Hybrid Probabilistic-Possibilistic Treatment of Uncertainty in Building Energy Models: A Case Study of Sizing Peak Cooling Loads

Fazel Khayatian[1*], Maryam MeshkinKiya[2], Piero Baraldi[3], Francesco Di Maio[4], Enrico Zio[5]

[1]PhD Student, Department of Architecture Built Environment and Construction Engineering, Politecnico di Milano, Via Ponzio, 31, 20133 Milano, Italy. E-mail: fazel.khayatian@polimi.it
[2]PhD Student, Department of Architecture Built Environment and Construction Engineering, Politecnico di Milano, Via Ponzio, 31, 20133 Milano, Italy. E-mail: maryam.meshkinkiya@polimi.it
[3]Associate Professor, Department of Energy, Politecnico di Milano, Via Ponzio, 34/3, 20133 Milano, Italy. E-mail: piero.baraldi@polimi.it
[4]Assistant Professor, Department of Energy, Politecnico di Milano, Via Ponzio, 34/3, 20133 Milano, Italy. E-mail: francesco.dimaio@polimi.it
[5]Full Professor, Chaire Systems Science and the Energy Challenge, Fondation Electricite' de France, Laboratoire Genie Industriel, CentraleSupélec/Université Paris-Saclay, Grande voie des Vignes, 92290 Chatenay-Malabry, France; Full Professor, Department of Energy, Politecnico di Milano, Via Ponzio, 34/3, 20133 Milano, Italy. E-mail: enrico.zio@centralesupelec.fr; enrico.zio@polimi.it

Abstract

Optimal sizing of peak loads has proven to be an important factor affecting the overall energy consumption of HVAC systems. Uncertainty quantification of peak loads enables optimal configuration of the system by opting for a suitable size factor. However, the representation of uncertainty in HVAC sizing has been limited to probabilistic analysis and scenario-based cases, which may limit and bias the results. This study provides a framework for uncertainty representation in building energy modeling, due to both random factors and imprecise knowledge. The framework is shown by a numerical case study of sizing cooling loads, in which uncertain climatic data is represented by probability distributions and human-driven activities are described by possibility distributions. Cooling loads obtained from the hybrid probabilistic-possibilistic propagation of uncertainty are compared to those obtained by pure probabilistic and pure possibilistic approaches. Results indicate that a pure possibilistic representation may not provide detailed information on the peak cooling loads, whereas a pure probabilistic approach may underestimate the effect of uncertain human behavior. The proposed hybrid

[*] Corresponding author. Fax: (+39)23999486.
E-mail address: fazel.khayatian@polimi.it

representation and propagation of uncertainty in this paper can overcome these issues by proper handling of both random and limited data.

1. Introduction

Energy efficient building design merits special attention as the construction sector holds the largest share of energy consumption in most countries (Birol 2010). The magnitude of energy consumption by the building sector has resulted in governmental concerns that has led to implementing global and national regulations for promoting energy efficiency in buildings (Guillén-Lambea, Rodríguez-Soria, and Marín 2016, Allouhi et al. 2015). To comply with these regulations, new buildings are designed with special attention to both indoor comfort and energy efficiency, while existing buildings undergo retrofits at envelope and systems levels. In either case, this practice is associated with careful (re)design of the Heating Ventilation and Air-Conditioning (HVAC) systems. Indeed, HVAC design is very sensitive to the implementation of optimal temperature and humidity control, which may account for up to 60% of the total electric energy consumption of a building (Pérez-Lombard, Ortiz, and Pout 2008, Zhao et al. 2013, Vakiloroaya et al. 2014). Studies show that cooling loads dominate the majority of HVAC energy consumption in office buildings (Wan Mohd Nazi et al. 2017) and optimal configuration of chillers can result in substantial energy saving (Salari and Askarzadeh 2015).

The first necessary step for optimal design of HVAC system (that eventually results in the optimal configuration of chillers/boilers) is to quantify the peak load on the heating/cooling system, which is commonly known as the sizing process. Sizing the cooling system is frequently conducted according to the ASHRAE (Mitchell and Braun 2013) procedure that estimates peak loads by means of a nominal day (Design-Day) representing the hottest climatic conditions throughout a year. The calculation procedure commonly known as the Radiant Time Series method is a simplified approximation of the Heat Balance method. This procedure is among the most conventional approaches for estimating peak cooling loads as it is reliable, easy to interpret, computationally inexpensive and is accompanied by climatic design conditions for thousands of locations around the world (Schuetter, DeBaillie, and Ahl 2014). However, the ASHRAE procedure is based on deterministic and conservative assumptions, which overlook the uncertainty in environmental and occupant-related variables. It is argued that this approach results in overestimating the peak loads (Djunaedy et al. 2011), since a common practice is to apply size factors to the calculated cooling loads to reduce the risk of an undersized system. Inadequate size factors can cause the HVAC system to rarely reach the intended load and result in inefficient energy performance (Yik et al. 1999).

Optimal characterization of the system by accounting for uncertainty in input quantities can be a reliable alternative to experimental-based application of safety factors. Various studies have considered uncertain quantities in the problem of sizing HVAC systems and calculating peak cooling loads (Sun, Huang, and Huang 2015, Cheng, Wang, and Yan 2016, Shen and Sun 2016, Gang, Wang, Xiao, et al. 2015, Burhenne et al. 2013, Yıldız and Arsan 2011, Cheng et al. 2015, Lee and Schiavon 2014, Cheng et al. 2017). This has allowed more accurate cooling load best estimates (Mui and Wong 2007), optimal cost-energy design (Rasouli et al. 2013) and ideal configuration of the cooling systems (Gang, Wang, Shan, et al. 2015). In (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010), the authors propose a method for calculating peak loads based on stochastic simulation, showing that calculating peak cooling loads while considering uncertainty can reduce the risk of oversizing the HVAC system. In (Sun et al. 2014), a new framework for sizing the HVAC systems considering uncertainty is introduced, combining actual weather data and random sampling of other uncertain variables to obtain the peak loads.

In the context of building energy modelling dissimilar levels of information are available for different uncertain input quantities, which should be handled with their respective appropriate representations (Wang et al. 2016, Corotis 2015). Considering for example, the uncertainty associated with occupancy, in some cases, measured historical data are available and occupancy patterns are represented through Markov Models (Page et al. 2008, Richardson, Thomson, and Infield 2008, Wang, Yan, and Jiang 2011, Tahmasebi and Mahdavi 2015) or clustered into a number of scenarios (D'Oca and Hong 2015, Miller, Nagy, and Schlueter 2015). Whereas in other cases, limited historical data on the peak number of occupants were treated by probability density functions (Eisenhower et al. 2012, Azar and Amoodi 2016, Kim 2016). As a result, representing occupant-related uncertainty in building energy simulation calls for a change of perspective towards a *fit-for-purpose* treatment (Gaetani, Hoes, and Hensen 2016). This challenge is specifically important in peak load calculations, as it can result in (under)oversizing the HVAC system.

In the present work, we distinguish between two types of uncertain quantities: (1) those affected by stochastic uncertainty, such as climatic parameters, whose randomness is due to their inherent variability, and (2) those affected by epistemic uncertainty, such as internal gains, whose uncertainty is due to lack of knowledge and information (Dubois and Prade 2009).

Stochastic uncertainty is typically represented by probability distributions whose parameters are estimated using experimental (Oberkampf et al. 2002). For example, large amount of data collected form weather stations (e.g.

temperature, relative humidity and wind speed) are available for estimating the parameters of the probability distributions representing the stochastic uncertainty of the climatic quantities. Specifically, in this study the probability distributions representing uncertainties in the urban microclimatic are based on a large dataset of hourly climatic data, collected from a weather station with close proximity to the studied building.

With respect to the epistemic uncertainty, in some cases, laboratory experiments are performed to quantify the uncertainty of the physical properties of building components (e.g. thermal conductivity, solar heat gain coefficient, moisture content, specific heat and mass). The repetition of experiments allows for a reliable information representation, properly characterizing the quantity variability. In other cases where very scarce information is available (e.g. internal gains of an unoccupied building which may be still in the design phase or evacuated for restoration) one may resort to the elicitation of expert knowledge to represent uncertainty. Expert elicitation is often of ambiguous quality in nature, and, therefore, may be difficult to describe through probability distributions. Let us, for example, consider the case, in which we are aware of the minimum and maximum values of an uncertain quantity. Since this information does not imply that the probability of occurrence of all intermediate values is the same, the use of a uniform probability distribution is questionable (Klir 1994). Similarly, the knowledge of the minimum, maximum and most probable values of an uncertain quantity, does not allow the use of a triangular probability distribution. To our opinion, the literature of building energy modelling have frequently (and inappropriately) represented the uncertainty on epistemic quantities, for which scarce knowledge is available through probability distributions (Tian 2013, D'Oca, Hong, and Langevin 2018).

Possibilistic representation of scarce information is an alternative to the conventional probabilistic quantification of uncertainty (Parsons 1996). This type of representation is particularly helpful in quantifying the uncertainty associated with incomplete knowledge, where opting for probability distributions may distort the actual information. In the practice of building energy modelling, a framework for handling both probabilistic and possibilistic representations of uncertainty is necessary.

In this study, we describe different representations of uncertainties involved in the problem of sizing HVAC loads, in support of a successive optimal design of the HVAC system. To handle both probabilistic and possibilistic uncertainty representations, we resort to a hybrid uncertainty propagation method (Guyonnet et al. 2003). A homogeneous post-processing approach is introduced to process the outputs obtained by the hybrid uncertainty propagation. To highlight the effectiveness of the hybrid method, fully probabilistic and fully possibilistic treatments of the uncertainties are presented in a comparative numerical case study.

The main original contributions of this study include:

- Introducing a possibilistic representation of occupant-related uncertainty in building energy modelling.
- Introducing the hybrid uncertainty treatment method for joint propagation of uncertainties represented by probability distributions (i.e. climatic data) and possibility distributions (i.e. internal gains).
- Contrasting the advantages and drawbacks of pure probabilistic and pure possibilistic treatments of uncertainty, compared to the introduced hybrid method.

The paper is structured as follows: Section 2 provides a detailed description of the possibilistic representation of scarce knowledge and introduces the hybrid method for uncertainty propagation. Section 3 applies the hybrid method to a case study of sizing cooling loads for an office building, and demonstrates the results of the presented method in comparison with pure probabilistic and pure possibilistic representations. Section 4 draws the conclusions and provides suggestions for future work.

2. Possibilistic representation of uncertainty

Uncertainty can be categorized into two classes, i.e. aleatory and epistemic. Aleatory uncertainty deals with randomness due to inherent variability in the system behavior (e.g. outdoor temperature fluctuation), while epistemic uncertainty is derived from lack of knowledge on the process or system (e.g. the state of an HVAC system) (Zio 2013). For example, lack of accessible information on the value of a quantity, which enters as a parameter of the system or process model, can result in epistemic uncertainty (e.g. due to difficulties in collecting accurate measurements or the lack of time for data collection).

Although one may argue that probability theory is sufficient for handling both aleatory and epistemic uncertainty (Lindley 1987, Zadeh 2008), recent studies have challenged the probabilistic framework, highlighting its limitations in representing incomplete knowledge (Dubois and Prade 2001, Cobb and Shenoy 2003, Haenni and Lehmann 2003). Studies have reasoned that a fully probabilistic approach can distort the actual scarce knowledge and impact the calculations obtained from the model (Dubois, Prade, and Smets 1996, Kohlas and Monney 2013). It has been shown that misrepresenting epistemic uncertainty - as a result of incomplete knowledge - can lead to faulty intuitions on the system's reliability (Chen et al. 2016, Zhang et al. 2017), and therefore, imprecise probabilistic frameworks have been introduced to properly handle both classes of uncertainty(Rocchetta and Patelli 2016). Take for example the uncertainty affecting the quantification of internal gains in a building, which is commonly represented by means of uniform, triangular or Gaussian probability density functions (Eisenhower et al. 2012, Hopfe and Hensen 2011, Heo, Choudhary, and Augenbroe 2012, Azar and Amoodi 2016, Kim 2016). Adopting a uniform probability distribution for occupant density does not correctly characterize the uncertain parameter, as we are not in complete ignorance of the number of occupants. On the other hand, assigning triangular or Gaussian distributions to occupant density – based on linguistic propositions of the number of occupants – will misrepresent the scarce information, as we do not know the frequency of occurrence, but rather, a range within which occupant density may vary (Cooper, Ferson, and Ginzburg 1996, Baudrit, Dubois, and Perrot 2008). In this study, we seek a reliable alternative for probabilistic treatment of epistemic uncertainty in building energy modelling, namely, a framework that can faithfully represent the imperfect knowledge on occupant behavior without distorting the information.

This goes under the framework of possibility theory for representing epistemic uncertainty (Zadeh 1999), where a possibility distribution value $\pi(x) \in [0,1]$ is allocated to each real value x in the range X. Expressing $\pi(x) = 0$, indicates that the value x is considered impossible, whereas $\pi(x) = 1$ implies that at least one interpretation of the value x is completely possible. Take for example the number of occupants in a bank at 10:00 a.m. of weekdays, where 10 employees work full-time and 7 to 10 visitors are anticipated. In this example, observing less than 10 occupants is unexpected and surprising i.e. $\pi(x) = 0$, while encountering 17 occupants is considered normal and the routine state of affairs i.e. $\pi(x) = 1$. Any number of occupants between 10 and 17, as well as 17 to 20, is characterized with a degree of certainty i.e. $0 < \pi(x) < 1$. According to the theory of possibility, the likelihood of an event A is described by two limiting measures, the possibility Π and the necessity N, defined as (Dubois et al. 2000):

$$\Pi(A) = \sup_{x \in A} \pi(x) \qquad \text{(Eq.1)}$$

$$N(A) = 1 - \Pi(\overline{A}) = \inf_{x \notin A} (1 - \pi(x)). \qquad \text{(Eq.2)}$$

Let $\mathcal{P}(\pi)$ be a family of probability distributions such that for any event A, the probability measure of that event $P(A)$ is within the assigned necessity and possibility limits, i.e. $N(A) \leq P(A) \leq \Pi(A)$; then,

$$N(A) = \inf P(A) \qquad \Pi(A) = \sup P(A) \qquad \text{(Eq.3)}$$

where the **inf**imum and **sup**remum probabilities represent the largest lower bound and the least upper bound of all probability measures in $\mathcal{P}$. This representation of uncertainty is particularly helpful when the available data is scarce or only the upper and lower bounds can be defined (e.g. uniform, triangular probability distributions). It is possible to transform a possibility distribution into a family of probability distributions (Figure 1). For this, a possibility distribution can be seen as a nested set of confidence intervals (Dubois and Prade 1992), which are the α-cuts of the distribution i.e. $[\underline{x}_\alpha, \overline{x}_\alpha] = \{x, \pi(x) \geq \alpha\}$. In this case, the necessity measure $N([\underline{x}_\alpha, \overline{x}_\alpha])$ gives the degree of certainty contained in the α-cuts $[\underline{x}_\alpha, \overline{x}_\alpha]$. Then, each interval is represented with a range of probability measures, such that $P(X \in [\underline{x}_\alpha, \overline{x}_\alpha]) \geq 1 - \alpha$ and $P(X \notin [\underline{x}_\alpha, \overline{x}_\alpha]) \approx \alpha$.

FIGURE 1. TRANSFORMATION OF POSSIBILITY DISTRIBUTION TO BELIEF FUNCTION.

2.1. Hybrid probabilistic-possibilistic uncertainty propagation

Uncertainty propagation is the process of numerically propagating the uncertainty associated to input quantities of the model to the outputs of that model. In this section, we describe how randomness (represented using probability distributions) and imprecision (represented using possibility distributions) can be jointly propagated (Guyonnet et al. 2003). Let us consider a model $Z = f(X_1, X_2, \dots, X_k, X_{k+1}, \dots, X_n)$, in which the output is a function of n uncertain quantities $X_i, i = 1,2, \dots n$. For ease of illustration, we consider that the first k quantities are aleatory with uncertainty represented by the probability distributions $p_{X_i}(x), i = 1,2, \dots, k$, whereas the remaining $n - k$ quantities are epistemic with uncertainty represented by the possibility distributions $\pi^{X_i}(x), i = k + 1, k + 2, \dots, n$. The procedure of propagating both types of uncertainty consists of two nested loops (Baraldi and Zio 2008): Monte Carlo sampling from the probabilistic distributions (outer loop) and approximation of the possibilistic distributions through α-cuts (inner loop). The following steps are to be performed:

1. A k dimensional vector of random realizations $(x_1, \dots, x_k)$ is generated by Monte Carlo sampling from the uncertain (probabilistic) quantities $(X_1, \dots, X_k)$.

2. α is set to zero and the related α-cuts of all possibility distributions $(\pi^{X_{k+1}}, \dots, \pi^{X_n})$ are found. The possibility distributions are intervals of possible values of the possibilistic quantities $(X_{k+1}, \dots, X_n)$.

3. The supremum and infimum values $[\overline{f}_\alpha, \underline{f}_\alpha]$ of $f(x_1, \dots, x_k, X_{k+1}, \dots, X_n)$ are calculated, where $(x_1, \dots, x_k)$ is the vector of Monte Carlo-sampled probabilistic quantities from step 1, and $(X_{k+1}, \dots, X_n)$ are the values of the possibilistic quantities obtained from step 2.

4. A small increment $(\Delta\alpha)$ is added to the value of α (e.g. $\Delta\alpha = 0.05$) and the new α-cuts are found.

5. Steps 3 and 4 are repeated while $\alpha \leq 1$.

6. Steps 1 to 5 are repeated until the desired number of Monte Carlo samples (m) are generated.

The outcomes of this procedure are m random realizations of n $\left(n = \left(\frac{1}{\Delta\alpha}\right)\right)$ possibility measures i.e. $(\pi_1, \dots, \pi_m)$. It is worth noting that the number of realizations ($m \,.\, n$) should come from a tradeoff between computational cost and desired accuracy in the uncertainty description. Choosing a large value for $\Delta\alpha$ will fail to adequately describe the possibilistic representations, while selecting a small value can result in a considerable increase in computation time. Similarly, a small value of m could fail to appropriately define the probabilistic representations, while a large value could lead to a large computational time.

Notice that a two nested loops uncertainty propagation scheme has been adopted in the context of building energy modelling (de Wilde and Tian 2009), but in a different case in which all uncertainties are treated probabilistically.

2.2. Post-processing analysis of uncertain model outputs

The outcomes of the hybrid procedure i.e. the m possibility distributions $(\pi_1, \dots, \pi_m)$ can be combined by using different methods, such as separate affectation of probability and possibility descriptions (Dubois, Fargier, and Fortin 2005), fuzzy prediction interval method (Guyonnet et al. 2003) and homogenous post-processing (Baudrit, Dubois, and Guyonnet 2006). The homogeneous post-processing method provides a tradeoff between the other two techniques and is based on the use of evidence theory, which is based on the allocation of basic probability assignments denoted by $v(E_i)$ to subsets ($E_i = 1,2, \dots, K$) of the uncertain quantity domain, with

$\sum_{i=1}^{K} v_i = 1$ (Shafer 1976). From the basic probability assignments $v(E_i)$, it is possible to obtain the belief (Bel) and plausibility (Pl) measures:

$$Bel(A) = \sum_{E,E \subseteq A} v(E_i) \qquad \text{(Eq.4)}$$

$$Pl(A) = \sum_{E,E \cap A \neq \emptyset} v(E_i) = 1 - Bel(\bar{A}). \qquad \text{(Eq.5)}$$

$Bel(A)$ measures the degree of belief that A will occur, while $Pl(A)$ measures the extent to which A evidence does not support the negation of A (Mauris et al. 2000). Notice that the possibility theory is a special case of the evidence theory through which we can interpret belief and plausibility measures as probability bounds similar to what is done in the possibility theory for the interpretation of the necessity and possibility measures (Eq. 1 - Eq.2). Therefore, the probability that A lies within the belief and plausibility interval satisfies:

$$\forall P \in \mathcal{P}, Bel(A) \leq P(A) \leq Pl(A). \qquad \text{(Eq.6)}$$

Since the basic probability assignments, v, of evidence theory generalizes both probability and possibility distributions, evidence theory provides a common framework for the hybrid modelling of uncertainty using both probability and possibility distributions. The homogeneous post processing method used in this work for combining the m possibility distributions $\pi_i, i = 1,2,..,m$, is based on the aggregation of the basic belief assignments $v_i, i = 1,2,..,m$, which can be obtained from the possibility distribution π_i (Masson and Denœux 2006), into a joint basic belief assignment. In (Baudrit, Dubois, and Guyonnet 2006) it is shown that this aggregation step can be performed by using the evidence theory laws and allows directly computing, for any subset A, the corresponding belief and plausibility measures:

$$Pl\ (A) = \frac{1}{m} \sum_{i=1}^{m} \Pi_i(A) \qquad \text{(Eq.7)}$$

$$Bel(A) = \frac{1}{m} \sum_{i=1}^{m} N_i(A) \qquad \text{(Eq.8)}$$

where $N_i(A)$ and $\Pi_i(A)$ are the necessity and possibility measures derived from the possibility distributions $(\pi_1, \ldots, \pi_m)$, and m is the number of Monte Carlo realizations.

3. A case study of cooling load design for an office building

The effectiveness of the hybrid propagation method is illustrated by evaluating the peak cooling loads of a 30'000 m^2 office building (Figure 2) located in the center of Milano (Italy), which is undergoing a Core and Shell retrofit. The renovations cover a wide range of properties i.e. internal architectural layout, building's envelope (walls, windows, and roofs), and both electrical and mechanical systems. The logic behind choosing the current building as the case study is that both aleatory and epistemic classes of uncertainty are available due to the in situ conditions of the building. Aleatory uncertainty is unavoidable due to the randomness in climatic data. Epistemic uncertainty is inevitable as the building is evacuated for renovation purposes, and therefore, the only information on occupant behavior is accessible through experts' knowledge on occupancy patterns. The building serves as the headquarters of a bank, consisting of six stories and two courtyards, as well as a ten story tower. To keep the simulation time below 30 seconds per simulation (i.e. the maximum time available to run the numerous simulations needed for performing the following analysis), each story is modelled as single thermal zone, except for the ground and fifth floors, which account for 3 and 2 thermal zones, respectively, giving rise to a 13 thermal zone model, because, to some extent, they have non-continuous (detached) thermal boundaries. Non-occupied spaces (i.e. toilets, staircases, and hallways are modelled as non-conditioned zones (Figure 3). Server rooms and Uninterrupted Power Supply rooms are also modelled as non-conditioned zones, as cooling for these spaces will be provided by separate Variable Refrigerant Flow (VRF) units. The surface to volume ratio of the building is 0.39, with a window to wall ratio of approximately 42 %. The properties of the renovated envelope components, are displayed in Table 1. Schedules of occupancy, lighting system, electric equipment and the HVAC system operation for the Design-Day calculation are provided in Table 2. It should be noted that keeping the number of thermal zones low (equal to 13) may lead to an underestimation of the peak cooling load, introducing a systematic error into the results obtained with the application of the uncertainty propagation methods that will follow. As a matter of fact, reliance on a detailed model (with 143 zones) and sizing peak cooling loads based on (already mentioned) overconservative ASHRAE method (with 0.4% summer design condition) returns a peak cooling load equal to 615.3 kW, which is 3.2% larger than the 595.9 kW peak cooling load that is obtained using the simplified model (with 13 zones) adopted in this study.

FIGURE 2. THE CASE-STUDY BUILDING AS MODELLED IN DESIGNBUILDER SOFTWARE.

FIGURE 3. ZONING SCHEMA OF A SAMPLE FLOOR. THE SHADED ZONES ARE CONSIDERED AS NON-CONDITIONED SPACES.

TABLE 1. ENVELOPE PROPERTIES OF THE CASE STUDY BUILDING (AFTER RETROFIT).

TABLE 2. PRESUMED PROPERTIES OF INTERNAL GAINS AND HVAC SYSTEMS. *THE METABOLIC RATE IS CONSIDERED TO BE 120 W/PERSON FOR LIGHT OFFICE WORK.

This latter cooling load (595.9 kW) follows the conventional Design-Day method as described in the ASHRAE Handbook of Fundamentals (HoF) (Handbook 2009), and is used as the baseline for comparison. In this method, a 24 hour temperature profile (representing the hottest day of the year) is generated based on the ASHRAE's "*Fraction of Daily Temperature Range*" Table. ASHRAE argues that the daily temperature variation is driven by the heat from the sun, therefore, the table assigns a series of weights (i.e. 24 weights corresponding to each hour of the day) to a single dry bulb temperature. This process generates a sinusoidal temperature profile that reaches minimum at early morning and hits peak in the afternoon. In ASHRAE's Design-Day method, a single dry bulb temperature value is chosen from the "*Climatic Design Conditions*" Table, which is published for each climate by ASHRAE. In this study however, ASHRAE's deterministic Design-Day dry bulb temperature is replaced with randomly sampled temperatures, whose probability of occurrence is obtained from a dataset gathered at a nearby weather station. Then, daily temperature profiles are created by applying ASHRAE's "*Fraction of Daily Temperature Range*" weights to the sampled dry bulb temperatures. A limitation of the method proposed in this study is that the weights used for generating daily temperature profiles are deterministic. Since the temperature transition from hour t to hour $t + 1$ in different days will always follow the same predefined pattern, the generated sinusoidal daily temperature profiles is an oversimplification of the reality, as it disregards the inevitable noise in the actual daily temperature fluctuation. Methods for simulating realistic time-series climatic profiles have been proposed in (Li and Zio 2012, Sansavini et al. 2014, Rocchetta, Li, and Zio 2015, Naseri et al. 2016), and can be considered for expanding the current study. The incident solar radiation on each surface is calculated based on the "*Clear-Sky Solar Radiation*" method in an hourly interval. ASHRAE's "*Clear-Sky Solar Radiation*" is the sum of beam, diffuse and ground reflected solar radiation received on each surface. The HoF design day sizing procedure provides climatic properties for calculating peak cooling loads, based on three possible choices of 0.4%, 1% and 2% design conditions. Each design condition is based on the maximum number of hours in a year, during which the HVAC system may not be able to fully maintain the desired internal conditions. The dry bulb temperature for 0.4% design condition can be obtained from the 0.4th percentile of the inverse cumulative distribution function of all temperatures that are collected during a 25 year span and scaled to one year. In other words, the 0.4% design condition tends to guarantee adequate cooling power for all conditions, except the most extremely hot 35 hours of a year (one year is considered to be 8760 hours).

The building is modelled using the DesignBuilder software (Tindale 2005), which performs sizing calculations according to the ASHRAE HoF procedure with the EnergyPlus calculation engine (Trčka and Hensen 2010, Crawley et al. 2001). The JEPlus simulation manager is utilized to facilitate the rapid initialization of the EnergyPlus software (Zhang 2009, Zhang and Korolija 2010, Zhang 2012). The same quantities selected in (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010, Sun et al. 2014) through a sensitivity analysis of the variables affecting the peak cooling loads have been considered as affected by uncertainty. Although the analysis in (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010, Sun et al. 2014) refers to different buildings and cities, the results of the sensitivity analysis are applicable to the current case study, given the similarity between the two buildings in terms of comparable characteristics, recurrent internal gain profiles and climatic design properties (Table 3). Note that the identification of the quantities whose uncertainty should be treated within the analysis is typically a complex process, which in a general case should be based on the application of sensitivity and uncertainty analysis techniques. Readers interested in this topic may refer to (Das et al. 2014) in the context of probabilistic uncertainty representation, (Zahiri, Tavakkoli-Moghaddam, and Pishvaee 2014) in the context of possibilistic uncertainty representation, and (Singh and Markeset 2014) in the context of hybrid probabilistic-possibilistic uncertainty treatment. The climatic variables with uncertain quantities consist of outdoor dry bulb temperature and relative humidity, while the internal gains are occupant-related variables, with uncertain quantities concerning the number of occupants (occupant density), lighting power and appliance power.

TABLE 3. ASHRAE CLIMATIC DESIGN PROPERTIES FOR MILANO COMPARED TO ATLANTA AND MALAGA

Concerning the former type of variables, HoF provides the required climatic information for sizing HVAC loads from databases of weather stations. For the current case study, the HoF resorts to Linate or Malpensa weather stations, where Linate is often more preferable considering its proximity to Milano city center, where the studied building is located. However, it is argued that weather stations located in open flat areas outside the city (e.g. airports), cannot adequately characterize the Urban Heat Island (UHI) intensity of the city center (Thevenard 2010, Paolini et al. 2016, 2017). To address UHI intensity in the cooling load calculations, in this work we use on-site climatic data from Brera weather station, which is located in the center of Milano city (Lombardia 2006). The extracted climatic data consist of dry bulb temperature and relative humidity. This data is used to calculate the design dry bulb temperature, the Mean Coincident Wet Bulb temperature (MCWB), the Mean Coincident Dry

Bulb temperature Range (MDBR) for 0.4%, 1% and 2% design conditions. Three "*Control Samples"* are proposed that correspond to 0.4%, 1% and 2% design conditions of Linate airport weather station. Similarly, 3 "*Case Studies"* are considered for Brera weather station corresponding to 0.4%, 1% and 2% design conditions (Table 4). It is found that for 0.4% design condition, the dry bulb temperature in city center is 0.9℃ higher than in Linate, while the MCWB and the MDBR in the city center are 1.9℃ and 1.5℃ lower, respectively. The obtained climatic differences between Linate airport and Brera weather station are in agreement with the UHI effects reported in literature (Thevenard and Cornick 2013).

The latter category of variables with uncertain quantities, namely occupant density, lighting and appliance power, are parameters that are difficult to quantify, as accurate measurements of human activities require a great amount of time and effort (Wang et al. 2016). In many cases, little to no information is available on occupant behavior and, therefore, designers often rely on nominal values provided by ASHRAE or occasionally resort to personal experience. Meanwhile, the reliability of ASHRAE recommended occupancy profiles for office buildings has also been subject to debate (Duarte, Van Den Wymelenberg, and Rieger 2013). Therefore, in this work, we sought experts' opinions on internal gains during the peak cooling load period. Experts suggested the presence of 1200 people during peak cooling loads, while recommending 150 kW and 180 kW for lighting and appliance power, respectively. The occupant density (0.04 person/m^2) suggested by the experts is slightly lower than the ASHRAE's nominal value for office spaces density (0.05 person/m^2). The presumed lighting power density (5 W/m^2) is notably lower than ASHRAE's recommendations for office spaces (10.5 W/m^2). This is due to the fact that the newly designed lighting configuration benefits from a fully dimmable LED system (2.2 - 9.6 W/m2). Since we are accounting for a clear sky in the ASHRAE Design-Day calculations and have access to adequate daylight, experts have set the lighting power density to almost half of the maximum available power.

TABLE 4. COMPARISON OF 0.4%, 1% AND 2% DESIGN CONDITIONS FOR LINATE AIRPORT AND BRERA CITY CENTER.

Considering that the process of sizing cooling loads for 0.4%, 1% and 2% design conditions follows a similar pattern, explaining each design condition individually is redundant. Therefore, in the following sections, the uncertainty quantification process is fully described for 0.4% design conditions, although it has been performed and analyzed for all three design conditions.

FIGURE 4. REPRESENTATION OF CLIMATIC VARIABLES WITH UNCERTAIN QUANTITIES: DRY BULB TEMPERATURE (TOP) AND MCWB TEMPERATURE (BOTTOM) THROUGH PROBABILISTIC

REPRESENTATIONS (BLUE-SOLID) AND THEIR TRANSFORMATION INTO POSSIBILISTIC MEASURES (RED-DASHED).

FIGURE 5. REPRESENTATION OF HUMAN-DOMINATED VARIABLES WITH UNCERTAIN QUANTITIES: OCCUPANT DENSITY (TOP), LIGHTING POWER (MIDDLE) AND APPLIANCE POWER (BOTTOM) THROUGH PROBABILISTIC REPRESENTATIONS (BLUE-SOLID) AND THEIR TRANSFORMATION INTO POSSIBILISTIC MEASURES (RED-DASHED).

3.1. Probabilistic representation

The most common approach for quantifying uncertainty in peak cooling load calculations is through a pure probabilistic approach. In this method, all measures (temperature, relative humidity, occupant density, lighting power and appliance power) are represented by means of a probability distribution.

Alongside UHI effects, the return period of extreme climatic conditions is also associated with uncertainty (Huang 2014). The collected weather data from Brera weather station cover a 17-year span from 1998 to 2015. Since this period may not be adequate to account for a reliable return period of 25 years, we incorporated a 0.5℃ standard deviation to the calculated dry bulb temperature of Brera station (Figure 4 - Top). This variation is recommended for 0.4% cooling design conditions at locations with climatic characteristics comparable to Milano (standard deviations of 0.4℃ and 0.3℃ have been suggested for 1% and 2% design conditions, respectively) (Thevenard 2010). The MCWB is calculated from the dry bulb temperature and relative humidity, and adopts a "Burr Type XII" probability distribution function (Figure 4 - Bottom) (Handbook 2009).

The probabilistic approach for characterizing scarce knowledge on occupant behavior is commonly characterized by probability density functions, either normal (Hopfe and Hensen 2011) or triangular (Heo, Choudhary, and Augenbroe 2012). Although, the experts recommended the presence of 1200 occupants, they did expect this value to vary between 1000 and 1250. Similarly, the peak lighting and appliance power are expected to have ±15 kW and ±30 kW variation respectively. Here, we adopt triangular probability distributions to characterize the uncertainties concerning internal gains (Figure 5).

A total of 1000 random samples have been generated by Latin hypercube sampling (Helton and Davis 2003) ($V^i = x_1^i, x_2^i, x_3^i, x_4^i, x_5^i, i = 1, \ldots, 1000$), and fed to JEPlus tool for sizing calculations. The output is a set of 1000 random cooling loads.

3.2. Possibilistic representation

A pure possibilistic approach is also studied. It is worth noting that this approach is not common in sizing HVAC loads. However, (Ruparathna, Hewage, and Sadiq 2017) performed a life cycle assessment of building energy retrofit by resorting to the theory of possibility. Furthermore, epistemic uncertainty has been implemented in HVAC control techniques by means of fuzzy parameters (Sunitha and Behera 2016, Keshtkar et al. 2015).

It is also important to note that using a possibilistic representation of an uncertain quantity takes a more conservative attitude. A possibility measure of 1 is a weaker statement compared to a probability of 1, as the possibilistic unit value states that the occurrence of an event is possible, expected and not surprising, whereas a probability of 1 states that the event is certain. For comparison purposes, we perform probability-possibility transformation to describe a probabilistic representation in possibility theory terms. Detailed descriptions on probability-possibility (possibility-probability) transformations can be found in (Dubois, Prade, and Sandri 1993, Dubois et al. 2004, Dubois, Prade, and Smets 2008, Flage et al. 2013). In this study, we adopt the Variable Transformation method described in (Mouchaweh et al. 2006), as it is easy to implement and provides a reasonable approximation of both representations. Figure 4 and 5 illustrate the possibilistic transformations of climatic variables and internal gains, respectively.

According to the theory of possibility, distributions may be transformed to a set of α-cuts (see section 2). The increment $\Delta\alpha$ is set to 0.001 and, therefore, 1001 values corresponding to lower bounds of each quantity, and another 1001 values as the upper limiting bounds are found. Eventually 2002 inputs are generated for all lower and upper bounds, where each input vector consists of 5 variables i.e. $\underline{V}^i = \underline{x}_1^i, \underline{x}_2^i, \underline{x}_3^i, \underline{x}_4^i, \underline{x}_5^i$ and $\overline{V}^i = \overline{x}_1^i, \overline{x}_2^i, \overline{x}_3^i, \overline{x}_4^i, \overline{x}_5^i,\ i = 1, \dots, 1001$. The inputs are fed into JEPlus tool and two sets of cooling loads are obtained. The outputs are presented through the possibility Π and necessity N measures.

3.3. Hybrid probabilistic – possibilistic representation

In this section, random (aleatory) variability is considered for climatic features i.e. dry bulb temperature and MCWB, which are described by probability distributions (see section 3.1). Since the building was under retrofit, not much information regarding human-dominated quantities can be obtained. In fact, resorting to experts' knowledge is the only way to obtain information on an evacuated building without any historical data on occupant behavior. This situation may be encountered in many cooling design scenarios, and therefore, dealing with scarce data merits careful consideration from two different points of view. First, the variability of occupant density and lighting/appliance power are prone to have dependencies between them. Therefore, representing each of these variables by a separate probability density function will neglect their dependencies and distort the original knowledge. Second, gathering information on human behavior can be associated with high imprecision, where the information may be categorized into an epistemic class. For this reason, the scarce data on occupant-related variables has been represented by possibilistic representations (see Section 3.2).

The vector of the input quantities which will be considered uncertain is formed by 5 features. Dry bulb temperature and MCWB uncertainty is described by probability distributions and the two quantities are indicated by X_1 and X_2, respectively, whereas occupant density, lighting power and appliance power uncertainty is described by possibility distributions and they are indicated by π^{X_3}, π^{X_4}, and π^{X_5} respectively. A set of $m = 1000$ random samples $\left(x_1^i, x_2^i\right), i = 1, \ldots, m$ are generated from their respective probability distributions $p_{x1}(x)$ and $p_{x2}(x)$. For the remaining 3 features (possibilistic quantities), the value of $\Delta\alpha$ is set to 0.01 and therefore 101 α-cuts are defined. The corresponding upper and lower boundaries of each α-cut are identified from the possibility distributions of occupant density (π^{X_3}), lighting power (π^{X_4}) and appliance power (π^{X_5}). One vector containing a temperature value (denoted x_1^1) and a MCWB value (denoted x_2^1) is selected from the matrix of m random samples. This vector is concatenated with the *lower limiting bounds* obtained from the first α-cut of the possibility distributions $\pi^{X_3}(x)$, $\pi^{X_4}(x)$ and $\pi^{X_5}(x)$, generating the vector $\underline{V}_1^1 = x_1^1, x_2^1, \underline{x}_3^1, \underline{x}_4^1, \underline{x}_5^1$. Inserting the vector $\underline{V}_1^1$ in the EnegyPlus *.idf script and executing the sizing calculation produces a single value, namely, the infimum bound of all plausible peak cooling loads that correspond to dry-bulb temperature x_1^1, MCWB x_2^1, and α-cut one (occupant density $\underline{x}_3^1$, lighting power $\underline{x}_4^1$ and appliances power $\underline{x}_5^1$). Similarly, we concatenate the same vector of random values (x_1^1 , x_2^1) with the *upper limiting bounds* of the first α-cut of possibility distributions $\pi^{X_3}(x)$, $\pi^{X_4}(x)$ and $\pi^{X_5}(x)$, generating vector $\overline{V}_1^1 = x_1^1, x_2^1, \overline{x}_3^1, \overline{x}_4^1, \overline{x}_5^1$. Once more, values of dry-bulb temperature, MCWB, occupant density, lighting power and appliance power is replaced in the EnergyPlus *.idf script with $\overline{V}_1^1$ and the sizing calculation is executed. The output of the second simulation returns the upper bound of all plausible peak cooling loads that correspond to dry-bulb temperature x_1^1, MCWB x_2^1 and α-cut one (occupant density $\overline{x}_3^1$, lighting power $\overline{x}_4^1$ and appliances power $\overline{x}_5^1$) . At this point we have obtained the two (upper and lower limiting) values, within which lies all plausible peak cooling loads corresponding to random vector i=1 (x_1^i , x_2^i) and α-cut j=1 (x_3^j, x_4^j, x_5^j). This procedure, based on the use of the extrema of the α-cuts of the epistemic quantities for the computation of the α-cut extrema of the corresponding peak cooling loads is derived from the intuition that the system's response to epistemic uncertain quantities (x_3^j, x_4^j, x_5^j) is monotonic, arguing that larger occupant density, lighting power and appliance power will result in larger internal heat gains, and therefore, larger cooling loads.

The process of obtaining the limiting bounds is repeated until the randomly generated vector *i*=1 (x_1^i , x_2^i) is concatenated with all n=101 lower bounds and n =101 upper bounds obtained from the α-cuts of the possibility distributions π^{X_3} , π^{X_4} and π^{X_5}. As a result, we obtain $\underline{V}_1^j = x_1^1, x_2^1, \underline{x}_3^j, \underline{x}_4^j, \underline{x}_5^j, j = 1, \dots, n$ and $\overline{V}_1^j = x_1^1, x_2^1, \overline{x}_3^j, \overline{x}_4^j, \overline{x}_5^j, j = 1, \dots, n$, through which 202 sizing calculations are executed. At this stage, we have defined all plausible ranges of peak cooling loads that correspond to temperature x_1^1, MCWB x_2^1 and all plausible internal gains. The same procedure is repeated m=1000 times for different random vectors of dry bulb temperature and MCWB (x_1^i , x_2^i), $i = 1, \dots, m$, and their corresponding n=101 α-cuts (x_3^j, x_4^j, x_5^j), $j = 1, \dots, n$, producing $m * n$ peak cooling loads for each limiting (upper/lower) bound. To facilitate this process through JEPlus, a batch of 1000 iterations are executed, each containing two sets of 101 simulations (1000*2*101 jobs). The described procedure is presented in the form of a flowchart in Figure 6. The flowchart consists of two loops, where the outer loop is responsible for generating random vectors and the inner loop handles the possibilistic variables. The two loops provide the n=101 α–cuts of m=1000 possibility distributions ($\underline{V_i^j}$, $\overline{V_i^j}$), $i = 1, \dots, m; \; j = 1, \dots, n$, which are aggregated by using the homogenous post-processing technique (see Section 2.2). This produces the the two limiting cumulative distributions, *Pl* and *Bel*, of the cooling load reported in Figure 7. In practice, the lower cumulative distribution, *Bel*, of the output is obtained by computing the average of the *m*=1000 necessity measures and the upper cumulative distribution, *Pl*, by computing the average of the *m*=1000 possibility distributions according to Eqs. 7 and 8.

FIGURE 6. FLOWCHART OF HYBRID PROBABILISTIC-POSSIBILISTIC UNCERTAINTY PROPAGATION DERIVED FROM (BARALDI AND ZIO 2008).

FIGURE 7. LIMITING PROBABILITY BOUNDS DERIVED FROM THE OUTPUTS OF THE HYBRID METHOD BY USING HOMOGENEOUS POST PROCESSING.

3.4. Results and comparison

In the post-processing stage, the outputs of all three methods (probabilistic, possibilistic and hybrid) are presented as cumulative distributions. Contrary to the probabilistic approach which returns a single percentile for each cooling load, the possibilistic and hybrid methods provide a range of percentiles. The range in the possibilistic approach is enclosed by the possibility (Π) and necessity (N) measures, whereas the hybrid method returns the boundary of the range through plausibility (Pl) and belief (Bel) functions. Figure 9 displays a comparison between the cumulative distributions of the hybrid probabilistic-possibilistic uncertainty propagation and the pure probabilistic and pure possibilistic methods for 0.4% design condition of the case study. It is observed that the outputs of the pure probabilistic representation (green continuous line denoted "MC") are contained between the plausibility function (blue dashed line denoted "Pl") and the belief functions (blue dotted line denoted "Bel") of the hybrid method. Also, the Pl and Bel functions extracted from the hybrid method are within the possibility function (red dashed line denoted "Π") and necessity function (red dotted line denoted "N") of the pure possibilistic representation. The effects of treating all uncertain variables probabilistically, versus treating climatic data probabilistically and internal gains possibilistically, can be seen by the distance between the Pl and MC distributions, as well as the distance between MC and Bel. This distance represents our incomplete knowledge on internal gains and, therefore, appears as a range with limiting bounds. Similarly, the effects of representing all data possibilistically, versus treating climatic data probabilistically and internal gains possibilistically, are shown by the distance between Π and Pl, as well as the distance between Bel and N.

FIGURE 8. MAPPING OUTPUTS FROM PURE PROBABILISTIC (MC), HYBRID (Pl, Bel) AND PURE POSSIBILISTIC (Π, N) UNCERTAINTY TREATMENTS FOR 0.4% DESIGN CONDITION.

From the computational point of view, the hybrid method is considerably more demanding than the pure probabilistic and possibilistic approaches. Since simulations are only conducted for the sizing process, each simulation lasts roughly 35 seconds on an Intel® Core™ i7-3610QM @ 2.30- 3.10 GHz processor with 8 logical cores and enabled multi-processing, executing mini-batches of 8 parallel jobs at a time. The overall simulation time for the pure probabilistic and the pure possibilistic methods are 1.2 and 2.5 hours respectively (35 seconds per simulation where 1000 and 2002 simulations are run for the pure probabilistic and possibilistic approach respectively). Execution of the hybrid method requires 250 hours (35 seconds per simulations for 202'000 total runs).

To compare the outputs of each representation, a quantitative assessment is provided (Table 5). Each value selected from the x axis (denoted "Peak cooling load"), will cross the Π, Pl, MC, Bel and N distributions in five different points along the y axis (denoted "ecdf"). Take Control sample A from Table 4 with a peak cooling load of 595.9 kW. This value intercepts the MC distribution at the 51st percentile, or in other words, would suffice to cover 51% of uncertain events (Figure 10). Mapping Control Sample A on the Pl and Bel measures of the hybrid method reveals that 595.9 kW would be able to cover between 23% and 73% of uncertain events. The observed difference between the pure probabilistic approach and the hybrid method is the result of forcing a probability density function onto human-dominated uncertain measures (i.e. internal gains). Mapping Control Sample A on the Π and N distributions will correspond to the 0th and 97th percentiles, implying that 595.9 kW would be able to cover anywhere between 0% and 97% of uncertain events. This, basically means that the pure possibilistic approach provides no information regarding the performance of Control Sample A. As expected, the pure probabilistic approach returns a crisp output and is the most sensitive of the three methods, whereas the pure possibilistic approach is the most cautious of the three approaches.

FIGURE 9. MAPPING "CONTROL SAMPLE A" AND "CASE STUDY A" ON THE CUMULATIVE DISTRIBUTIONS OF UNCERTAIN PEAK COOLING LOADS FOR THE 0.4% DESIGN CONDITION. TOP: PURE PROBABILISTIC. MIDDLE: HYBRID PROBABILISTIC-POSSIBILISTIC. BOTTOM: PURE POSSIBILISTIC.

TABLE 5. ADEQUACY ASSESSMENT OF PEAK COOLING LOADS FOR LINATE AND BRERA STATIONS BASED ON PROBABILISTIC (PROB.), POSSIBILISTIC (POSS.) AND HYBRID UNCERTAINTY TREATMENTS.

In practice, HVAC designers often make up for the uncertainty by applying sizing factors to the estimated peak cooling load. However, based on the building type and designer's experience the magnitude of the safety margin may greatly vary. Previous studies argued that the application of uncertainty quantification is a reliable alternative to the experimental-based application of safety factors, consequently, lowering the risk of oversizing the system (Domínguez-Muñoz, Cejudo-López, and Carrillo-Andrés 2010, Sun et al. 2014). In the case study, we are looking for a value which can cover 90% of uncertain events for the 0.4% design condition. Therefore, we seek the cooling load corresponding to the 90th percentile on the cumulative distributions. Also, we seek the suitable size factor that enables Control Sample A (595.9 kW) to cover all uncertain events except the worst 10%. The pure probabilistic approach reaches the 90th percentile at 607.2 kW and, therefore, a size factor of 1.019 is assigned.

To achieve the same level of confidence on the outputs of the hybrid method we intercept the 90th percentile on the Bel distribution, which returns a peak cooling load of 612.3 kW and a 1.028 size factor. The desired confidence from the pure possibilistic method is obtained by intercepting the 90th percentile at the N distribution, returning a peak cooling load of 619.6 kW and a 1.04 size factor. It is worth noting that these results are effected by the systematic error introduced by the simplification of the building's thermal zones (i.e. 3.2% for the 0.4% summer design condition). Therefore, designers should take caution when simplifying the thermal zoning, to obtain a reasonable tradeoff between the overhead systematic error due to building model simplifications and computational cost of simulations, to be run within the uncertainty propagation method proposed, that are shown to be methodologically suitable to address these problems of epistemic and aleatory uncertainty propagation.

Improper representation of internal gains through probability distributions neglects the lack of accurate measurements, and the inherent epistemic nature of uncertainty associated with the peak lighting and equipment power, as well as the maximum number of occupants. In each random generation, the pure probabilistic approach forces a single probability of occurrence on each internal gain component. Therefore, the pure probabilistic approach is the most risky among the three uncertainty representation and propagation methods. It is observed that the output of the pure possibilistic approach is the most conservative of the three methods. Consequently, a possibilistic representation of climatic variables is prone to overestimate the peak cooling load and is not recommended for sizing cooling loads under uncertainty. The hybrid method on the other hand, provides a somewhat conservative range of confidence that is less risky compared to the pure probabilistic approach and less conservative than the pure possibilistic approach. Therefore, the outputs of the hybrid method are less likely to undersize the system with respect to the pure probabilistic approach and to oversize it in comparison with the pure possibilistic approach. Since the pure probabilistic approach returns a crisp output for every percentile, it can be a suitable starting point for estimating the size factor. In the meantime, the belief and plausibility measures obtained from the hybrid method can work as the support of the pure probabilistic method, by quantifying the magnitude of confidence due to incomplete knowledge on internal gains.

Figure 10. Assigning size factors for different uncertainty representations. Top: 0.4% design condition, Middle: 1% design condition, Bottom: 2% design condition.

Figure 11 displays the results of all three uncertainty quantification methods i.e. probabilistic (denoted "Probability Theory), possibilistic (denoted "Possibility Theory") and hybrid (denoted "Evidence Theory") for 0.4%, 1% and 2% cooling design conditions. It is observed that the pure probabilistic method and the hybrid method demonstrate close performances, specifically in high percentiles. This phenomenon is generally evident after the 90th percentile, regardless of the type of design condition (0.4%, 0.1% or 2%). It also indicates that in our case study, a probabilistic representation of internal gains will result in a slightly undersized system. Therefore, we suggest the application of pure probabilistic uncertainty treatment for sizing cooling loads, only in buildings with deterministic occupancy profiles. The slopes of the necessity (N) and possibility (Π) measures reveal the effect of treating climatic variables (dry bulb, MCWB) as scarce knowledge and is mainly evident at 2% design conditions. The fixed distance between the Pl and Bel measures indicate that the magnitude of effect of uncertainties in human-dominated variables (i.e. internal gains) is constant at all climatic conditions. This is related to the fact that in our case study, all design conditions are mainly dominated by climatic variables rather than internal gains.

The probabilistic treatment of uncertainty provides a crisp output for each percentile, which may deem risky as it overlooks the lack of knowledge on activities dominated by human-behavior. The hybrid approach on the other hand provides a range of values (cooling loads) which are equally plausible. This means that even though the real value of the peak cooling load is unknown, one may assume that encountering any value outside the provided range is unlikely (implausible). To contrast how the provided range may assist designers in decision support, a comparative assessment of resorting to each method is provided. Let us assume that the designer tends to select a single chiller by using ASHRAE's 1% summer design conditions, while seeking a reliability of 99%. In this case, choosing the 99th percentile from the pure probabilistic framework returns a peak cooling load of 609 kW. In this case, designers often opt for the closet option that is larger than the estimated load, i.e. a chiller with a reference cooling capacity of 650 kW. The hybrid method returns two values for the 99th percentile which define the range of most plausible occurrences, i.e. encountering any peak cooling load between 594 kW and 610 kW is equally plausible. By resorting to the 99th percentile of the Pl measure (594 kW), one can opt for a chiller with a reference cooling power of 600 kW. It is also possible to rely on the 99th percentile of the Bel measure (610 kW) and select a chiller with a 650 kW reference cooling capacity, which will be identical to the outcome of a purely probabilistic-based decision. Therefore, it can be inferred that forcing unavailable information on occupant density, lighting power and appliance power in the form of probability distributions, results in overestimating the peak cooling load and eventually eliminating some design choices. It is observed that opting different frameworks for handling uncertainty can provide dissimilar options, and therefore adds to the degrees of freedom provided to the designer for decision making. This level of versatility will explicitly come in handy when sequencing multiple chillers for achieving optimal operational COP. Meanwhile, gathering further information on the aforementioned epistemic uncertain quantities can prevent overestimation of peak cooling loads and provide more reliable cooling capacity.

4. Conclusion

The existing literature on uncertainty assessment of HVAC systems assumes random variability for all uncertain parameters. This approach can be challenged in situations of scarce and incomplete knowledge. In this paper, we address this concern by characterizing aleatory uncertainties with probability and epistemic uncertainty with possibility representations. The study offers a hybrid uncertainty propagation method so that both aleatory and epistemic classes of uncertainty are properly introduced to the model. The proposed method is able to propagate the uncertainty through the model with minimal information loss. Results are presented in the form of plausibility and belief functions.

A numerical case study is provided to compare the effects of adopting the hybrid uncertainty treatment to the pure probabilistic and possibilistic representations. In this study, climatic data (i.e. temperature and humidity) are characterized with probability density functions while human-dominated events (i.e. occupant density, appliance power and lighting power) are represented by possibility distributions. The introduced hybrid treatment of uncertainty can be useful in support of the optimal design of chillers' configuration (balancing operating expenses and capital expenditure). The hybrid treatment of uncertainty is particularly important from the HVAC system design point of view, when designers have limited access to complete information regarding building characteristics. The hybrid approach may be well fitting for buildings with unpredictable occupancy patterns (e.g. hospitals), where quantifying the exact number of occupants is extremely difficult to quantify. Therefore, opting for a hybrid probabilistic-possibilistic framework for Bayesian calibration of building energy models is a potential for future studies (Pedroni et al. 2015). The hybrid treatment of uncertainty could also be evaluated when dealing with control regimes of autonomous building components (shading and lighting systems), where incomplete knowledge over occupant behavior and a system's state may have dependencies. In other situations, i.e. buildings with more predictable occupancy patterns (e.g. schools), results may be less sensitive to the hybrid approach, when compared to pure probabilistic representations.

In this study, cooling loads are estimated by means of dry bulb and MCWB temperature. However, resorting to dew point temperature, humidity ratio and mean coincident dry bulb temperature (for dehumidification purposes) is not expected to alter the methodological scheme followed for the analysis. Also, it is advised to perform simplifications on the thermal zoning with cautious, and seek for a tradeoff that does not oversimply the model while maintaining a reasonable simulation time. As a last remark, scarce information (such as occupant density) can occasionally contain more information than the studied example, although not enough to fit a probability density function. In such cases, the application of Chebyshev's inequality can provide a suitable fit for all family of probability distributions and, therefore, is a potential for expanding the provided framework.

References

Allouhi, A, Y El Fouih, T Kousksou, A Jamil, Y Zeraouli, and Y Mourad. 2015. "Energy consumption and efficiency in buildings: current status and future trends." *Journal of Cleaner Production* 109:118-130.

Azar, Elie, and Ahmed Al Amoodi. 2016. "Quantifying the impact of uncertainty in human actions on the energy performance of educational buildings." Proceedings of the 2016 Winter Simulation Conference.

Baraldi, Piero, and Enrico Zio. 2008. "A combined Monte Carlo and possibilistic approach to uncertainty propagation in event tree analysis." *Risk Analysis* 28 (5):1309-1326.

Baudrit, Cédric, Didier Dubois, and Dominique Guyonnet. 2006. "Joint propagation and exploitation of probabilistic and possibilistic information in risk assessment." *IEEE transactions on fuzzy systems* 14 (5):593-608.

Baudrit, Cédric, Didier Dubois, and Nathalie Perrot. 2008. "Representing parametric probabilistic models tainted with imprecision." *Fuzzy sets and systems* 159 (15):1913-1928.

Birol, Fatih. 2010. "World energy outlook 2010." *International Energy Agency* 1.

Burhenne, Sebastian, Olga Tsvetkova, Dirk Jacob, Gregor P Henze, and Andreas Wagner. 2013. "Uncertainty quantification for combined building performance and cost-benefit analyses." *Building and Environment* 62:143-154.

Chen, Ning, Dejie Yu, Baizhan Xia, and Michael Beer. 2016. "Uncertainty analysis of a structural–acoustic problem using imprecise probabilities based on p-box representations." *Mechanical Systems and Signal Processing* 80:45-57.

Cheng, Qi, Shengwei Wang, and Chengchu Yan. 2016. "Robust optimal design of chilled water systems in buildings with quantified uncertainty and reliability for minimized life-cycle cost." *Energy and Buildings* 126:159-169.

Cheng, Qi, Shengwei Wang, Chengchu Yan, and Fu Xiao. 2015. "Probabilistic approach for uncertainty-based optimal design of chiller plants in buildings." *Applied Energy*.

Cheng, Qi, Shengwei Wang, Chengchu Yan, and Fu Xiao. 2017. "Probabilistic approach for uncertainty-based optimal design of chiller plants in buildings." *Applied Energy* 185 (Part 2):1613-1624. doi: https://doi.org/10.1016/j.apenergy.2015.10.097.

Cobb, Barry R, and Prakash P Shenoy. 2003. "A comparison of Bayesian and belief function reasoning." *Information Systems Frontiers* 5 (4):345-358.

Cooper, J Arlin, Scott Ferson, and Lev Ginzburg. 1996. "Hybrid processing of stochastic and subjective uncertainty data." *Risk Analysis* 16 (6):785-791.

Corotis, Ross B. 2015. "An Overview of Uncertainty Concepts Related to Mechanical and Civil Engineering." *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering* 1 (4):040801.

Crawley, Drury B, Linda K Lawrie, Frederick C Winkelmann, Walter F Buhl, Y Joe Huang, Curtis O Pedersen, Richard K Strand, Richard J Liesen, Daniel E Fisher, and Michael J Witte. 2001. "EnergyPlus: creating a new-generation building energy simulation program." *Energy and buildings* 33 (4):319-331.

Das, Payel, Clive Shrubsole, Benjamin Jones, Ian Hamilton, Zaid Chalabi, Michael Davies, Anna Mavrogianni, and Jonathon Taylor. 2014. "Using probabilistic sampling-based sensitivity analyses for indoor air quality modelling." *Building and environment* 78:171-182.

de Wilde, Pieter, and Wei Tian. 2009. "Identification of key factors for uncertainty in the prediction of the thermal performance of an office building under climate change." *Building Simulation* 2 (3):157-174. doi: 10.1007/s12273-009-9116-1.

Djunaedy, Ery, Kevin Van den Wymelenberg, Brad Acker, and Harshana Thimmana. 2011. "Oversizing of HVAC system: Signatures and penalties." *Energy and Buildings* 43 (2):468-475.

Domínguez-Muñoz, Fernando, José M Cejudo-López, and Antonio Carrillo-Andrés. 2010. "Uncertainty in peak cooling load calculations." *Energy and Buildings* 42 (7):1010-1018.

Duarte, Carlos, Kevin Van Den Wymelenberg, and Craig Rieger. 2013. "Revealing occupancy patterns in an office building through the use of occupancy sensor data." *Energy and Buildings* 67:587-595.

Dubois, Didier, Hélene Fargier, and Jérôme Fortin. 2005. "The empirical variance of a set of fuzzy intervals." The 14th IEEE International Conference on Fuzzy Systems, 2005. FUZZ'05.

Dubois, Didier, Laurent Foulloy, Gilles Mauris, and Henri Prade. 2004. "Probability-possibility transformations, triangular fuzzy sets, and probabilistic inequalities." *Reliable computing* 10 (4):273-297.

Dubois, Didier, Etienne Kerre, Radko Mesiar, and Henri Prade. 2000. "Fuzzy interval analysis." In *Fundamentals of fuzzy sets*, 483-581. Springer.

Dubois, Didier, and Henri Prade. 1992. "When upper probabilities are possibility measures." *Fuzzy sets and systems* 49 (1):65-74.

Dubois, Didier, and Henri Prade. 2001. "Possibility theory, probability theory and multiple-valued logics: A clarification." *Annals of mathematics and Artificial Intelligence* 32 (1):35-66.

Dubois, Didier, and Henri Prade. 2009. "Formal representations of uncertainty." *Decision-Making Process: Concepts and Methods*:85-156.

Dubois, Didier, Henri Prade, and Sandra Sandri. 1993. "On possibility/probability transformations." In *Fuzzy logic*, 103-112. Springer.

Dubois, Didier, Henri Prade, and Philippe Smets. 1996. "Representing partial ignorance." *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans* 26 (3):361-377.

Dubois, Didier, Henri Prade, and Philippe Smets. 2008. "A definition of subjective possibility." *International Journal of Approximate Reasoning* 48 (2):352-364.

D'Oca, Simona, and Tianzhen Hong. 2015. "Occupancy schedules learning process through a data mining framework." *Energy and Buildings* 88:395-408.

D'Oca, Simona, Tianzhen Hong, and Jared Langevin. 2018. "The human dimensions of energy use in buildings: A review." *Renewable and Sustainable Energy Reviews* 81:731-742.

Eisenhower, Bryan, Zheng O'Neill, Vladimir A Fonoberov, and Igor Mezić. 2012. "Uncertainty and sensitivity decomposition of building energy models." *Journal of Building Performance Simulation* 5 (3):171-184.

Flage, Roger, Piero Baraldi, Enrico Zio, and Terje Aven. 2013. "Probability and Possibility ‐ Based Representations of Uncertainty in Fault Tree Analysis." *Risk Analysis* 33 (1):121-133.

Gaetani, Isabella, Pieter-Jan Hoes, and Jan LM Hensen. 2016. "Occupant behavior in building energy simulation: towards a fit-for-purpose modeling strategy." *Energy and Buildings* 121:188-204.

Gang, Wenjie, Shengwei Wang, Kui Shan, and Diance Gao. 2015. "Impacts of cooling load calculation uncertainties on the design optimization of building cooling systems." *Energy and Buildings* 94:1-9.

Gang, Wenjie, Shengwei Wang, Fu Xiao, and Dian-ce Gao. 2015. "Robust optimal design of building cooling systems considering cooling load uncertainty and equipment reliability." *Applied Energy* 159:265-275.

Guillén-Lambea, Silvia, Beatriz Rodríguez-Soria, and José M Marín. 2016. "Review of European ventilation strategies to meet the cooling and heating demands of nearly zero energy buildings (nZEB)/Passivhaus. Comparison with the USA." *Renewable and Sustainable Energy Reviews* 62:561-574.

Guyonnet, Dominique, Bernard Bourgine, Didier Dubois, Hélène Fargier, Bernard Côme, and Jean-Paul Chilès. 2003. "Hybrid approach for addressing uncertainty in risk assessments." *Journal of environmental engineering* 129 (1):68-78.

Haenni, Rolf, and Norbert Lehmann. 2003. "Implementing belief function computations." *International Journal of Intelligent Systems* 18 (1):31-49.

Handbook, ASHRAE Fundamentals. 2009. "American society of heating, refrigerating and air-conditioning engineers." *Inc.: Atlanta, GA, USA*.

Helton, Jon C, and Freddie Joe Davis. 2003. "Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems." *Reliability Engineering & System Safety* 81 (1):23-69.

Heo, Yeonsook, Ruchi Choudhary, and GA Augenbroe. 2012. "Calibration of building energy models for retrofit analysis under uncertainty." *Energy and Buildings* 47:550-560.

Hopfe, Christina J, and Jan LM Hensen. 2011. "Uncertainty analysis in building performance simulation for design support." *Energy and Buildings* 43 (10):2798-2805.

Huang, Yu Joe. 2014. "An Evaluation of ASHRAE's Climatic Design Conditions Against Actual Long-Term Recorded Weather Data." *ASHRAE Transactions* 120:V1.

Keshtkar, Azim, Siamak Arzanpour, Fazel Keshtkar, and Pouria Ahmadi. 2015. "Smart residential load reduction via fuzzy logic, wireless sensors, and smart grid incentives." *Energy and Buildings* 104:165-180.

Kim, Young-Jin. 2016. "Comparative study of surrogate models for uncertainty quantification of building energy model: Gaussian Process Emulator vs. Polynomial Chaos Expansion." *Energy and Buildings* 133:46-58.

Klir, George J. 1994. "On the alleged superiority of probabilistic representation of uncertainty." *IEEE Transactions on Fuzzy Systems* 2 (1):27-31.

Kohlas, Jürg, and Paul-André Monney. 2013. *A mathematical theory of hints: An approach to the Dempster-Shafer theory of evidence*. Vol. 425: Springer Science & Business Media.

Lee, Kwang Ho, and Stefano Schiavon. 2014. "Influence of three dynamic predictive clothing insulation models on building energy use, HVAC sizing and thermal comfort." *Energies* 7 (4):1917-1934.

Li, Yanfu, and Enrico Zio. 2012. "Uncertainty analysis of the adequacy assessment model of a distributed generation system." *Renewable Energy* 41:235-244.

Lindley, Dennis V. 1987. "The Probability Approach to the Treatment of Uncertainty in Artificial Intelligence and Expert Systems."17-24. doi: 10.1214/ss/1177013427.

Lombardia, ARPA. 2006. "Rapporto sullo stato dell'ambiente in Lombardia." *Milan: Regione Lombardia*.

Masson, Marie-Hélène, and Thierry Denœux. 2006. "Inferring a possibility distribution from empirical data." *Fuzzy sets and systems* 157 (3):319-340.

Mauris, Gilles, Lamia Berrah, Laurent Foulloy, and Alain Haurat. 2000. "Fuzzy handling of measurement errors in instrumentation." *IEEE Transactions on instrumentation and measurement* 49 (1):89-93.

Miller, Clayton, Zoltán Nagy, and Arno Schlueter. 2015. "Automated daily pattern filtering of measured building performance data." *Automation in Construction* 49:1-17.

Mitchell, John W, and James E Braun. 2013. *Principles of heating, ventilation, and air conditioning in buildings*: Wiley.

Mouchaweh, M Sayed, Mohamed Said Bouguelid, Patrice Billaudel, and Bernard RiERA. 2006. "Variable probability-possibility transformation." *International Journal of Approximate Reasoning*.

Mui, KW, and LT Wong. 2007. "Cooling load calculations in subtropical climate." *Building and environment* 42 (7):2498-2504.

Naseri, Masoud, Piero Baraldi, Michele Compare, and Enrico Zio. 2016. "Availability assessment of oil and gas processing plants operating under dynamic Arctic weather conditions." *Reliability Engineering & System Safety* 152:66-82.

Oberkampf, William L, Sharon M DeLand, Brian M Rutherford, Kathleen V Diegert, and Kenneth F Alvin. 2002. "Error and uncertainty in modeling and simulation." *Reliability Engineering & System Safety* 75 (3):333-357.

Page, Jessen, Darren Robinson, Nicolas Morel, and J-L Scartezzini. 2008. "A generalised stochastic model for the simulation of occupant presence." *Energy and buildings* 40 (2):83-98.

Paolini, Riccardo, Andrea Zani, Maryam MeshkinKiya, Veronica Lucia Castaldo, Anna Laura Pisello, Florian Antretter, Tiziana Poli, and Franco Cotana. 2016. "The hygrothermal performance of residential buildings at urban and rural sites: sensible and latent energy loads and indoor environmental conditions." *Energy and Buildings*.

Paolini, Riccardo, Andrea Zani, Maryam MeshkinKiya, Veronica Lucia Castaldo, Anna Laura Pisello, Florian Antretter, Tiziana Poli, and Franco Cotana. 2017. "The hygrothermal performance of residential buildings at urban and rural sites: Sensible and latent energy loads and indoor environmental conditions." *Energy and Buildings* 152 (Supplement C):792-803. doi: https://doi.org/10.1016/j.enbuild.2016.11.018.

Parsons, Simon. 1996. "Current approaches to handling imperfect information in data and knowledge bases." *IEEE Transactions on knowledge and data engineering* 8 (3):353-372.

Pedroni, Nicola, E Zio, A Pasanisi, and M Couplet. 2015. "Empirical comparison of two methods for the Bayesian update of the parameters of probability distributions in a two-level hybrid probabilistic-possibilistic uncertainty framework for risk assessment." *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering* 2 (1):04015015.

Pérez-Lombard, Luis, José Ortiz, and Christine Pout. 2008. "A review on buildings energy consumption information." *Energy and buildings* 40 (3):394-398.

Rasouli, Mohammad, Gaoming Ge, Carey J Simonson, and Robert W Besant. 2013. "Uncertainties in energy and economic performance of HVAC systems and energy recovery ventilators due to uncertainties in building and HVAC parameters." *Applied Thermal Engineering* 50 (1):732-742.

Richardson, Ian, Murray Thomson, and David Infield. 2008. "A high-resolution domestic building occupancy model for energy demand simulations." *Energy and buildings* 40 (8):1560-1566.

Rocchetta, R, YF Li, and Enrico Zio. 2015. "Risk assessment and risk-cost optimization of distributed power generation systems considering extreme weather conditions." *Reliability Engineering & System Safety* 136:47-61.

Rocchetta, R., and E. Patelli. 2016. "Imprecise probabilistic framework for power grids risk assessment and sensitivity analysis." In *Risk, Reliability and Safety: Innovating Theory and Practice*, 2789-2796. CRC Press.

Ruparathna, Rajeev, Kasun Hewage, and Rehan Sadiq. 2017. "Economic evaluation of building energy retrofits: A fuzzy based approach." *Energy and Buildings* 139:395-406.

Salari, Elahe, and Alireza Askarzadeh. 2015. "A new solution for loading optimization of multi-chiller systems by general algebraic modeling system." *Applied Thermal Engineering* 84:429-436.

Sansavini, Giovanni, Roberta Piccinelli, LR Golea, and Enrico Zio. 2014. "A stochastic framework for uncertainty analysis in electric power transmission systems with wind generation." *Renewable Energy* 64:71-81.

Schuetter, Scott, Lee DeBaillie, and Doug Ahl. 2014. "Future climate impacts on building design." *ASHRAE Journal* 56 (9):36-44.

Shafer, Glenn. 1976. *A mathematical theory of evidence*. Vol. 1: Princeton university press Princeton.

Shen, Limei, and Yongjun Sun. 2016. "Performance comparisons of two system sizing approaches for net zero energy building clusters under uncertainties." *Energy and Buildings* 127:10-21.

Singh, Maneesh, and Tore Markeset. 2014. "Hybrid models for handling variability and uncertainty in probabilistic and possibilistic failure analysis of corroded pipes." *Engineering Failure Analysis* 42:197-209.

Sun, Yongjun, Pei Huang, and Gongsheng Huang. 2015. "A multi-criteria system design optimization for net zero energy buildings under uncertainties." *Energy and Buildings* 97:196-204.

Sun, Yuming, Li Gu, CF Jeff Wu, and Godfried Augenbroe. 2014. "Exploring HVAC system sizing under uncertainty." *Energy and Buildings* 81:243-252.

Sunitha, KA, and Saswati Behera. 2016. "Comparision of Conventional Control Techniques For An Energy Efficient HVAC Systems." *International Journal of Applied Engineering Research* 11 (6):4258-4263.

Tahmasebi, Farhang, and Ardeshir Mahdavi. 2015. "The sensitivity of building performance simulation results to the choice of occupants' presence models: a case study." *Journal of Building Performance Simulation*:1-11.

Thevenard, Didier. 2010. "Influence of Long-Term Trends and Period of Record Selection on the Calculation of Cilmatic Design Conditions and Degree Days." *ASHRAE Transactions* 116 (1).

Thevenard, Didier, and Steve Cornick. 2013. "Revising ASHRAE Climatic Data for Design and Standards–Part 1: Overview and Data." *ASHRAE Transactions* 119 (2).

Tian, Wei. 2013. "A review of sensitivity analysis methods in building energy analysis." *Renewable and Sustainable Energy Reviews* 20:411-419.

Tindale, A. 2005. "Designbuilder software." *Stroud, Gloucestershire, Design-Builder Software Ltd*.

Trčka, Marija, and Jan LM Hensen. 2010. "Overview of HVAC system simulation." *Automation in Construction* 19 (2):93-99.

Vakiloroaya, Vahid, Bijan Samali, Ahmad Fakhar, and Kambiz Pishghadam. 2014. "A review of different strategies for HVAC energy saving." *Energy Conversion and Management* 77:738-754.

Wan Mohd Nazi, Wan Iman, Mohammad Royapoor, Yaodong Wang, and Anthony Paul Roskilly. 2017. "Office building cooling load reduction using thermal analysis method – A case study." *Applied Energy* 185 (Part 2):1574-1584. doi: https://doi.org/10.1016/j.apenergy.2015.12.053.

Wang, Chuang, Da Yan, and Yi Jiang. 2011. "A novel approach for building occupancy simulation." Building simulation.

Wang, Qinpeng, Godfried Augenbroe, Ji-Hyun Kim, and Li Gu. 2016. "Meta-modeling of occupancy variables and analysis of their impact on energy outcomes of office buildings." *Applied Energy* 174:166-180.

Yik, FWH, WL Lee, J Burnett, and P Jones. 1999. "Chiller plant sizing by cooling load simulation as a means to avoid oversized plant." *HKIE Transactions* 6 (2):19-25.

Yıldız, Yusuf, and Zeynep Durmuş Arsan. 2011. "Identification of the building parameters that influence heating and cooling energy loads for apartment buildings in hot-humid climates." *Energy* 36 (7):4287-4296.

Zadeh, Lotfi A. 1999. "Fuzzy sets as a basis for a theory of possibility." *Fuzzy sets and systems* 100:9-34.

Zadeh, Lotfi A. 2008. "Is there a need for fuzzy logic?" *Information sciences* 178 (13):2751-2779.

Zahiri, Behzad, Reza Tavakkoli-Moghaddam, and Mir Saman Pishvaee. 2014. "A robust possibilistic programming approach to multi-period location–allocation of organ transplant centers under uncertainty." *Computers & Industrial Engineering* 74:139-148.

Zhang, Hao, Loc Ha, Quanwang Li, and Michael Beer. 2017. "Imprecise probability analysis of steel structures subject to atmospheric corrosion." *Structural Safety* 67:62-69.

Zhang, Yi. 2009. "Parallel EnergyPlus and the development of a parametric analysis tool." 11th Conference of International Building Performance Association IBPSA, Glasgow, UK.

Zhang, Yi. 2012. "Use jEPlus as an efficient building design optimisation tool." CIBSE ASHRAE technical symposium, London.

Zhang, Yi, and Ivan Korolija. 2010. "Performing complex parametric simulations with jEPlus." 9th SET Conference Proceedings, Shanghai, China.

Zhao, Peng, Gregor P Henze, Sandro Plamp, and Vincent J Cushing. 2013. "Evaluation of commercial building HVAC systems as frequency regulation providers." *Energy and Buildings* 67:225-235.

Zio, Enrico. 2013. *The Monte Carlo simulation method for system reliability and risk analysis*: Springer.