EXERCISE SESSION 1:

BASICS OF PROBABILITY THEORY FOR APPLICATIONS TO RELIABILITY AND RISK ANALYSIS

Exercise 1

Ten compressors, each one with a failure probability of 0.1, are tested independently.

1. What is the expected number of compressors that are found failed?
2. What is the variance of the number of compressors that are found failed?
3. What is the probability that none will fail?
4. What is the probability that two or more will fail?

Exercise 2

An aircraft flight panel is fitted with two types of artificial horizon indicators. The times to failure of each indicator from the start of a flight follow an exponential distribution with a mean value of 15 hours and 30 hours, respectively. A flight lasts for a period of 3 hours.

1. What is the probability that the pilot will be without any artificial horizon indication by the end of a flight?
2. The mean time of this event in the case in which the aircraft is used for long duration flights?

Exercise 3

In considering the safety of a building, the total force acting on the columns of the building must be examined. This would include the effects of the dead load \( D \) (due to the weight of the structure), the live load \( L \) (due to human occupancy, movable furniture, and the like), and the wind load \( W \).

Assume that the load effects on the individual columns are statistically independent Gaussian variates with

\[
\begin{align*}
\mu_D &= 4.2 \text{ kips} \\
\sigma_D &= 0.3 \text{ kips} \\
\mu_L &= 6.5 \text{ kips} \\
\sigma_L &= 0.8 \text{ kips} \\
\mu_W &= 3.4 \text{ kips} \\
\sigma_W &= 0.7 \text{ kips}
\end{align*}
\]

1. Determine the mean and standard deviation of the total load acting on a column.
2. If the strength \( R \) of a column is also Gaussian with a mean equal to 1.5 times the total mean force, what is the probability of failure of the column? Assume that the coefficient of variation of the strength \( \delta_R \) is 15% and that the strength and load effects are statistically independent.
Exercise 4

The following relationship arises in the study of earthquake-resistant design:

\[ Y = ce^X \]

where \( Y \) is ground-motion intensity at the building site, \( X \) is the magnitude of an earthquake and \( c \) is related to the distance between the site and center of the earthquake. If \( X \) is exponentially distributed,

\[ f_X(x) = \lambda e^{-\lambda x} \quad x \geq 0 \]

find the cumulative distribution function of \( Y \), \( F_Y(y) \).

Exercise 5

The air pollution in a city is caused mainly by industrial (I) and automobile (A) exhausts. In the next 5 years, the chances of successfully controlling these two sources of pollution are, respectively, 75% and 60%. Assume that if only one of the two sources is successfully controlled, the probability of bringing the pollution below acceptable level would be 80%.

• What is the probability of successfully controlling air pollution in the next 5 years?
• If, in the next 5 years, the pollution level is not sufficiently controlled, what is the probability that is entirely caused by the failure to control automobile exhaust?
• If pollution is not controlled, what is the probability that control of automobile exhaust was not successful?