Importance analysis considering time-varying parameters and different perturbation occurrence times

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Abstract

Importance measures are integral parts of risk assessment for risk-informed decision making. Because the parameters of a risk model, such as the component failure rates, are functions of time, and a perturbation (change) in their values can occur during the mission time, time dependence must be considered in the evaluation of the importance measures. In this paper, it is shown that the change in system performance at time *t*, and consequently the importance of the parameters at time *t*, depends on the parameters perturbation time and their value functions during the system mission time. We consider a non-homogeneous continuous time Markov model of a series-parallel system to propose the mathematical proofs and simulations, while the ideas are also shown to be consistent with general models having non-exponential failure rates. Two new measures of importance and a simulation scheme for their computation are introduced to account for the effect of perturbation time and time-varying parameters.

KEYWORDS importance measures; time-varying parameters; perturbation time; time-varying importance analysis; Markov modeling; non-Poisson process

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Nomenclature

Symbol	Description
IM	Importance measures
PDF	Probability distribution function
MC	Markov chain
T	Mission time of the system
$A(t)$	Instantaneous availability at time t
Parameter	The system risk model parameters including failure rates, repair rates and so on.
$SDP_{failure}^{T}$	The system downtime percent (SDP) from time *zero* to T, i.e. the sum of downtimes of the system from time *zero* to T, divided by T, when all parameters are at their nominal values
$SDP_{failure}^{T,\lambda_i,t}$	The downtime percent of the system from time *zero* to T, when parameter λ_i is perturbed from its nominal value at time t
$UA_{\lambda_i}^{t} = SDP_{failure}^{T,\lambda_i,t} - SDP_{failure}^{T}$	Difference in the system downtime percents during its mission time, due to the perturbation in the value of λ_i at time t
$F(t)$	System unavailability at time t
$F_j(t)$	Component j unavailability at time t
$F_{\lambda_i,t'}(t)$	System unavailability at time t when parameter λ_i is perturbed at time t'
$\bar{F}(T)$	System average unavailability in time period $[0,T]$
$\pi(t)$	Markov chain state probability row vector at time t
$Q(t)$	Markov chain infinitesimal generator matrix
$\Phi(t,t_0)$	Markov chain state transition probability matrix which maps $\pi(t_0)$ to $\pi(t)$
E	The perturbation matrix that is added to $Q(t)$, whose elements are equal to *zero* except the element that corresponds to the perturbed parameter, which is equal to *one*
Ω	The vertical vector having same size as the transpose of $\pi(t)$. The elements that correspond to states in which the system is unavailable are equal to *one*, and the other elements are equal to *zero*
$SPM(t)$	System performance metric at time t
$SPM_{\lambda_i,t'}(t)$	System performance metric at time t when parameter λ_i is perturbed at time t'

1 | INTRODUCTION

Systems consist of subsystems, elements and components, whose parameters, e.g. failure and repair rates, are the input of the system risk model. In many studies, these parameters are considered to be constant over time. However, the assumption of constant failure and repair rates is an approximation that does not hold for many real world systems[1,2]. For this reason, the interest is increasing for the analysis of system performance metrics, such as reliability and availability, considering time-varying parameters[3-7]. On the other hand, importance measures (IMs) play a significant role in risk-informed decision-making applications, because they can be used for identifying the most important components with respect to system performance metrics such as safety, availability, maintainability, and reliability[8].

Local IMs, such as differential importance measure (DIM), provide the importance of risk model parameters with respect to an infinitesimal change occurring at a single instance in their value domain[9]. To obtain the importance of a parameter at time t, the approach is to calculate the difference between the values of a chosen system performance metric under two conditions: 1- assuming a value for the parameter, constant during the system mission time, 2- assuming another value, equal to the parameter value at condition 1 plus a perturbation (change), also constant during the system mission time. Accordingly, the importance of the parameter at time t is obtained calculating the difference[10-13].

The local importance measures consider the effect of a given perturbation of just one value of one parameter, whereas it is known that a parameter can take different values. The global importance measures have been developed to take into account different values of a parameter for calculating its importance. The approach of the local measures is repeated for several values of the parameter. Consequently, the global importance of the parameter at time t is the average (or another function) of the performance metric differences between the perturbed and unperturbed conditions[14-19].

The local and global importance measures, as already shown, always consider two assumptions: 1- the parameters values are constant during the system mission time, 2- the perturbation occurs at a single time point, usually at time *zero* or at the present time[20,21].

Contrary to the approaches of the local and global importance measures, in real-world systems with degrading components, the risk model parameters values are functions of time, i.e. they change continuously or at discrete time instances during the system mission time. The effect of time-varying parameters and degrading components on the system reliability has been already considered[22-28]. Besides, it is known that thermal shocks, vibration changes, humidity changes, maintenance actions and etc. can affect the risk model parameters[29-33]. Because these events can occur at any time instances during the system mission time, perturbations (changes) in the values of the system risk model parameters do as well.

With an emphasis on time-varying risk model parameters, and the fact that perturbations can occur at any time during the system mission time, the present paper attempts to answer the question: what is the effect of perturbing time-varying risk model parameters values at different time instances on their importance at time *t*? In particular, the effects, their magnitude and the factors influencing the magnitude are thoroughly disclosed. Furthermore, two new importance measures are introduced to evaluate the importance of time-varying system risk model parameters, respecting the perturbation time. A simulation scheme is provided for the calculation of the two new measures.

The paper is organized as follows. The considered system performance metric is proposed in section 2. Section 3 explains the effect of the time of parameters perturbation on the importance analysis. The two new importance measures are introduced in section 4. Section 5 provides a computation scheme for calculating these new importance measures. The results and discussion are presented in section 6. Finally, conclusions are drawn in section 7.

2 | THE CONSIDERED SYSTEM PERFORMANCE METRIC

Reliability and availability are important system performance metrics in reliability, availability, maintainability and safety (RAMS). Reliability is the probability that the system accomplishes its required function for a determined period of time without failure. Availability is the probability that the system is working correctly when it is demanded for use. System availability (or unavailability) is the system performance metric that is utilized in this research, because it considers repair actions and is more general than reliability[34].

Point, or instantaneous, availability at time *t*, $A(t)$, is the probability that the system (or component) is operational at time, *t*. Mean availability is the proportion of time that the system is available for use during a specified period of time. It is the average of the instantaneous availability function over the specified period of time. Equation (1) represents the formula of the mean availability, A_m, over the time period $\left[0,T\right]$:

$$A_m\left(T\right)=\frac{1}{T}\int_0^T A\left(t\right)dt \tag{1}$$

The steady-state availability is the limit of the instantaneous availability function $A(t)$ as time *t* goes to infinity. Finally, operational availability is a measure of availability that incorporates administrative downtime, logistic downtime and all other sources of downtime:

$$\text{Operational availability } = \frac{\text{Uptime}}{\text{Operating cycle}} \tag{2}$$

In (2), the overall period of operation time considered is the operating cycle, and the total time that the system is functioning during the operating cycle is the uptime. Equation (2) returns the mean availability of the system, when there is no logistic downtime or preventive maintenance specified.

System average uptime availability (or system uptime average unavailability) is widely used for analyzing the performance of a system in its mission time[35,36]. The complement of the mentioned measure, i.e. system mean unavailability, is used in this research as the system performance metric, and the importance of a parameter is measured in terms of the effect that a change in the parameter value has on the system average unavailability. Although one can acknowledge the validity of the new ideas on the other system performance metrics like: reliability and availability, but we use system mean unavailability in the whole paper to have a consistent presentation of the ideas.

3 | CONSIDERING PERTURBATION TIME IN IMPORTANCE ANALYSIS

In this section, the reason for considering the time of parameters values perturbations in importance analysis is further explained, and the necessity of introducing the new importance measures is clarified. To evaluate the importance of a parameter in a system risk model, IMs consider a system performance metric (such as system unavailability or unreliability) in two different conditions: 1- when the parameter is at its nominal value, 2- when the parameter value is perturbed from its nominal value. Large difference of the system performance metric in the above two conditions imply large importance of the parameter with respect to its influence on the system performance metric[11,12]. Evidently, for different nominal values of the parameter, different parameter importance values are obtained. Therefore, knowing that in real systems the parameters values change in time[1,2], and a perturbation can occur to their values at any time during the system mission time, it can be deduced that the amount of change in the value of the performance metric depends on the parameters values perturbation times.

In the literature, the importance of the risk parameters for system performance metrics like unavailability, at different times, has been considered. It has been shown that the importance changes with time, i.e. the importance of a parameter like failure rate λ for the system unavailability at time t_1 ($F(t_1)$) is different from its importance in the system unavailability at time $t_2, t_2 \neq t_1$, ($F(t_2)$)[27]. This idea is shown abstractly in Figure 1.

In Figure 1, a perturbation is imposed to the value of λ at time *zero*. The hypothetical system unavailability with the baseline value of λ is showed as solid line, while the dotted line represents the system unavailability with perturbed parameter value, i.e. $\lambda + \varepsilon$ (i.e. the dotted line is hypothetical perturbation of the solid line that is used to show the idea more clearly). The difference between the solid line and the dotted line at time *t* measures the local importance of λ at time *t*. It is shown that the local importance of λ can be different at different time instances $t_1, t_2, \ldots$: In the other words, the local importance measures are time-varying (time-dependent).

Global importance measures calculate the importance based on several parameter values, as presented in the hypothetical example of Figure 2.

Figure 2 shows that the global importance measures apply the same idea as the local importance measures, but using various baseline parameter values. Just like the local measures, they calculate the difference between the system unavailability at time *t* considering a constant value (λ) for the parameter during the system mission time, and the system unavailability at the same time *t* considering another constant but perturbed value for the parameter ($\lambda+\varepsilon$) during the system mission time ($\lambda+\varepsilon$ is the parameter value that is perturbed at time *zero* by an amount ε). Afterward, the mentioned difference is calculated for many values of the parameter $\lambda,\lambda',\lambda'',\ldots$. The importance is, then, obtained by taking the average or the variance of a function (e.g. the absolute value) of the obtained differences. In other words, the global measures are averages or variances of the local measures over the value domain of the parameter.

The concept of lifetime importance measures that are time-independent measures, then, introduced[37]. Because, the importance of the individual components varies with time, we need to take into account the system's entire mission time to get a comprehensive understanding of a component's contribution to the system's unavailability. As time-independent importance measures aim to evaluate the importance of a component or a failure mode over the whole system mission time, a common way to obtain a time-independent importance measure is to integrate a time-dependent importance measure over the system mission time[20,21,38,39].

Contrary to the above, in this paper we consider the importance of a time-varying parameter λ with respect to a system performance metric at time t, considering the time at which λ changes value. For example in the hypothetical example of this section, if the value of λ is perturbed at time $t_1, t_1<t$ the system unavailability at time *t* will be $F_1(t)$, while if the value of λ is perturbed at time $t_2, t_1<t_2<t$ the system unavailability at time *t* will be $F_2(t)$, where $F_1(t)\neq F_2(t)\neq F(t)$. The schematic representation of this phenomenon is provided in Figure 3, where the solid line represents the system unavailability with parameter λ, and the dotted lines show the system unavailability with parameter λ perturbed at various time instances. The variation in the value of the system unavailability at time *t*, $F(t)$, depends on the time of perturbing the value of λ and, thus, the importance of λ also changes depending on the perturbation time. To show this fact, two approaches are employed, as explained in the following subsections.

Hereafter, the importance of a parameter at time *t* refers to the importance of the parameter when its value is perturbed at time *t*, and the system performance metric is system mean unavailability.

3.1 | Simulation approach

Consider the series-parallel system of Figure 4 and a mission time of *T*=200 hours. To show the effect of time on the importance of the parameters, the failure rates are considered as functions of time. On the contrary, the repair rates are assumed constant, without loss of generality.

The considered system performance metric is the average unavailability over the mission time. Actually, to analyze the results we consider the system downtime percent during the mission time, $SDP_{failure}^{T}$, which is defined as the sum of downtimes of the system from time *zero* to *T*, divided by *T*, when all parameters are at their nominal values. In fact, $SDP_{failure}^{T}$ is the operational unavailability that, as was said in section II, is equal to the average unavailability of the system when there is no logistic downtime or preventive maintenance specified in the time period $[0,T]$, while expressing the idea using $SDP_{failure}^{T}$ is easier. The similar behavior of $SDP_{failure}^{T}$ and the average unavailability of the system in time period $[0,T]$ is discussed in the Appendix.

We analyze the importance of λ_1 considering that at time t a perturbation occurs to λ_1 (i.e. an infinitesimal positive value ε is added to λ_1) and the value of λ_1 becomes $\lambda_1+\varepsilon$ from time t to T. Due to this perturbation, there is a difference between $SDP_{failure}^{T,\lambda_1,t}$ (the system downtime percent during its mission time when λ_1 is perturbed at time *t*) and $SDP_{failure}^{T}$. The amount of the difference, $UA_{\lambda_1}^{t}=SDP_{failure}^{T,\lambda_1,t}-SDP_{failure}^{T}$, gives the magnitude of the effect of perturbing λ_1 at time *t* on the system downtime percent from time *zero* to T. So, $UA_{\lambda_1}^{t}$ can be used to represent the importance of perturbing λ_1 at time *t* on the average unavailability of the system in time period $[0,T]$. It will be shown that when the time of perturbation changes, the importance of λ_1 in the system downtime percent from time *zero* to *T* varies too, which is not considered in the previous investigations. This phenomenon becomes even more critical when the studied risk model includes dynamic logic gates, in which the system unavailability depends on the sequence of components failure. For example in the PAND (priority and) gate of Figure 5, if component A fails before component B, the PAND gate will fire, resulting in failure of the respective subsystem, whereas the subsystem does not fail if B fails before A. So, an increase in the failure rate of B after the failure of A increases the system unavailability, whereas increasing the failure rate of B before component A failure will raise the probability that B fails before A, and subsequently will decrease the subsystem unavailability. So the time of perturbation influences the importance of the risk model parameters.

Now we consider an identical perturbation in a different parameter λ_2 at time t and the obtained value of $UA_{\lambda_2}^{t}$. If $UA_{\lambda_1}^{t}$ is greater than $UA_{\lambda_2}^{t}$, λ_1 is more important than λ_2 at time t, because a perturbation in the value of λ_1 at t imposes more variation in the system downtime percent (i.e. the system average unavailability) from time *zero* to *T*, than a perturbation in the value of λ_2. The critical point is that when the time of the perturbation changes, $UA_{\lambda_1}^{t}$ and $UA_{\lambda_2}^{t}$ will also change. For example, at time t_k we can have $UA_{\lambda_1}^{t_k}>UA_{\lambda_2}^{t_k}$ (i.e. λ_1 is more important than

λ_2) whereas at time $t_m, t_m \neq t_k$, we can have $UA_{\lambda_2}^{t_k} > UA_{\lambda_1}^{t_k}$ (i.e. λ_2 is more important than λ_1). So, the importance ranking of the parameters can change, depending on when the perturbation in their values occurs.

To illustrate this discussion more thoroughly, we simulate the series-parallel system of Figure 4 on the basis of its non-homogeneous continuous time Markov chain (MC) model (Figure 6). Note that, to alleviate the burden of proposing proofs and simulations, exponential distribution for the failure and repair times of the components is considered in the subsequent lines of this subsection and the next subsection, although one can verify that the ideas proposed by the paper are not limited to exponential distributions and Markov chains.

Table 1 gives the states of the MC of Figure 6. Note that the exponential distribution and, consequently, the Markov approach are assumed in this example to show the ideas in full clarity, while the assumptions can be easily relaxed to have a more general case.

The simulation method to solve the MC model is the next-reaction method (NRM)[40-42]. In the simulation, the next system state of the MC is determined based on the calculation of the passage time from the present state to all possible states: the next state reached by the system is that with the smallest passage time. The calculation of the passage time from a state to another is done based on a Poisson process. If the transition rate between two states is constant over time, the passage time between the two states is a random value from an exponential distribution. However, when the transition rate is varying over time, the passage time between the two states is a random value from a non-homogeneous exponential distribution[43,44]. For this example, the transition rates are assumed to be: $\lambda_1 = t^2 + 0.5$ per hour, $\lambda_2 = t + 0.65$ per hour (note that any other time-varying functions can be used, as shown in (11) of the next subsection), $\mu_1 = 70$ per hour, and $\mu_2 = 90$ per hour, where *t* represents time in hours. The simulation proceeds as follows:

1. The system behavior is simulated for $T = 200$ hours, with all system risk model parameters set at their nominal values. The system is in state 1 at the initial time. The first passage time is when the system goes from state 1 to the reachable state 2 or state 3. The first passage time is, then, the minimum of the two random realizations, the first from a non-homogeneous exponential distribution with rate $2\lambda_1 = 2\left(t^2 + 0.5\right)$ (i.e. the time to go to state 2), and the second from a non-homogeneous exponential distribution with rate $2\lambda_2 = 2\left(t + 0.65\right)$ (i.e. the time to go to state 3). If the first random value is less than the second one, the next state is state 2 and vice versa. The simulation continues for the passage to the next state and the next, until the mission time T=200 hours to obtain the value of $SDP_{failure}^{T}$.
2. The simulation is repeated exactly as in Step 1 above, but at t_1=first discrete point in time, a perturbation is imposed to λ_1, so, the value of $SDP_{failure}^{T,\lambda_1,t_1}$ is calculated. This process is repeated for λ_2, and $SDP_{failure}^{T,\lambda_2,t_1}$ is obtained.

3. Step 2 is repeated for each discrete point in time separately and the values of $SDP_{failure}^{T,\lambda_1,t_k}, k=1,2,...,dp$ and $SDP_{failure}^{T,\lambda_2,t_k}, k=1,2,...,dp$ are obtained, wherein each t_k is a discrete point in time from 0 to 200 hours, and *dp* is the number of discrete points in time with which the mission time *T* has been discretized. Note that the discrete points in time are exactly the same for the two parameters.
4. Finally, we calculate also the values of $UA_{\lambda_1}^{t_k}, k=1,2,...,dp$ and $UA_{\lambda_2}^{t_k}, k=1,2,...,dp$.

The simulation results are depicted in Figure 7.

In Figure 7, the horizontal axis is time in hours, and the vertical axis is the value of $UA_{\lambda_i}^{t_k}$. The solid line shows $UA_{\lambda_1}^{t_k}$ and the dotted line shows $UA_{\lambda_2}^{t_k}$. Note that the values in Figure 7 are discrete, but we have connected the points to have a more clear illustration. We observe, for example, that $UA_{\lambda_1}^{t_k} - UA_{\lambda_2}^{t_k} = 0.0026$ at time $t_k = 20$ hours, which means that the system downtime from time *zero* to 200 hours, when λ_1 is perturbed at time $t_k = 20$ hours is $(200-20)\times 0.0026 = 0.468$ hours, more than the system downtime from time *zero* to 200 hours, when λ_2 is perturbed at time $t_k = 20$ hours: this result implies a larger importance of λ_1 than λ_2. Note that as the time of occurring the perturbation, i.e. t_k, increases, the perturbation affects the system on a smaller time period. So, because according to the performance metric, the difference in the system unavailability due to a perturbation at time t_k is divided by the length of mission time, as time passes value of UA^{t_k} reduces for the two components.

Clearly, the values of $UA_{\lambda_1}^{t_k}$ and $UA_{\lambda_2}^{t_k}$ are changed by changing the value of t_k. More importantly, the value of $UA_{\lambda_1}^{t_k}$ is more than the value of $UA_{\lambda_2}^{t_k}$ at some points in time, which implies a larger importance of λ_1 than λ_2, and vice versa. These phenomena occur due to different perturbation times, different parameters value functions during the mission time, and the interaction effects of these, which will be clarified using the analytical approach.

3.2 | Analytical approach

In a non-homogeneous continuous time Poisson process with intensity function $\lambda(t)$, the interarrival times of the events are functions of $m(t)$ where $m(t) = \int_0^t \lambda(\tau)d\tau$ [44]. For example, T_1 and T_2 are the first and the second interarrival times with probability density functions[45-47]:

$$f_{T_1}(t) = \lambda(t)e^{-m(t)} \tag{3}$$

$$f_{T_2}(t)=\int_0^{\infty}\lambda(\tau)\lambda(t+\tau)e^{-m(t+\tau)}d\tau \tag{4}$$

Evidently, if a perturbation occurs in the different time instances in $[0,t]$, the change in the values of $f_{T_1}(t)$ and $f_{T_2}(t)$ will be different. Considering what happens in a non-homogeneous continuous time Poisson process, it can be deduced that the system unavailability change at time *t*, using the non-homogeneous continuous time Markov chain, is a function of the perturbation occurrence time; this assertion is proved as follows.

The state probability vector of a non-homogeneous continuous time Markov chain model is obtained by solving the differential equation[47]:

$$\dot{\pi}(t)=\pi(t)Q(t) \tag{5}$$

wherein each element of $Q(t)$ is a function of time-varying parameters $\lambda(t)$,[46]. Equation (5) is a linear time-varying system (LTV), which has been proven to have a unique solution in the form of [48]:

$$\pi(t)=\pi(t_0)\Phi(t,t_0) \tag{6}$$

A general solution for (6) is given by the Peano-Baker series in (7), which if for all *t*, $Q(t)$ and $\int_{t_0}^{t}Q(\tau)d\tau$ commute each other, reduces to (8)[47,48]:

$$\begin{aligned}\Phi(t,t_0)=I&+\int_{t_0}^{t}Q(\tau_1)d\tau_1+\int_{t_0}^{t}Q(\tau_1)\int_{t_0}^{\tau_1}Q(\tau_2)d\tau_2d\tau_1+\ldots\\&+\int_{t_0}^{t}Q(\tau_1)\int_{t_0}^{\tau_1}Q(\tau_2)\ldots\int_{t_0}^{\tau_{k-1}}Q(\tau_k)d\tau_kd\tau_{k-1}\ldots d\tau_2d\tau_1+\ldots\end{aligned} \tag{7}$$

$$\Phi(t,t_0)=e^{\int_{t_0}^{t}Q(\tau)d\tau} \tag{8}$$

Now consider that at time t' a perturbation occurs in the value of a parameter, i.e. a scalar value ε is added to the parameter. The perturbation matrix is shown by *E*, whose elements are equal to *zero* except the element that corresponds to the perturbed parameter, which is equal to *one*. So, the perturbed $Q(t)$ is:

$$Q^{\varepsilon}(t)=\begin{cases}Q(t) & t<t'\\ Q(t)+\varepsilon\times E & t\geq t'\end{cases} \tag{9}$$

Subsequently, the perturbed state probability vector of the non-homogeneous continuous time Markov chain model, $\pi^{\varepsilon}(t)$, is obtained based on the perturbed $\Phi(t,t_0)$, i.e. $\Phi^{\varepsilon}(t,t_0)$, as[47]:

$$\pi^{\varepsilon}(t) = \pi(t_0)\Phi^{\varepsilon}(t,t_0) \tag{10}$$

As (9) shows, each perturbation in the value of the parameter over the system mission time imposes a discontinuity in $Q^{\varepsilon}(t)$; the following lemma is, then, needed to proceed.

Lemma 1. $Q^{\varepsilon}(t)$ is Riemann integrable.

Proof. Because the number of perturbations in the system mission time is a finite number, the set of discontinuity times of $Q^{\varepsilon}(t)$ is a finite set too. So, the Lebesgue measure of the set of $Q^{\varepsilon}(t)$ discontinuity times is *zero*. Because a bounded function with *zero* Lebesgue measure of the set of its discontinuity points is Riemann integrable, then the $Q^{\varepsilon}(t)$ is Riemann integrable.

The importance of the parameter in the system unavailability at time *t*, due to the occurred perturbation at time t', is calculated using the value of $\left(\pi^{\varepsilon}(t) - \pi(t)\right)\Omega = \pi(t_0)\left(\Phi^{\varepsilon}(t,t_0) - \Phi(t,t_0)\right)\Omega$, which shows the imposed variation in the system unavailability value at time *t* due to the perturbation in the parameter value at time t'. Using lemma 1 and some matrix calculations we have:

$$
\begin{aligned}
&\Phi^{\varepsilon}(t,t_0)-\Phi(t,t_0)\\
&=\left(I+\int_{t_0}^{t}Q^{\varepsilon}(\tau_1)\,d\tau_1+\int_{t_0}^{t}Q^{\varepsilon}(\tau_1)\int_{t_0}^{t}Q^{\varepsilon}(\tau_2)\,d\tau_2 d\tau_1+\ldots\right)\\
&-\left(I+\int_{t_0}^{t}Q(\tau_1)\,d\tau_1+\int_{t_0}^{t}Q(\tau_1)\int_{t_0}^{t}Q(\tau_2)\,d\tau_2 d\tau_1+\ldots\right)\\
&=I+\int_{t_0}^{t'}Q(\tau_1)\,d\tau_1+\int_{t'}^{t}\left(Q(\tau_1)+\varepsilon\times E\right)d\tau_1+\int_{t_0}^{t'}Q(\tau_1)\int_{t_0}^{\tau_1}Q(\tau_2)\,d\tau_2 d\tau_1\\
&+\int_{t'}^{t}\left(Q(\tau_1)+\varepsilon\times E\right)\left(\int_{t_0}^{t'}Q(\tau_2)+\int_{t'}^{\tau_1}\left(Q(\tau_2)+\varepsilon\times E\right)\right)d\tau_2 d\tau_1\\
&+\int_{t_0}^{t'}Q(\tau_1)\int_{t_0}^{\tau_1}Q(\tau_2)\int_{t_0}^{\tau_2}Q(\tau_3)\,d\tau_1 d\tau_2 d\tau_3\\
&+\int_{t'}^{t}\left(Q(\tau_1)+\varepsilon\times E\right)\left(\int_{t_0}^{t'}Q(\tau_2)\int_{t_0}^{\tau_2}Q(\tau_3)+\int_{t'}^{\tau_1}\left(Q(\tau_2)+\varepsilon\times E\right)\left(\int_{t_0}^{t'}Q(\tau_3)+\int_{t'}^{\tau_2}\left(Q(\tau_3)+\varepsilon\times E\right)\right)\right)d\tau_3 d\tau_2 d\tau_1\\
&-\left(I+\int_{t_0}^{t}Q(\tau_1)\,d\tau_1+\int_{t_0}^{t}Q(\tau_1)\int_{t_0}^{t}Q(\tau_2)\,d\tau_2 d\tau_1+\ldots\right)\\
&=\varepsilon\times E\times(t-t')+\int_{t'}^{t}\varepsilon\times E\int_{t_0}^{\tau_1}Q(\tau_2)\,d\tau_2 d\tau_1+\int_{t'}^{t}\left(Q(\tau_1)+\varepsilon\times E\right)\int_{t'}^{\tau_1}\varepsilon\times E\,d\tau_2 d\tau_1\\
&+\int_{t'}^{t}\left(Q(\tau_1)+\varepsilon\times E\right)\left(\int_{t'}^{\tau_1}\varepsilon\times E\int_{t_0}^{\tau_2}Q(\tau_3)+\int_{t'}^{\tau_1}\left(Q(\tau_2)+\varepsilon\times E\right)\int_{t'}^{\tau_2}\varepsilon\times E\right)d\tau_3 d\tau_2 d\tau_1\\
&+\int_{t'}^{t}(\varepsilon\times E)\int_{t_0}^{\tau_1}Q(\tau_2)\int_{t_0}^{\tau_2}Q(\tau_3)\,d\tau_3 d\tau_2 d\tau_1+\ldots
\end{aligned}
\tag{11}
$$

Referring to the results of (11), it can be seen that the value of $\Phi^{\varepsilon}(t,t_0)-\Phi(t,t_0)$ depends on: 1- the time of occurrence of the perturbation, 2- the time-varying function of the system perturbed parameters values, i.e. $Q(t)$, from time t' to t and in the whole mission time, and 3- the interaction effects of the two previous cases. Therefore, it is proved that $\left(\pi^{\varepsilon}(t)-\pi(t)\right)\Omega$, which determines the parameter importance in the system unavailability at time *t*, is contingent upon the time of occurrence of the perturbation, that verifies the idea of Figure 3. It is, then, deduced that the system average unavailability in its mission time, i.e. $\int_0^T\left(\pi(t)\times\Omega\right)dt \big/ T$, depends on when a perturbation occurs in the value of the parameter. So the ideas of this paper hold for system unavailability, average unavailability, and equivalently for system availability, average availability and the reliability metrics of systems.

4 | THE NEW IMPORTANCE MEASURES

The idea underlying the new importance measures is to take the average and the variance of the importance values of each parameter, over perturbation times based on a performance metric at time *t*. The new measures are proposed in (12) and (13). Equation (12) is the average of variations of the performance metric at time *T* calculated over all perturbations that occur in the value of a parameter before time *T*, while (13) is the variance of the variations over the mentioned perturbation times.

$$E\left(\lambda_i(T)\right)=\frac{\overbrace{\int_0^T\left(SPM_{\lambda_i,t}(T)-SPM(T)\right)dt}^{\text{The importance of parameter }\lambda_i\text{ in the system performance metric at time T}}}{T}=\frac{\overbrace{\int_0^T\left(\frac{\int_0^T F_{\lambda_i,t}(\tau)d\tau}{T}-\frac{\int_0^T F(\tau)d\tau}{T}\right)dt}^{\text{The importance of parameter }\lambda_i\text{ in the system performance metric of this paper}}}{T} \tag{12}$$

$$V\left(\lambda_i(T)\right)=\frac{\overbrace{\int_0^T\left(\left(SPM_{\lambda_i,t}(T)-SPM(T)\right)-E(\lambda_i)\right)^2 dt}^{\text{The importance of parameter }\lambda_i\text{ in the system performance metric at time T}}}{T}=\frac{\overbrace{\int_0^T\left(\left(\frac{\int_0^T F_{\lambda_i,t}(\tau)d\tau}{T}-\frac{\int_0^T F(\tau)d\tau}{T}\right)-E(\lambda_i)\right)^2 dt}^{\text{The importance of parameter }\lambda_i\text{ in the system performance metric of this paper}}}{T} \tag{13}$$

Considering the performance metric of this paper, the importance at time *t* is the variation in the average system unavailability in the whole system mission time, due to an infinitesimal perturbation in the value of the parameter at time *t*, i.e. $\int_0^T F_{\lambda_i,t}(\tau)d\tau \big/ T-\int_0^T F(\tau)d\tau \big/ T$. Accordingly, (12) gives the average importance of λ_i over all perturbation times. The definition of variance is used in (13) to calculate the variance of the importance of λ_i over all perturbation times in the mission time.

If the importance of λ_i is more than the importance of λ_j at most instants in the mission time, the value of (12) will be larger for λ_i than λ_j, which indicates a larger importance for λ_i than λ_j. Equation (13) will be high for λ_i, if the importance of λ_i has high variation in time (i.e. at some points in time, a change in λ_i causes a very high change in system average unavailability, and in some other points it causes a very low variation in system average unavailability).

To compare two parameters, say λ_i and λ_j, if (12) and (13) are larger for λ_i than λ_j, it means that in most time points in the mission time a perturbation in the value of λ_i has induced more variation in the system average unavailability than a variation in the value of λ_j (from (12)). Furthermore, the importance of λ_i is more sensitive than the importance of λ_j, which means a more unstable behavior of λ_i than λ_j in the mission time (from (13)). So, we can come to a conclusion that λ_i is more important than λ_j. If the value of (12) is larger for λ_i than λ_j but the value of (13) is larger for λ_j than λ_i, we can again conclude that λ_i is still more important than λ_j. Because, although the importance of λ_j has had more unstable behavior in the whole mission time than the importance of λ_i, and perhaps in some time points λ_j has been even more important than λ_i, but λ_i has been more influential than λ_j in most points in time, which means that a perturbation in the value of λ_i has had a larger effect on the system average unavailability. If the value of (12) for the two parameters is approximately equal, but the value of (13) for λ_i is larger than that of λ_j, we can deduce that λ_i is more important than λ_j, because while their average effect on the system average unavailability is the same, but the importance of λ_i has had more unstable behavior than the importance of λ_j in most time points.

5 | A COMPUTATION METHOD FOR CALCULATING (12) AND (13)

Obtaining the closed-form solution for (12) and (13) is not possible in most cases, even considering exponential distribution that is an easy distribution to work with[1]. Hence, a simulation approach is proposed to calculate (12) and (13). In this respect, $\int_0^T F(\tau)\,d\tau$ is the total downtime of the system from time *zero* to *T*, when all parameters are at their nominal values. So, $\int_0^T F(\tau)\,d\tau \big/ T$ is the percent of time from *zero* to *T*, that the system is not working. We can conclude that $\int_0^T F(\tau)\,d\tau \big/ T$ is equal to $SDP_{failure}^{T}$ (more discussion is brought in the Appendix). By the same argument, we can deduce that $\int_0^T F_{\lambda_i,t}(\tau)\,d\tau \big/ T$ is equal to $SDP_{failure}^{T,\lambda_1,t}$. Thereupon, we can find the value of $\int_0^T F_{\lambda_i,t}(\tau)\,d\tau \big/ T - \int_0^T F(\tau)\,d\tau \big/ T$ using $UA_{\lambda_1}^{t} = SDP_{failure}^{T,\lambda_1,t} - SDP_{failure}^{T}$ at time *t*. If we calculate $UA_{\lambda_1}^{t} = SDP_{failure}^{T,\lambda_1,t} - SDP_{failure}^{T}$ at many points in time (i.e. for many *t*) and, then, compute their average, the obtained value can be used as an estimate for

$\int_0^T \left(\int_0^T F_{\lambda_i,t}(\tau) d\tau \Big/ T - \int_0^T F(\tau) d\tau \Big/ T \right) dt / T$. Thus, an estimate for (12) and (13) can be generated using the same simulation as the one presented in the previous section.

6 | RESULTS AND DISCUSSION

Using the data from the previous simulation, (12) and (13) are applied for λ_1 and λ_2 to obtain $E(\lambda_1) = 7.0050 \times 10^{-5}$, $E(\lambda_2) = 5.5882 \times 10^{-5}$, $V(\lambda_1) = 7.1340 \times 10^{-8}$, and $V(\lambda_2) = 7.9975 \times 10^{-8}$. In order to have a more precise comparison, the simulation was run for 60 times. Using statistical analysis to examine H0: average of (12) for 60 runs is equal for the two parameters against H1: H0 does not hold, and H0: average of (13) for 60 runs is equal for the two parameters against H1: H0 does not hold, the Z-statistic is obtained as 17.1898 and 17.9782, respectively, which results in rejecting both null hypotheses on an absolutely high confidence level. The results show that the variation in the importance of λ_2 is more than the variation in the importance of λ_1 in the 200 hours of system mission. Additionally, the average importance of λ_1 is more than the average importance of λ_2, which shows that most of the times a variation in the value of λ_1 has affected the system average unavailability more than a variation in the value of λ_2. According to the provided comparison scheme in the final paragraph of the section 4 and due to the larger value of (12) for λ_1 than that of λ_2, we can conclude that λ_1 is more important than λ_2 in the studied mission time.

To show another case for comparison, we changed the parameters, $\mu_1 = 50$ and $\mu_2 = 35$, to obtain $E(\lambda_1) = 2.2759 \times 10^{-5}$, $E(\lambda_2) = 2.2760 \times 10^{-5}$, $V(\lambda_1) = 1.6257 \times 10^{-8}$, and $V(\lambda_2) = 5.0892 \times 10^{-8}$, which indicates approximately equal (12) values for parameters λ_1 and λ_2, but a larger (13) value for λ_2 than for λ_1, known from the fact that the Z-statistic for examining H0: average of (13) in 60 runs for λ_2 is equal or less than that for λ_1, against H1: H0 does not hold is obtained as 25.0035 and the null hypothesis is rejected under an absolutely high confidence level. The result of the simulation for *T*=200 hours is showed in Figure 8, wherein the solid line shows $UA_{\lambda_1}^{t_k}$ and the dotted line shows $UA_{\lambda_2}^{t_k}$. These results show that in the mission time, a change in the value of both parameters has approximately equal average effect on the system average unavailability. On the other hand, the effect of a change in the value of λ_2 on the system average unavailability is more unstable than that of λ_1. For example, two of the highest values of $UA_{\lambda_2}^{t_k}$ are 0.0014 and 0.0009 whereas the highest values of $UA_{\lambda_1}^{t_k}$ are 0.0006 and 0.0002. It means that although the two parameters have equal average effect on the system average unavailability (i.e. equal values of (12)), in some points in the mission time λ_2 has higher effect on the system average unavailability than λ_1. So, we can conclude that λ_2 is more important than λ_1.

As another case, a real world electrical power supply system for nuclear power plants is considered[35]. The system is showed in Figure 9,A , wherein the grid supply is the primary component, the diesel supply is a redundant component for supplying the power, and the sensing and control circuitry is grid supply failure detector to put the diesel supply in operation. Figure 9,B presents the fault tree of the system that can be easily mapped into its representative Markov chain considering exponential failure and repair rates[35].

The results of implementing the new ideas on the Markov chain of the power supply system for 2×10^6 hours are shown in Figure 10, with same notations and interpretations as Figures 7 and 8. The simulation has been repeated for 500 times to omit the effect of stochastic behavior of the system on the results.

The importances of λ_1 and λ_2 using the method provided by Do Van et al.[12,13] are calculated to be 0.7315 and 0.2685 respectively. The importances of the two parameters calculated using the new methods are $E(\lambda_1)=1.9542\times10^{-8}$, $E(\lambda_2)=8.8744\times10^{-9}$, $V(\lambda_1)=6.8487\times10^{-17}$, and $V(\lambda_2)=2.3324\times10^{-17}$. The importance of λ_1 is 2.7244 times the importance of λ_2 obtained using the method of Do Van et al.[12,13] that is based on the perturbation of the parameters at time zero, while according to Figure 10, the importance of λ_1 decreases due to its perturbation in the other time points that is considered in the new measures to obtain the importance of λ_1 about 2.2 times of the importance of λ_2 using (12). According to the results of Figure 10 and the interpretation method provided in section IV and used in the early paragraphs of this section, λ_1 is more important than λ_2. Furthermore, according to the sequence of failure that is imposed by the cold spare gate to the system of Figure 9, whenever grid supply fails later, the diesel supply component is operated later and has less time for failure, so the whole system has less chance to fail in the system mission time. The same condition happens in the PAND gate of Figure 10. These conditions are completely captured by the new idea, to obtain increasing importance for λ_1 by decreasing the perturbation time.

To validate the ideas for a general real world system, the system of Figure 9 is again considered wherein the cold spare gate is substituted with a hot spare gate and the failure function of the grid supply and the control circuitry are changed to: Weibull distribution with scale parameter 500 and shape parameter 2, normal distribution with mean parameter 600 and standard deviation parameter 10. System failure and functioning simulation, and the calculation of system downtime percent over the mission time of 2×10^6 hours are performed based on the Monte Carlo method of Rao et al.[35] The simulation results are in Figure 11, with same notations and interpretations as Figure 10. Figure 11 shows the system downtime percent variation based on the different perturbation times of the grid supply scale parameter, the diesel supply failure rate, and the control circuitry mean parameter. The importances of the mentioned parameters using (12) are 0.1363×10^{-3}, 0.0764×10^{-3}, and 0.0596×10^{-3} while using (13) the importances are obtained 0.1373×10^{-7}, 0.0416×10^{-7}, and 0.0478×10^{-7}, respectively. According to the interpretation method of section IV, the scale parameter of the grid supply is the most important parameter while the control circuitry mean parameter is the second important one.

To compare the new measures and the other importance measures, we consider the unavailability of the system of Figure 12.

In Figure 12, non-repairable components 1 and 2 have exponential PDFs $f_1(t)=\lambda_1 e^{-\lambda_1\times t}$, $f_2(t)=\lambda_2 e^{-\lambda_2\times t}$ respectively, wherein $\lambda_1=0.05$ failures per hour, and $\lambda_2=0.04$ failures per hour. The system unavailability at time *t* is:

$$F(t)=F_1(t)\times F_2(t) \tag{14}$$

In (14), $F_j(t)=1-e^{-\lambda_j\times t}$ is component *j* unavailability at time *t*. Table 2 presents the formulas of the IMs for computing the importance of component *j*, wherein the source column provides the reference for formulas' explanations and notations. The failure rates are considered constant because the IMs of Table 2 (except the two new IMs) are proposed for constant risk model parameters. So, only the effect of perturbation occurrence time is analyzed here, and not its interaction effect with time-varying parameters which was studied in the previous examples. The value of the perturbation in the components unavailabilities is $\varepsilon=0.00001$.

For comparing the IMs, their performance metrics, other than those of the new measures, are multiplied by the perturbation value, because the new measures are based on the difference between the performance metric in the normal and the perturbed conditions while the other measures divide the difference by the perturbation value. The results of calculating the equations of Table 2 for the system of Figure 12 are presented in Figures 13 to 15.

Figure 13, A to C shows the plots of I_B, I_{B-P}, and I_N for the two components over time. Note that in the literature I_{B-P} and I_N are calculated for an infinite mission time, while we have calculated them at different mission times to have a more clear understanding of them. The derivative-based global sensitivity measures of the two components are presented in Figure 14, A to F. The two new measures are presented in Figure 15, A and B.

As the results show, the global IMs and the Natvig measure show almost constant behavior in time for both components, while the other measures depend on time. Figure 13, A and B and Figure 15, A show different value and trend for *E* in comparison with those of I_B^j and I_{B-P}^j for both components. This happens because I_B^j calculates the importance of each component at time *t* considering a perturbation in their failure time PDFs at time *zero* and I_{B-P}^{j} just simply calculates the average of I_B^j, while measure *E* at time *t*, averages the different importances of the component at time *t* each of which obtained assuming different perturbation occurrence times. It is evident that in the system of Figure 12, whatever a perturbation occurs later in a component failure time PDF, its effect on the system unavailability at time *t* decreases (because of constant parameters and absence of dynamic gates), which results in the smaller value of *E* than that of I_B^j and also a decrease in *E* for long mission times that is analyzed in the following paragraphs. So while some IMs are approximately time independent, and time dependent IMs like I_B^j and I_{B-P}^{j} do not consider the perturbation occurrence time, this concept is considered in *E*, then if the perturbation time distribution (i.e. PDF of shocks occurrence time) exists, we can apply that to obtain the weighted forms of (12) and (13), wherein $w(t)$ represents the PDF of the perturbation time. In (15) and (16) the probability that a perturbation occurs at time *t* is multiplied by its importance.

$$E(\lambda_i)=\int_0^T w(t)\left(\frac{\int_0^T F_{\lambda_i,t}(\tau)d\tau}{T}-\frac{\int_0^T F(\tau)d\tau}{T}\right)dt \tag{15}$$

$$V(\lambda_i)=\int_0^T\left(w(t)\left(\frac{\int_0^T F_{\lambda_i,t}(\tau)d\tau}{T}-\frac{\int_0^T F(\tau)d\tau}{T}\right)-E(\lambda_i)\right)^2 dt \tag{16}$$

For example, Figure 16 considers $w_1(t)=0.1\times e^{-0.1\times t}, 0\le t$ for the perturbation of the component 1 and $w_2(t)=0.02\times e^{-0.02\times t}, 0\le t$ for that of the component 2 of the system of Figure 12.

Therefore, based on (15) and (16) that capture the probability and the magnitude of the effects simultaneously, the plots of the new measures in different mission times are obtained as Fig. 17, A and B. Fig. 17, A and B show that unlike Fig. 15, A and B, component 1 is more important than component 2, because the probability that a perturbation occurs in component 1 during the early hours of the system operation is more than that of the component 2 according to the PDFs of Fig. 16, and in the system of Fig. 12 the perturbations that occur earlier have higher effect on the importance of their respective components. Considering higher effect and occurrence probability for the early perturbations of component 1 in (15) and (16), component 1 is more important that component 2 in Fig. 17, A and B unlike Fig. 15, A and B.

Figure 15, A shows another characteristic of the new measure, that is, reduction in the importance of the components in system mission time more than 50 hours. The reason is that the unavailability of the system is very high at long mission times, whether in perturbed or unperturbed conditions. Therefore, perturbing the components failure time PDFs (especially at times close to the mission time) has very small effect, i.e. importance, on the system unavailability at long mission times. Consequently, *E* will decrease, because it is the average of all these effects from *zero* to the mission time. This phenomenon does not happen in Fig. 17, A. The reason is that the perturbations with the highest effect on the system performance metric, i.e. the perturbations that occur in the early times of the system mission time, have higher occurrence probability, due to Fig. 16, in comparison with the other perturbations. This higher occurrence probability is multiplied by their respective perturbations' high effect on the system performance metric using (15) to concurrently capture the effect of magnitude and possibility of the effects and to produce non decreasing Fig. 17, A. Figure 15, B shows that the variation in the effect of different perturbation times on the system unavailability at long mission times is higher for component 2 than for component 1, which means more unstable behavior of the system unavailability due to a perturbation in the unavailability of the component 2.

7 | CONCLUSION

In this paper, we have investigated the importance of system risk model parameters with respect to system unavailability, considering two assumptions: 1- a perturbation in the values of the parameters can occur at any time during the system mission time, 2- the parameters values are functions of time. We have showed, using simulation and analytical formulation, that the time of parameter value perturbation has undeniable effects on its importance. It has also been shown, that the magnitude of the effects depends on the perturbation time of the parameter value, and the parameter value function in the mission time. Besides, it has been illustrated that while a parameter, due to a perturbation in a time instance, can be more important than another one at time *t*, this order can be reversed if the perturbation time changes. Furthermore, two new importance measures have been introduced to consider the two assumptions, and a simulation scheme has been proposed for their computation. All claims and measures were examined and confirmed using a non-homogeneous continuous time Markov chain of a series-parallel system and a real world case study, while the validation of the ideas for general models were indicated using the simulation of a real world system having non-exponential failure rates. The new measures were compared with some traditional IMs, to show their capability of considering the effects of parameters perturbation occurrence time in their importances, while extending new efficient IMs to consider the ideas can be studied in future researches.

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APPENDIX | The relationship between system average unavailability and $UA_{\lambda_i}^t$

$F(t)$ is the sum of the state probabilities of a Markov chain model at time *t*, wherein the system is unavailable.[11-13] The system average unavailability in a time period $[0,T]$ can be obtained using (17)[12] below:

$$\bar{F}(T)=\frac{1}{T}\int_0^T F(t)\,dt \tag{17}$$

In (17), $F(t)dt$ is a multiplication of system unavailability at time *t*, i.e. $F(t)$, and an infinitesimal time period, i.e. dt. So, $F(t)dt$ is the system expected downtime in time period dt. Therefore, considering the definition of the integral, $\int_0^T F(t)\,dt$ is the system expected downtime in time period $[0,T]$. Thus, one can conclude that $\frac{1}{T}\int_0^T F(\tau)\,d\tau$ is equal to $SDP_{failure}^{T}$. By the same logic $SDP_{failure}^{T,\lambda_i,t}$ is equal to $\frac{1}{T}\int_0^T F_{\lambda_i,t}(\tau)\,d\tau$. Therefore, $UA_{\lambda_i}^t$ is equal to the change in system average unavailability when λ_i is perturbed at time *t*, i.e. $UA_{\lambda_i}^t=\frac{1}{T}\int_0^T F_{\lambda_i,t}(\tau)\,d\tau-\frac{1}{T}\int_0^T F(\tau)\,d\tau$. Accordingly, when one uses $UA_{\lambda_i}^t$ to analyze the results, $\frac{1}{T}\int_0^T F_{\lambda_i,t}(\tau)\,d\tau-\frac{1}{T}\int_0^T F(\tau)\,d\tau$ can be intended. Obviously, as the number of considered perturbation times increases, a more accurate estimation for (12) and (13) can be obtained using $UA_{\lambda_i}^t$.

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TABLE 1 Description of Figure 6.

Node	State	Failed components in each state of Figure 6	System description
1	1	No component is failed	Working
2	2	Component 1 or component 2	Working
3	3	Component 3 or component 4	Working
4	4	(Component 1 or component 2) and (Component 3 or component 4)	Working
5	5	Component 1 and component 2	Failed
6	6	Component 3 and component 4	Failed
7	7	(Component 1 and component 2) and (Component 3 or component 4)	Failed
8	8	(Component 1 or component 2) and (Component 3 and component 4)	Failed

TABLE 2 IMs that are used for calculating the components importances of the system in Figure 12.

Measure	Source	Formula
Birnbaum lifetime IM	Kuo and Zhu[37]	$I_B^j(t)=\dfrac{\partial F(t)}{\partial F_j(t)}$
Derivative-based global sensitivity measures	Kuo and Zhu[37]	$I_{D\mu}^j=\int_0^1 \dfrac{\partial F(t)}{\partial F_j(t)}dF_j(t)$
		$I_{D\tilde{\mu}}^j=\int_0^1 \left\lvert\dfrac{\partial F(t)}{\partial F_j(t)}\right\rvert dF_j(t)$
		$I_{D\sigma}^j=\left[\int_0^1 \left(\dfrac{\partial F(t)}{\partial F_j(t)}-I_{D\mu}^j\right)^2 dF_j(t)\right]^{1/2}$
		$I_{D\tilde{\sigma}}^j=\left[\int_0^1 \left(\left\lvert\dfrac{\partial F(t)}{\partial F_j(t)}\right\rvert-I_{D\tilde{\mu}}^j\right)^2 dF_j(t)\right]^{1/2}$
		$I_{D^2}^j=\int_0^1 \left(\dfrac{\partial F(t)}{\partial F_j(t)}\right)^2 dF_j(t)$
		$I_{D_A^2}^j=\int_0^1 \left(\dfrac{\partial F(t)}{\partial F_j(t)}\right)^2 \dfrac{1-3F_j(t)+3F_j(t)^2}{6}dF_j(t)$
Barlow-Proschan IM	Bergman[38]	$I_{B-P}^j=\int_0^\infty \left[h(1_j,F(t))-h(1_j,F(t))\right]f_i(t)dt$
Natvig IM	Norros[39]	$I_N^j=\dfrac{E(Z_j)}{\sum_{i=1}^n E(Z_i)}$ where $E(Z_j)=\int_0^\infty \sum_{(\cdot_j,x)}\prod_{j\neq i}F_j(t)^{x_j}(1-F_j(t))^{1-x_j}\int_0^\infty\left[E\left(\vec{H}_t^{(1_j,x)}(u)\right)-E\left(\vec{H}_t^{(0_j,x)}(u)\right)\right]du\, f_j(t)dt$ and $\vec{H}_t^X(u)=\left(H_{1,t}^{x_1}(u),\ldots,H_{n,t}^{x_n}(u)\right),H_{j,t}^1(u)=\dfrac{F_j(t+u)}{F_j(t)},H_{j,t}^0(u)=0$
$E(F_j(t))$	Using (3) and (12)	$E(F_j(t))=\dfrac{\int_0^t \left(F_{\lambda_i,t'}(t)-F(t)\right)dt'}{t}$
$V(F_j(t))$	Using (3) and (13)	$V(F_j(t))=\dfrac{\int_0^t \left(F_{\lambda_i,t'}(t)-F(t)-E(F_j(t))\right)^2 dt'}{t}$