,B\},\{C,D\}$;

$$I_F^B = F^+(1000) - F^-(1000) = P(\{A,B\} \cup \{C,D\} \cup \{D,E\}) - P(\{A,B\} \cup \{C,D\}) =$$

$$(0.0952^2 + 0.0952^2 + 0.0952^2) - (0.0952^2 + 0.0952^2) = 0.0091$$

Fussel-Vesely:

$$I_F^{FV} = \frac{P(\{\text{D,E,F}\})}{P(\cup_i MCS_i)} = \frac{[(0.0952)^3]}{0.0190} = \frac{8.628 \cdot 10^{-4}}{0.0190} = 0.0454$$

RRW:

$$I_E^{RRW} = F(1000)/F^-(1000) = \frac{(0.0952^2 + 0.0952^2 + 0.0952^3)}{(0.0952^2 + 0.0952^2)} = 1.0476$$

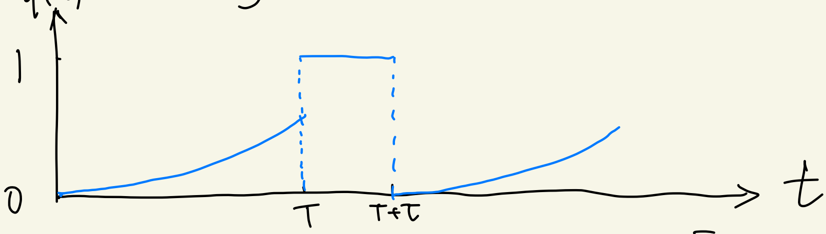
Comments:

According to all the three measures component D is more important than components E and F. This is due to the structure of the system.

If we want to reduce the system failure probability, it is much more efficient to upgrade component D than components E and F (RRW very close to 1 indicates that it would be almost useless to reduce the failure rates of E and F).

Availability:

1.



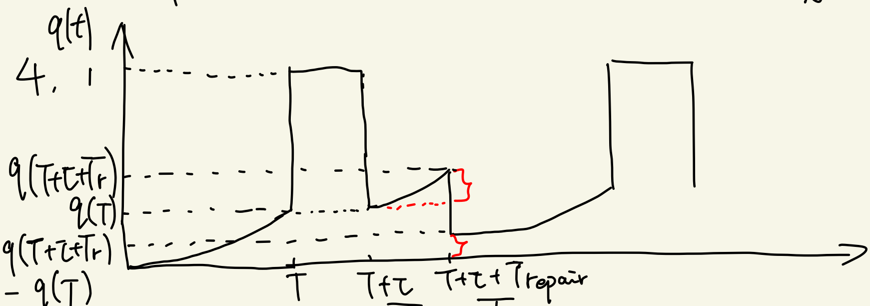
2. $$\bar{q} = \frac{\int_0^T q(t)\,dt + \int_T^{T+\tau} q(t)\,dt}{T+\tau} = \frac{\int_0^T (1-e^{-\lambda t})\,dt + \tau}{T+\tau}$$

$$\cong \frac{\int_0^T \lambda t\,dt + \tau}{T+\tau} = \frac{\frac{\lambda T^2}{2} + \tau}{T+\tau} \cong \frac{\frac{\lambda T^2}{2} + \tau}{T}$$

$$= \frac{\lambda T}{2} + \frac{\tau}{T} = \frac{1}{1000} \times \frac{50}{2} + \frac{\frac{1}{4}}{50} = 0.03$$

3. $$\frac{d\bar{q}}{dT} = \frac{\lambda}{2} - \frac{\tau}{T^2} = 0 \Rightarrow T_{opt} = \sqrt{\frac{2\tau}{\lambda}} = 22.3 \text{ days.}$$

4.



$T_{repair} << T$ $\quad T$ is considered as the total time.

$$\bar{q} = \frac{\int_0^T q(t)\,dt + \tau + (1-e^{-\lambda T})\cdot T_{repair}}{T} = \frac{\lambda T}{2} + \frac{\tau}{T} + \lambda T_{repair}$$

$$= 0.0303$$

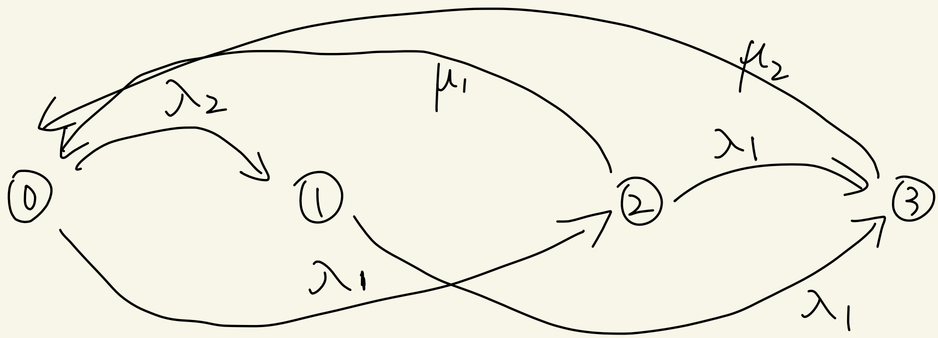
2. Markov.

A. state 0: both not fail

1: operation unit working, standby unit fail

2: operation unit repair, standby unit working

3: both fail



B. $A = \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_2 & \lambda_1 & 0 \\ 0 & -\lambda_1 & 0 & \lambda_1 \\ \mu_1 & 0 & -(\lambda_1+\mu_1) & \lambda_1 \\ \mu_2 & 0 & 0 & -\mu_2 \end{bmatrix}$

$$\frac{d\vec{P}(t)}{dt} = \vec{p}(t) \cdot A$$

C.

$$A^* = \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_2 & \lambda_1 \\ 0 & -\lambda_1 & 0 \\ \mu_1 & 0 & -(\lambda_1+\mu_1) \end{bmatrix}$$

$$(A^*)^{-1} = -\frac{1}{\lambda_1(\lambda_1^2+\lambda_1\lambda_2+\lambda_2\mu_1)} \begin{bmatrix} \lambda_1(\lambda_1+\mu_1) & \lambda_2(\lambda_1+\mu_1) & \lambda_1^2 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$MTTF = -\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (A^*)^{-1} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{2\lambda_1^2 + \lambda_1\mu_1 + \lambda_1\lambda_2 + \lambda_2\mu_1}{\lambda_1(\lambda_1^2+\lambda_1\lambda_2+\lambda_2\mu_1)}$$

D.