POLITECNICO DI MILANO

Basic notions of probability theory

Part 2: EXERCISES

EXERCISE 1

A contractor is planning the purchase of equipment, including bulldozers, needed for a new project in a remote area. Suppose that from his previous experience, he figures there is a 50% chance that each bulldozer can last at least 6 months without any breakdown.

- If he purchased 3 bulldozers, what is the probability that there will be only 1 bulldozer left operative in 6 months?

EXERCISE 2

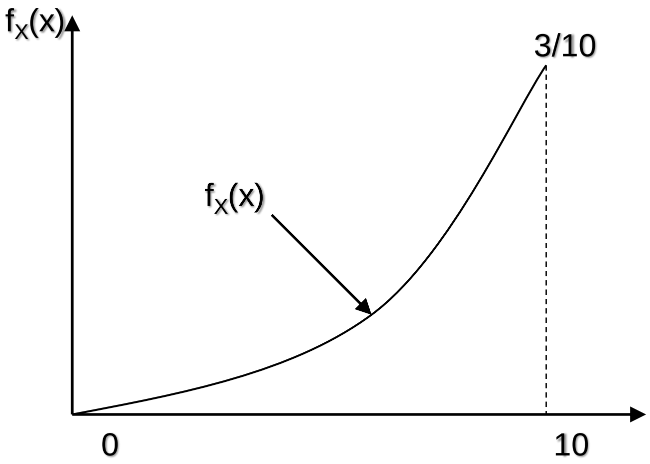
- Let X be the random variable whose values represent the number of good bulldozers after 6 months. The probability that a bulldozer will remain operational after 6 months is $p = 0.8$. Using the above information, plot the probability mass function (PMF) as well as the cumulative distribution function (CDF) of X.
- Using the obtained information, compute the following:
  - Mean of X
  - Variance of X
  - Standard Deviation of X

EXERCISE 3

Suppose that a random variable X is described by a PDF of the form

$$f_X(x) = \begin{cases} \alpha x^2 & 0 \le x \le 10 \\ 0 & \text{elsewhere} \end{cases}$$

1. Find the value of α for which $f_X(x)$ is a PDF?
2. What is $P(X > 5)$?
3. Compute the following:
   - Mean of X
   - Variance of X
   - Standard Deviation of X
   - Median of X



EXERCISE 4

- The occurrences of flood may be modelled by a Poisson process with rate υ. Let $p(k; t, \upsilon)$ denote the probability of k flood occurrences in t years.

1. If the mean occurrence rate of floods for a certain region A is once every 8 years, determine the probability of no floods in a 10-year period; of 1 flood; of more than 3 floods.

EXERCISE 5

- Suppose, from historical data that the total annual rainfall in a catch basin is estimated to be normal (gaussian) N(60cm,15cm),

What is the probability that in future years the annual rainfall will be between 40 and 70 cm?

EXERCISE 7:

- With reference to previous Exercise, assume that the total annual rainfall is log-normally distributed (instead of normally) with the same mean and standard deviation of 60 cm and 15 cm, respectively.

What is the probability that in future years the annual rainfall will be between 40 and 70 cm, under this assumption?

EXERCISE 8

- The daily concentration of a certain pollutant in a stream has the exponential distribution

1. If the mean daily concentration of the pollutant is 2 mg/10^3 liter, determine the constant *c* in the exponential distribution.

2. Suppose that the problem of pollution will occur if the concentration of the pollutant exceeds 6mg/10^3 liter. What is the probability of a pollution problem resulting from this pollutant in a single day?

3. What is the return period (in days) associated with this concentration level of 6 mg/10^3 liter? Assume that the concentration of the pollutant is statistically independent between days.

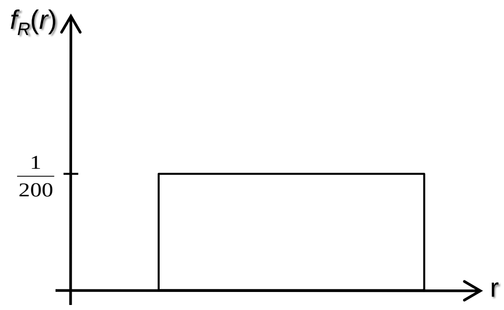
4. What is the probability that this pollutant will cause a pollution problem at most once in the next 3 days?

5. If instead of the exponential distribution, the daily pollutant concentration is Gaussian with the same mean and variance, what would be the probability of pollution in a day in this case?

EXERCISE 9

- Let R be the electric resistance of an equipment, a random variable uniformly distributed between 900Ω - 1100 Ω.

  R is characterized by its distribution:



We are interested in the conductance Y=1/R, where Y is also a random variable that must be characterized by its CDF and PDF.

EXERCISE 10

Problem statement:

- If X has a normal distribution with parameters μ_Z and σ_Z, what is the distribution of $Y=e^X$?

EXERCISE 11

- The strain energy in a linearly elastic bar subjected to a force S is given by

$$\boxed{U = \frac{L}{2AE} S^2}$$

where L = length of the bar
A = cross-sectional area of the bar
E = modulus of elasticity of the elastic material

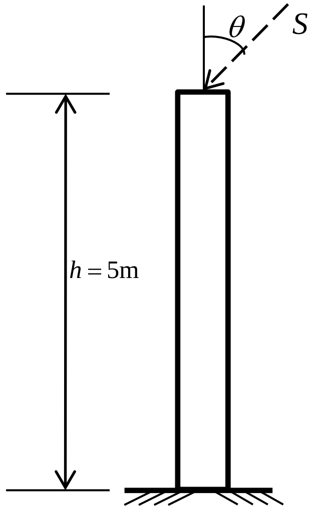
If S is a standard normal variate $N(0,1)$, what is the density function of U?

EXERCISE 12

- Consider a 5-meter-high column supporting a load S, which is inclined at an angle Θ from the vertical as shown in the Figure. Here S and Θ are random variables with respective means and standard deviations.

$$\bar{S} = 100 \text{ Newtons}, \qquad \sigma_S = 20 \text{ Newtons}$$
$$\bar{\theta} = 30° \text{ (0.524 rad)}, \qquad \sigma_\theta = 5° \text{ (0.087 rad)}$$

Determine the mean value and standard deviation of the maximum bending moment on the column induced by the inclined load. Assume that S and Θ are statistically independent.



EXERCISE 13

- With reference to previous Exercise, assume that the total annual rainfall is log-normally distributed (instead of normally) with the same mean and standard deviation of 60 cm and 15 cm, respectively.

What is the probability that in future years the annual rainfall will be between 40 and 70 cm, under this assumption?