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System dynamic reliability assessment and failure prognostics

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ABSTRACT

Traditionally, equipment reliability assessment is based on failure data from a population of similar equipment, somewhat giving an average description of the reliability performance of an equipment, not capturing the specificity of the individual equipment. Monitored degradation data of the equipment can be used to specify its behavior, rendering dynamic the reliability assessment and the failure prognostics of the equipment, as shown in some recent literature. In this paper, dynamic reliability assessment and failure prognostics with noisy monitored data are developed for a system composed of dependent components. Parallel Monte Carlo simulation and recursive Bayesian method are integrated in the proposed modelling framework to assess the reliability and to predict the Remaining Useful Life (RUL) of the system. The main contribution of the paper is to propose a framework to estimate the degradation state of a system composed of dependent degradation components whose conditions are monitored (even without knowing the initial system degradation state) and to dynamically assess the system risk and RUL. As case study, a subsystem of the residual heat removal system of a nuclear power plant is considered. The results shows the significance of the proposed method for tailored reliability assessment and failure prognostics.

1. Introduction

Traditional reliability assessment computes the reliability of an equipment based on failure data from a (large) number of similar equipment [1,10,17,19,9]. This provides the reliability for the somewhat average equipment, without taking into account the specificity of the physical process of degradation and the related monitored data of the individual equipment under assessment.

Recently, dynamic reliability assessment and failure prognostics have been investigated. Following Liu et al. [12], dynamic reliability assessment in this paper is interpreted as the dynamic updating or modification of the reliability model of a specific equipment, when additional information becomes available, related to the state and degradation process of the equipment. The failure prognostics of the equipment can also be dynamic, within the reliability assessment framework.

Dynamic reliability assessment and failure prognostics have been investigated with reference to single components. The works of Dong and He [2], Ghasemi et al. [4], Ye et al. [23] and Si et al. [20] are some examples.

The dynamic reliability assessment and failure prognostics of a system of multiple components have not yet been explored in depth, because of the complications due to the interactions and dependences of behavior of the components constituting the system. The work of Liu et al. [12] and Moghaddass et al. [14] represent recent efforts in developing methods for the dynamic reliability assessment and failure prognostics of a system, where the observations on the system state are considered (although without noise).

In this paper, the dynamic reliability assessment and failure prognostics of a system with dependent multi-state/continuous degrading components is addressed. To the authors' knowledge, this is the first time that such type of system is considered for dynamic risk assessment and prognostics. Some main practical considerations are:

i) Degradation is usually described as a continuous process by physics-based or data-driven models [11,18,5,6,8]. In this paper, the authors propose a framework for dynamic reliability assessment and prognostics of a system composed of a pump and a valve, whose degradation processes are multi-state and continuous, respectively. The degradation model for each component is given.
ii) As the system can be operated in different conditions and environments, uncertainties affect the degradation models.

Acronyms: AE, Absolute Error; MC simulation, Monte Carlo simulation; PDF, Probability Density function; PDMP, Piecewise Deterministic Markov Process; RUL, Remaining Useful Life

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Symbols

t	time
C	number of components in a system
$s_{i,t}$	state of component i at time t
$M_{i,j}$	j-th degradation state of component i
$s_{i,t,M_{i,j}}$	degradation state of component i at time t is $M_{i,j}$
$\boldsymbol{s}_{s,t}$	state of the system at time t
x_t	measured value of the system state at time t
Δs_i	difference between two consequent states of component i
$\Delta \boldsymbol{s}_{s,t}$	difference between two consequent discrete system states at time t
N_i	number of possible degradation states of component i
N_{all}	number of possible degradation states of the system
N_{MC}	replication times of Monte Carlo simulation
Th_i	threshold for failure of component i
$g_i(\bullet)$	degradation function of component i
$h(\bullet)$	relation between the system state and the measured value
$\boldsymbol{\delta}_t$ and γ_t	uncertainty in the degradation function at time t
$\boldsymbol{X}_{1:t}$	vector including the values monitored from inspection time 1 to inspection time t
$p(A\|B)$	conditional probability of A given B
$p(A, B\|C)$	conditional probability of A and B given C
$p(A\|B, C)$	conditional probability of A given B and C
$p(A)$	unconditional probability of A
RUL_s	true RUL of the system
$\hat{RUL}_s$	estimated RUL of the system
$R(t)$	estimated reliability of the system at time t
λ_{mn}	transition rate from state m to state n
$N(\mu, \sigma)$	Gaussian distribution with a mean of μ and a standard deviation of σ
$\boldsymbol{\varepsilon}_t$	noise in the measured value at time t

iii) As the system can be operated in different conditions and environments, uncertainties are included in the degradation model of each component.
iv) The data monitored by the sensors is noisy.

To the authors' knowledge, this is the first time that such a system is considered for dynamic risk assessment and prognostics.

In Lin et al. [10], the simulation of a Piecewise Deterministic Markov Process (PDMP) model is performed for the reliability assessment of multiple dependent components with multi-state and continuous degradation processes, where the system degradation state is supposed to be precisely known and no uncertainty is considered. PDMP gives a same average result for different systems, without considering information on the specific system. If the precise system degradation state is not available, how can one assess the system reliability and predict the Remaining Useful Life (RUL) of the system with monitored noisy data? If the system degradation state is available for some time instances, how can one update the results on the reliability assessment and prognostics with the monitored value? This paper proposes a modelling framework to answer these questions.

Given a known system degradation state, PDMP can update the reliability and RUL of a multi-component system based on data monitored on the system. To exploit the information in the monitored data, especially in the case that the true system state is not known, a modelling framework combining recursive Bayesian method and parallel Monte Carlo simulation is proposed in this paper.

Specifically, the general recursive Bayesian method for system degradation state estimation with monitored data is firstly established for the considered system. As in Tombuyses and Aldemir [21], the system degradation state is discretized into a finite number of states. The recursive Bayesian method for the considered system with a finite number of degradation states is derived. The strategy for numerically implementing the established Bayesian framework for dynamic reliability assessment and failure prognostics is a parallel Monte Carlo simulation: one Monte-Carlo simulation is carried out for each possible system degradation state. The reliability and RUL of the considered system are calculated based on the results of the parallel Monte Carlo simulation. The dynamic reliability assessment and failure prognostics of an illustrative system with two dependent components are carried out to show the application of the proposed modelling framework. The case study in this paper concerns the degradation of a subsystem, composed of a pneumatic valve and a centrifugal pump, belonging to the residual heat removal system of a nuclear power plant.

The remainder of the paper is organized as follows. The generic formulation of the modelling problem considered is presented in Section 2. Section 3 details the framework for the proposed dynamic

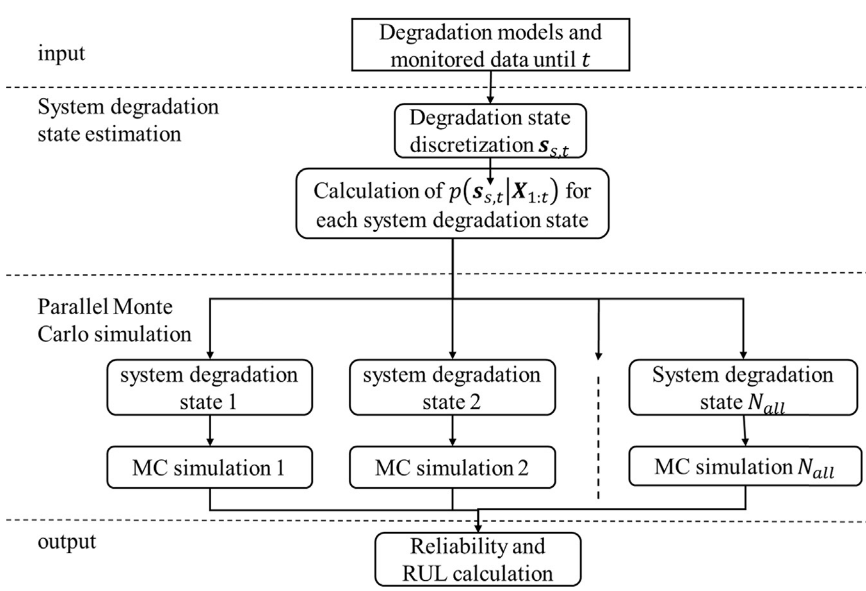

Fig. 1. Flowchart of the proposed modelling framework for dynamic reliability assessment and failure prognostics.

reliability assessment and failure prognostics. The pump and valve subsystem of the case study are described in Section 4. The results of the case study are presented in Section 5. Some conclusions and perspectives are given in Section 6.

2. Target system configuration

Engineering systems are normally composed of multiple components. The degradation process of one component may influence those of others. Often, it is difficult to observe directly the degradation states of all components and the observation of the system degradation process is only available at a system level, or for some of the components in the system. Generally, the measurable degradation variables are monitored at a fixed time interval.

Before establishing the method for dynamic reliability assessment and failure prognostics based on monitored data, some general specifications regarding the system considered are presented as follows:

1. The system contains C components. The state of component i at time t is noted as $s_{i,t}$. The state of the system at time t, noted as $\boldsymbol{s}_{s,t}$, is composed of the state of all the components, i.e $\boldsymbol{s}_{s,t}=\{s_{1,t}, s_{2,t},\ldots,s_{c,t}\}$. When the degradation of component i follows a multi-state process and the degradation state of the component at time t is $M_{i,j}$, it is noted as $s_{i,t,M_{i,j}}$, and j is an integer between 1 and N_i, with N_i the number of possible degradation states of the component i. For estimating the Remaining Useful Life (RUL) of the system, the threshold Th_i for failure of the component i in the system is assumed known. For guaranteeing a proper safety and reliability level of the component and system, the failure threshold of a component can be given by the field experts, experiments, etc. For example, in Vitanza [22], the fuel enthalpy failure threshold is calculated by an equation derived from the post-test strain data. The failure threshold of leakage from the first seal of reactor coolant pump in a nuclear power plant is determined by the field experts [13]. Degradation thresholds are calculated in Pope et al. by minimizing the total maintenance cost of the monitored component, or to minimize the downtime of the machine due to maintenance on the monitored component. If, instead of each component, the threshold for the system performance is available, the proposed framework for dynamic prognostics is still applicable, as the thresholds are only used to decide the RUL of the system in each replication of Monte Carlo simulation. For a discrete degradation process, the threshold is simply the failure state. The failure threshold can also be dynamic according to different environments as in Emani-Maeini et al. and Jiang et al. [7]. It is not the objective of this work to decide the system failure threshold, and we just assume that the system failure threshold is known. Uncertain failure thresholds can also be treated, within a double-loop Monte Carlo simulation.
2. In this paper, we assume that the degradation rate of the continuous degrading component i at time t is dependent on the system state at time t. The dependency is given by $s'_{i,t} = g_i(\boldsymbol{s}_{s,t})$. The state of the component i at time $t+t'$ can be expressed as $s_{i,t+t'} = \int_t^{t+t'} s'_{i,t}dt + \gamma_t$. For a component with multi-state degradation process, its transition rates at time t are dependent on the state of the system at time t, and the transition rates can be given by $\boldsymbol{\lambda}_{i,t+t'} = g_i(\boldsymbol{s}_{s,t}, \boldsymbol{\lambda}_{i,t}) + \boldsymbol{\delta}_t$. The degradation functions $g_i(\bullet)$ and the uncertainty elements γ_t, $\boldsymbol{\delta}_t$ are known or can be estimated from historical data.
3. The degradation of all components may not always be possible to observe. Suppose there are some measurable variables related to system degradation, whose values are monitored at discrete times $t = 1,2, \ldots$ and are noted as $\boldsymbol{x}_t$. We suppose that the measured value $\boldsymbol{x}_t$ is only dependent on the system state at time t and it can be expressed as $\boldsymbol{x}_t=h(\boldsymbol{s}_{s,t}) + \boldsymbol{\varepsilon}_t$. The noise $\boldsymbol{\varepsilon}_t$ from the sensors follows a constant or time-dependent distribution. The function $h(\bullet)$ and the noise $\boldsymbol{\varepsilon}_t$ are known or can be estimated from historical data. The vector $\boldsymbol{X}_{1:t}=\{\boldsymbol{x}_1, \boldsymbol{x}_2,\ldots,\boldsymbol{x}_t\}$ includes the values monitored from inspection time 1 to inspection time t.

These specifications in Section 2 are not especially for a parallel, series or mixed system. A system that satisfies the previous specifications can use the proposed framework for dynamic reliability assessment and failure prognostics, independently of the specific logic configuration. In the experiments of Section 5, the system considered is a series system where the failure of one component can result in the failure of the system. However, the relation between the component failure and the system failure can also be different.

3. Dynamic reliability assessment of a system made of multiple dependent degrading components

The proposed modelling framework is composed of two parts, as shown in Fig. 1, i.e. the system degradation state estimation and the parallel Monte Carlo simulation of the evolution of the system degradation state. With the monitored data and system degradation model, the recursive Bayesian method introduced in Section 3.1 is used to estimate the possible system degradation states and their probabilities. Then, for each possible system state with non-zero probability, a Monte Carlo simulation is carried out whose number of replications is proportional to the probability of this system state. The reliability of the system is calculated based on the RUL of each replication in a parallel Monte Carlo simulation. When more monitored data on the system degradation are available, the probability of each system degradation state is updated and parallel Monte Carlo simulation is repeated to calculate dynamically the new system reliability and RUL.

3.1. Estimation of the probability of the system degradation state at time t

Nonlinear Bayesian filtering presented in Orchard [15] is used in this Section for estimating the conditional probability of a system degradation state given the measured data until time t and the degradation functions in Section 2.

The values of the measurable variables are monitored at a discrete time t. This section establishes the procedure for calculating the conditional probability of the system at a specific degradation state $\boldsymbol{s}_{s,t}$, which can be noted as

$$p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t}) = \frac{p(\boldsymbol{X}_{1:t}\boldsymbol{s}_{s,t})p(\boldsymbol{s}_{s,t})}{p(\boldsymbol{X}_{1:t})} = \frac{p(\boldsymbol{x}_t, \boldsymbol{X}_{1:t-1}\boldsymbol{s}_{s,t})p(\boldsymbol{s}_{s,t})}{p(\boldsymbol{X}_{1:t})}. \tag{1}$$

Eq. (1) can be reformulated as

$$p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t}) = \frac{p(\boldsymbol{x}_t, \boldsymbol{X}_{1:t-1}\boldsymbol{s}_{s,t})p(\boldsymbol{s}_{s,t})}{p(\boldsymbol{x}_t \mid \boldsymbol{X}_{1:t-1})p(\boldsymbol{X}_{1:t-1})}. \tag{2}$$

The conditional probability $p(\boldsymbol{x}_t, \boldsymbol{X}_{1:t-1}\boldsymbol{s}_{s,t})$ equals to $p(\boldsymbol{x}_t \mid \boldsymbol{X}_{1:t-1}, \boldsymbol{s}_{s,t})p(\boldsymbol{X}_{1:t-1} \mid \boldsymbol{s}_{s,t})$. From the assumptions in Section 2, we know that the measured value at time t is only dependent on the system state at time t. Thus, we have $p(\boldsymbol{x}_t\boldsymbol{X}_{1:t-1}, \boldsymbol{s}_{s,t})=p(\boldsymbol{x}_t\boldsymbol{s}_{s,t})$. The Bayes' rule gives that $p(\boldsymbol{X}_{1:t-1}\boldsymbol{s}_{s,t}) = \frac{p(\boldsymbol{s}_{s,t} \mid \boldsymbol{X}_{1:t-1})p(\boldsymbol{X}_{1:t-1})}{p(\boldsymbol{s}_{s,t})}$. Thus, the Eq. (2) can be rewritten as

$$p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t}) = \frac{p(\boldsymbol{x}_t \mid \boldsymbol{s}_{s,t})p(\boldsymbol{s}_{s,t} \mid \boldsymbol{X}_{1:t-1})}{p(\boldsymbol{x}_t \mid \boldsymbol{X}_{1:t-1})}, \tag{3}$$

with

$$p(\boldsymbol{x}_t\boldsymbol{X}_{1:t-1}) = \int p(\boldsymbol{x}_t \mid \boldsymbol{s}_{s,t})p(\boldsymbol{s}_{s,t} \mid \boldsymbol{X}_{1:t-1})d\boldsymbol{s}_{s,t}. \tag{4}$$

In Eqs. (3) and (4), $p(\boldsymbol{x}_t \mid \boldsymbol{s}_{s,t})$ can be calculated from the given degradation function $h(\bullet)$ and noise distribution ε_t. The other two conditional probabilities in the right part of Eq. (3) are calculated as follows:

$$p(\mathbf{s}_{s,t}\mathbf{X}_{1:t-1}) = \int p(\mathbf{s}_{s,t} \mid \mathbf{s}_{s,t-1})p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})d\mathbf{s}_{s,t-1} \tag{5}$$

where the conditional probability $p(\mathbf{s}_{s,t}\mathbf{s}_{s,t-1}) = \prod_{i=1}^{C} p(s_{i,t} \mid \mathbf{s}_{s,t-1})$, and $p(s_{i,t} \mid \mathbf{s}_{s,t-1})$ can be calculated directly with the degradation functions of assumption 2 in Section 2;

$p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})$ in Eq. (5) is the conditional probability of $\mathbf{s}_{s,t-1}$ given all the measured values until time $t-1$. Then,

$$p(\mathbf{s}_{s,t}\mathbf{X}_{1:t}) = \frac{p(\mathbf{x}_t \mid \mathbf{s}_{s,t}) \int p(\mathbf{s}_{s,t} \mid \mathbf{s}_{s,t-1})p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})d\mathbf{s}_{s,t-1}}{\iint p(\mathbf{x}_t \mid \mathbf{s}_{s,t})p(\mathbf{s}_{s,t} \mid \mathbf{s}_{s,t-1})p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})d\mathbf{s}_{s,t-1}d\mathbf{s}_{s,t}}. \tag{6}$$

Thus, as shown in Eq. (6), $p(\mathbf{s}_{s,t}\mathbf{X}_{1:t})$ is, eventually, expressed as a function of $p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})$. Given that $p(\mathbf{s}_{s,1}\mathbf{X}_{1:1})$ can be estimated with the assumption 3 in Section 2, the conditional probability $p(\mathbf{s}_{s,t}\mathbf{X}_{1:t})$ can be calculated recursively.

The Eqs. (1)–(6) are from classic Bayesian filtering methods and are kept for completeness and self-containment of the paper, while they are adapted to the considered system for dynamic prognostics and risk assessment.

3.2. Parallel Monte Carlo simulation for dynamic reliability assessment and prognostics

In order to carry out the recursive Bayesian method established in Section 3.1, the discretization of the continuous degradation state. For example, the component i is one such component and the whole range of degradation states is $[0,Th_i]$, which is divided into N_i different states and the distance between two consequent states is Δs_i which is $\frac{Th_i}{N_i-1}$. The first state $M_{i,1}$ is the failure state and the N_i-th state M_{i,N_i} is the perfect functioning state. For a multi-state degrading component i, Δs_i is 1 in arbitrary unit. Suppose that the volume between two consequent discrete system states is noted as $\Delta \mathbf{s}_{s,t}$, then, with the assumed the second factor of the numerator in Eq. (6) can be approximated as

$$\int p(\mathbf{s}_{s,t}\mathbf{s}_{s,t-1})p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})d\mathbf{s}_{s,t-1} \cong [\sum_{i_{t-1,C}=1}^{N_C} \sum_{i_{t-1,c-1}=1}^{N_{c-1}} \cdots \sum_{i_{t-1,1}=1}^{N_1} f(\bullet)]\Delta \mathbf{s}_{s,t-1},$$

with

$$f(\bullet) = p(\{s_{1,t,M_{1,i_{t,1}}}, \quad s_{2,t,M_{2,i_{t,2}}}, \ldots, \quad s_{C,t,M_{C,i_{t,C}}}\}\{s_{1,t-1,M_{1,i_{t-1,1}}}, s_{2,t-1,M_{2,i_{t-1,2}}}, \ldots, \quad s_{C,t-1,M_{C,i_{t-1,C}}}\})*p(\{s_{1,t-1,M_{1,i_{t-1,1}}}, \quad s_{2,t-1,M_{2,i_{t-1,2}}}, \ldots, s_{C,t-1,M_{C,i_{t-1,C}}}\}\mathbf{X}_{1:t-1}). \tag{7}$$

where $s_{i,t,M_{i,j}}j = 1, \ldots, N_i$ means that the state of the component i at time t is $M_{i,j}$.

The denominator can be approximated similarly following the previous steps, and becomes

$$\iint p(\mathbf{x}_t\mathbf{s}_{s,t})p(\mathbf{s}_{s,t}\mathbf{s}_{s,t-1})p(\mathbf{s}_{s,t-1}\mathbf{X}_{1:t-1})d\mathbf{s}_{s,t-1}d\mathbf{s}_{s,t} \cong \Delta \mathbf{s}_{s,t-1}\Delta \mathbf{s}_{s,t} \sum_{i_{t,C}=1}^{N_c} \sum_{i_{t,C-1}=1}^{N_{C-1}} \cdots \sum_{i_{t,1}=1}^{N_1} p(\mathbf{x}_t\{s_{1,t,M_{1,i_1}}, s_{2,t,M_{2,i_{t,2}}}, \ldots, \quad s_{C,t,M_{C,i_{t,C}}}\}) \sum_{i_{t-1,C}=1}^{N_C} \sum_{i_{t-1,c-1}=1}^{N_{c-1}} \cdots \sum_{i_{t-1,1}=1}^{N_1} f(\bullet). \tag{8}$$

For reliability assessment and failure prognostics of the system at time t, Monte Carlo simulation is used. The states of the continuous degrading components are discretized as before and, thus, the system degradation states can have $N_{all} = \prod_{i=1}^{C} N_i$ possible configurations. The conditional probability for each possible state of the system is recursively calculated with the monitored data using the Eq. (6). A Monte Carlo simulation, as shown in Fig. 1, is carried out in parallel for each possible system degradation state, for a total of N_{MC} replications. The number of replications $N_{MC,i}$ in the i-th Monte Carlo simulation is proportional to the probability of the corresponding system degradation state $\mathbf{s}_{s,t}$ at time t, i.e. $N_{MC,i}{=}p(\mathbf{s}_{s,t}\mathbf{X}_{1:t})*\Delta \mathbf{s}_{s,t}*N_{MC}$ and $N_{MC} = \sum_{i=1}^{N_{all}} N_{MC,i}$. Note that, with Eqs. (6)–(8), $\Delta \mathbf{s}_{s,t}$ is eliminated in the Monte Carlo simulation and $p(\mathbf{s}_{s,t}\mathbf{X}_{1:t})*\Delta \mathbf{s}_{s,t}$ is the likelihood that the system state at time t is $\mathbf{s}_{s,t}$. For a system state, if $p(\mathbf{s}_{s,t}\mathbf{X}_{1:t})$=0, there is no need of a Monte Carlo simulation with this initial system state.

To account for the uncertainties in the degradation models, during a replication of Monte Carlo simulation, one value is randomly selected as realization of the degradation state at the next time step from the probability density distribution given by the degradation model.

For each replication of Monte Carlo simulation, a system degradation evolution is generated and the RUL of the system for this replication is noted as $R\hat{U}L_{i,s}, i = 1,2, \ldots, N_{MC}$. The estimated RUL of the system, noted as $R\hat{U}L_s$, given by the parallel Monte Carlo simulation is calculated as the mean of the $RUL_{i,s}$ of all the degradation evolutions simulated, i.e. $R\hat{U}L_s = \frac{1}{N_{MC}}\sum_{i=1}^{N_{MC}} R\hat{U}L_{i,s}$. The reliability of the system at time t+t' is calculated as $R(t+t') = \frac{N(R\hat{U}L_{i,s} \geq t')}{N}$, with $N(R\hat{U}L_{i,s} \geq t')$ the number of iterations with $R\hat{U}L_{i,s} \geq t'$.

When new monitored data, e.g. $\mathbf{x}_{t+1}$, are available, the probability of the each possible system state can be updated, i.e. $p(\mathbf{s}_{s,t+1}\mathbf{X}_{1:t+1})$ and Monte Carlo simulation can be used to dynamically calculate the reliability and RUL of the system.

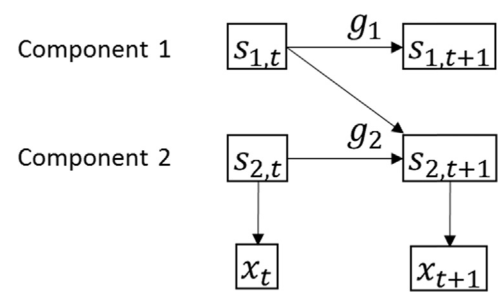

Fig. 2. Schematic explanation of the illustrative system.

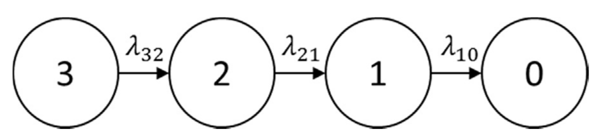

Fig. 3. Degradation process of the pump.

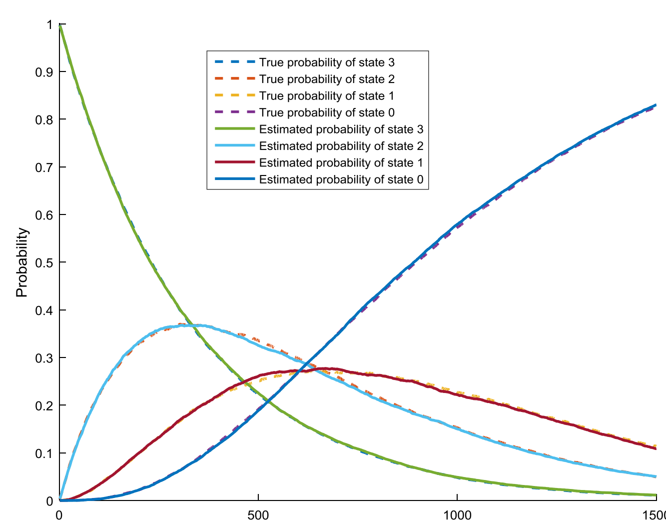

Fig. 4. The true and estimated probabilities of the pump at different states.

4. An illustrative case study

In this section, a system composed of two dependent components is considered as illustrative example to explain the proposed modelling framework, as shown in Fig. 2.

The system is characterized as:

Table 1
Details of the four considered failure scenarios.

	Pump holding time			failure time	failure type
	state 3	state 2	state 1		
Scenario 1	22	433	291	559	valve failure
Scenario 2	333	104	700	658	valve failure
Scenario 3	65	172	32	268	pump failure
Scenario 4	49	81	267	397	pump failure

- component 1 follows a N_1-state degradation process and component 2 undertakes a continuous degradation process;
- the monitored data x_t is the sensor measurement of the degradation of the component 2, noted as $s_{2,t}$, with a known sensor noise ε following a fixed distribution, i.e. $x_t = s_{2,t}+\varepsilon$. The measurement is carried out at a fixed time interval of 1 (with an arbitrary unit);
- the degradation rate of component 2 is dependent on the state of components 1 and 2 at that time, i.e. $s'_{2,t} = g_2(s_{1,t}, s_{2,t})$ and $s_{2,t+1} = \int_t^{t+1} g_2(s_{1,t}, s_{2,t})+\gamma$;
- component 1 follows a multi-state Markov process and $s_{1,t+1} = g_1(s_{1,t}, \lambda_t)$;
- the failure threshold of component 1 is Th;
- if one component in the system fails, the system fails.

In order to carry out the recursive Bayesian method presented in Sections 2 and 3, the discretization of the degradation state of component 2 is necessary. Suppose the continuous degradation state of component 2, ranged in interval $[0, Th]$, is divided into N_2 states and

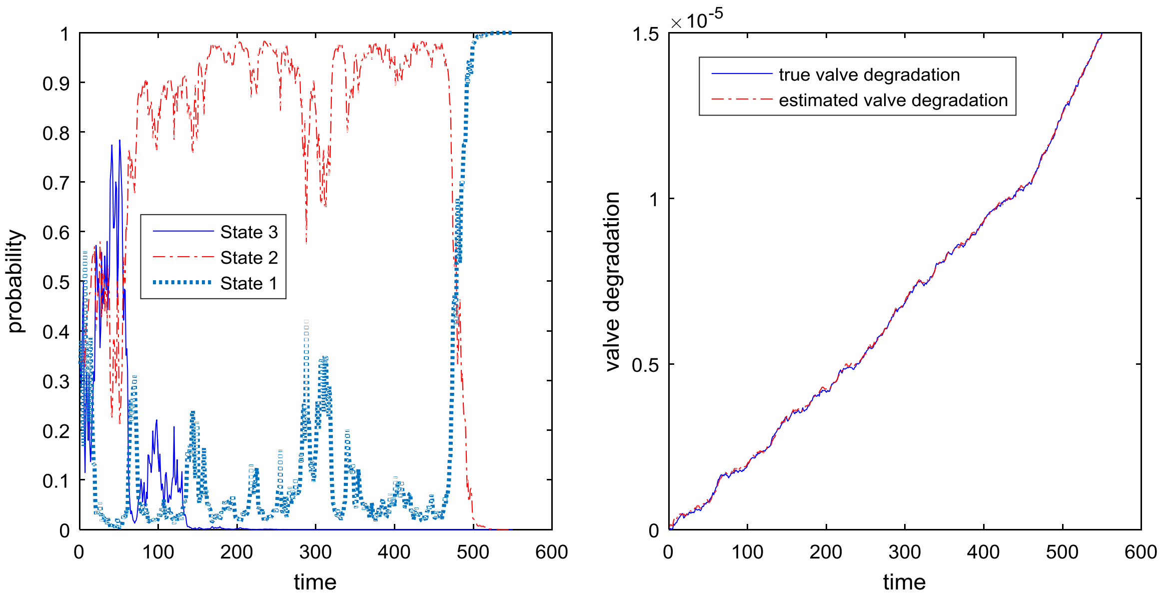

Fig. 5. True and estimated pump (left) and valve (right) degradation state of scenario 1.

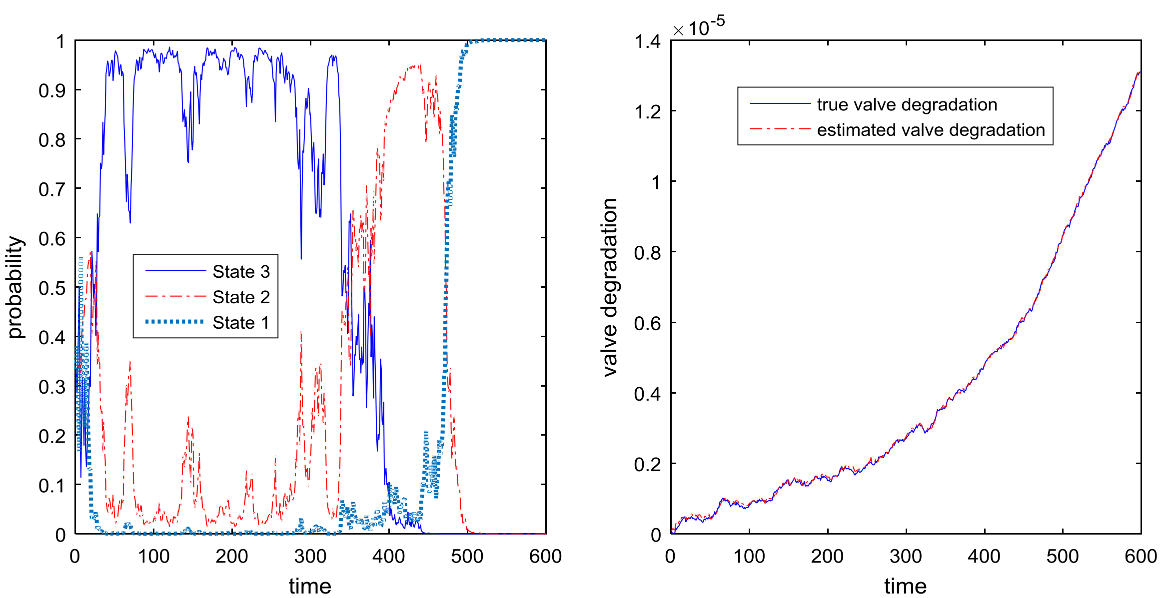

Fig. 6. True and estimated pump (left) and valve (right) degradation state of scenario 2.

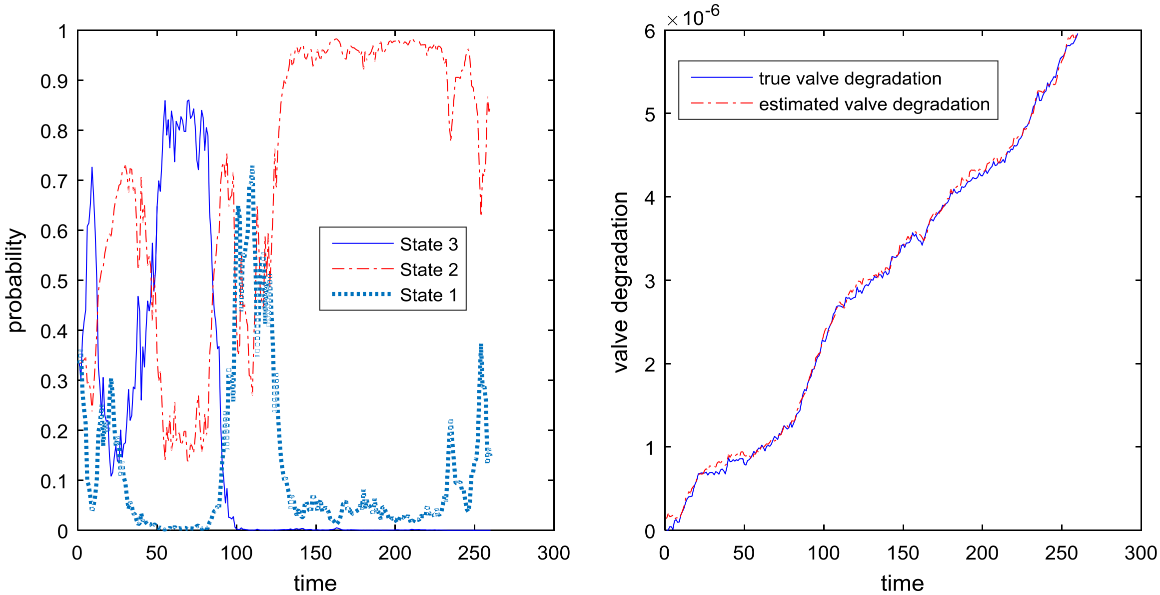

Fig. 7. True and estimated pump (left) and valve (right) degradation state of scenario 3.

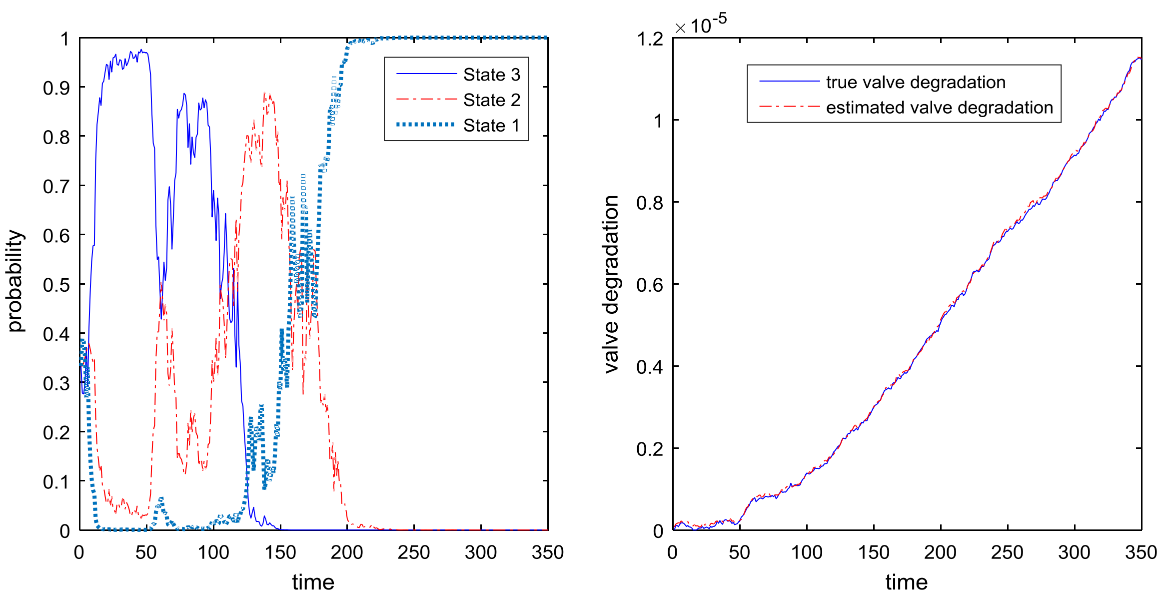

Fig. 8. True and estimated pump (left) and valve (right) degradation state of scenario 4.

the distance between two consequent states is $\Delta s_2 = \frac{Th}{N_2 - 1}$. With $s_{s,t} = \{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\}$, $i_{t,1}=1, \ldots, N_1 andi_{t,2}=1, \ldots, N_2$, Eqs. (6)–(8) give that the probability that the system state is $s_{s,t}$ at time t is

$$p(\{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\}|\boldsymbol{X}_{1:t}) == \frac{p(x_t|\{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\})l(\bullet)}{\Delta s_{s,t}\sum_{i_{t,2}=1}^{N_2}\sum_{i_{t,1}=1}^{N_1} p(x_t|\{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\})l(\bullet)},$$

with

$$l(\bullet) = \sum_{i_{t-1,2}=1}^{N_2}\sum_{i_{t-1,1}=1}^{N_1} p(\{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\}|\{s_{1,t-1,M_{1,i_{t-1,1}}}, \quad s_{2,t-1,M_{2,i_{t-1,2}}}\}) \\ p(\{s_{1,t-1,M_{1,i_{t-1,1}}}, \quad s_{2,t-1,M_{2,i_{t-1,2}}}\}|\boldsymbol{X}_{1:t-1}). \tag{9}$$

As the measuring data is only dependent on the degradation state of component 2, the recursive Bayesian Eq. (9) can be simplified as

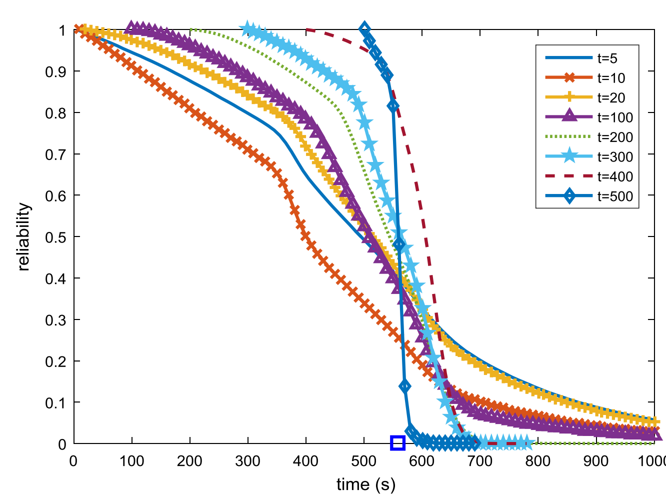

Fig. 9. Reliability of the monitored system, given thedata monitored until different observation timesr for Scenario 1. (The true failure time is marked as a square on x axis.).

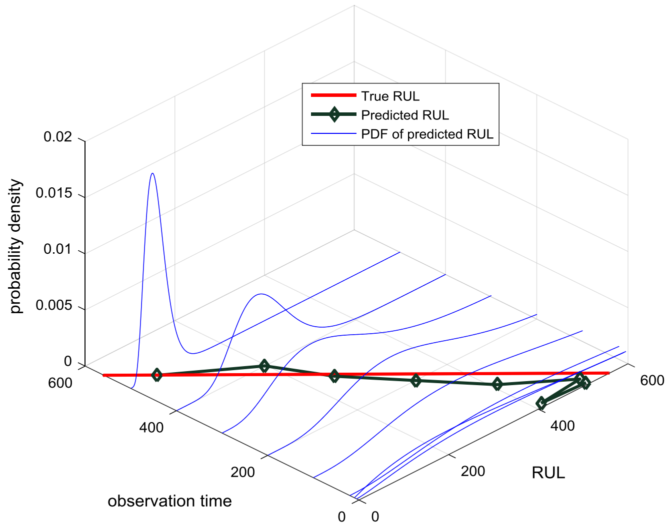

Fig. 10. Estimated system RUL PDFs, given data monitored until different observation times*t* for Scenario 1.

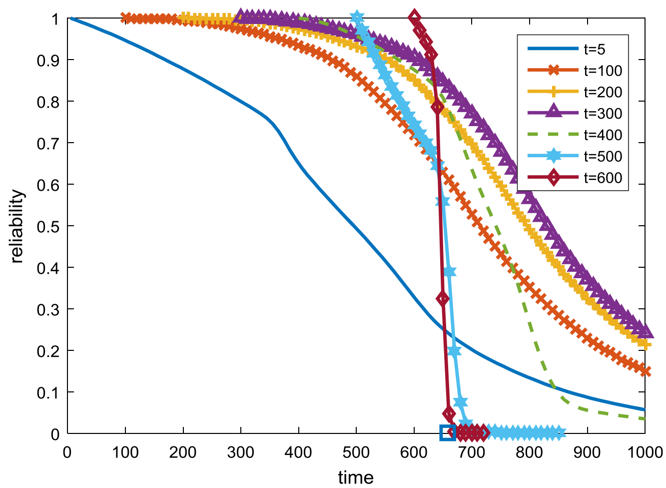

Fig. 11. Reliability of the monitored system, given the data monitored until different observation times *t* for Scenario 2. (The true failure time is marked as a square on x axis.).

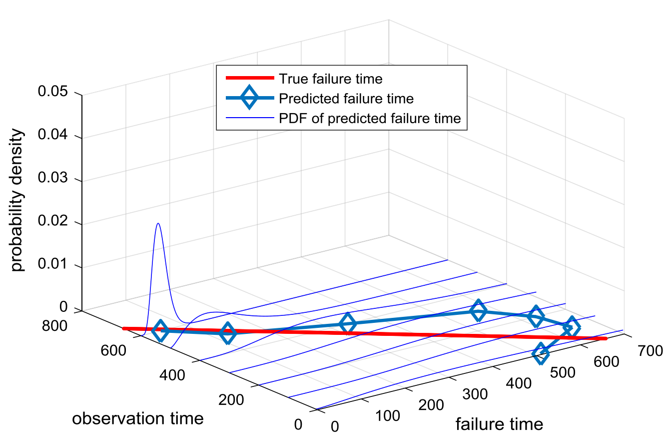

Fig. 12. Estimated system RUL PDFs, given data monitored until different observation times *t* for Scenario 2.

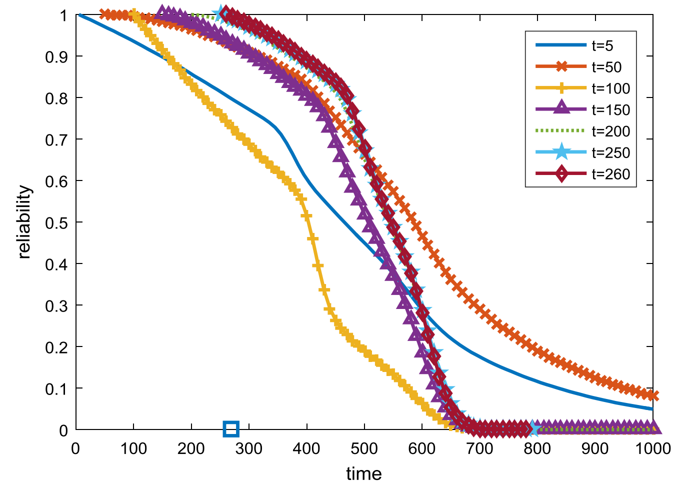

Fig. 13. Reliability of the monitored system, given the data monitored until different observation times *t* for Scenario 3. (The true failure time is marked as a square on x axis.).

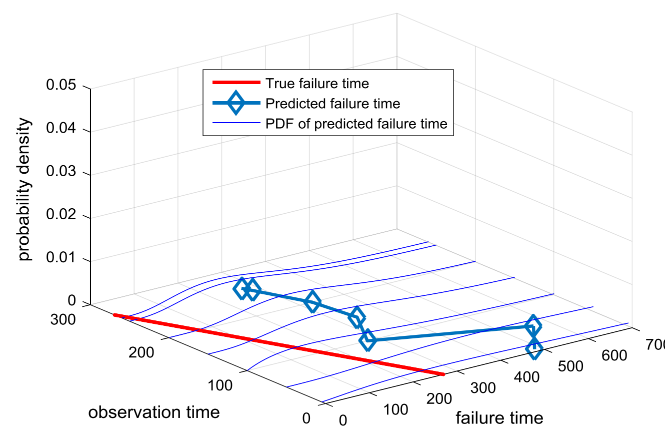

Fig. 14. Estimated system RUL PDFs, given data monitored until different observation times *t* for Scenario 3.

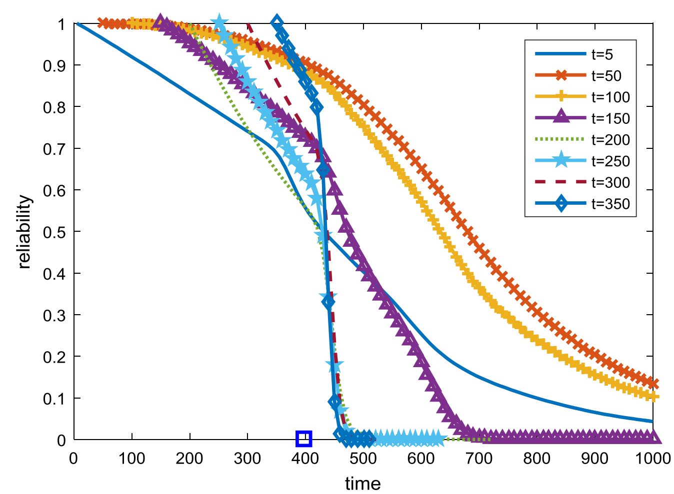

Fig. 15. Reliability of the monitored system, given the data monitored until different observation times *t* for Scenario 4. (The true failure time is marked as a square on x axis.).

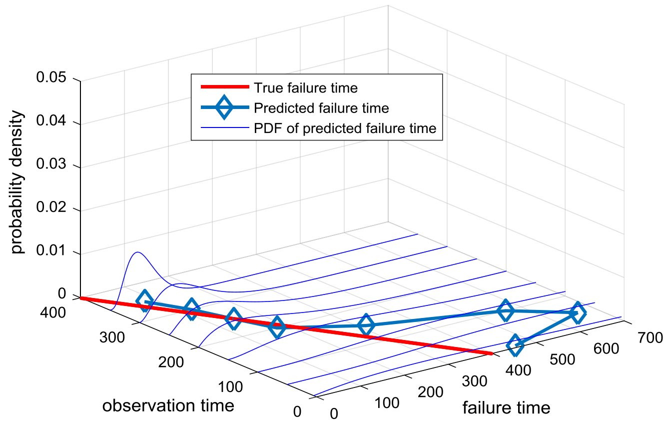

Fig. 16. Estimated system RUL PDFs, given data monitored until different observation times t for Scenario 4.

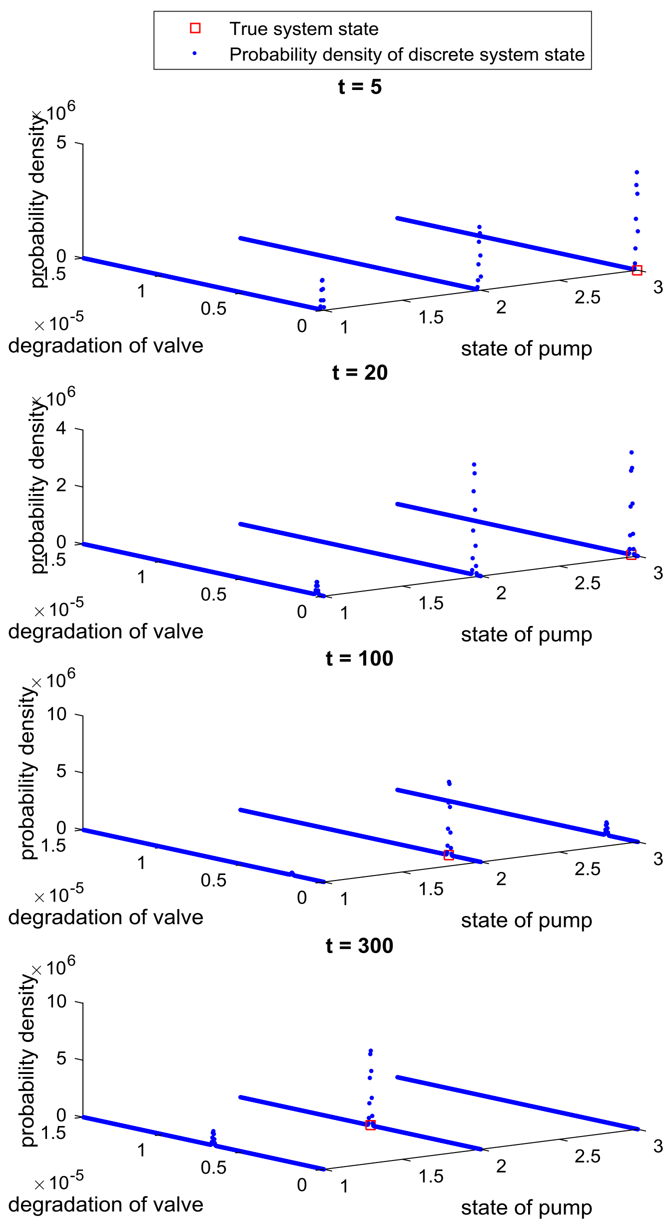

Fig. 17. True system state and estimated probability of the system state at observation time t = 5, 20, 100, 300, using the proposed method for Scenario 1.

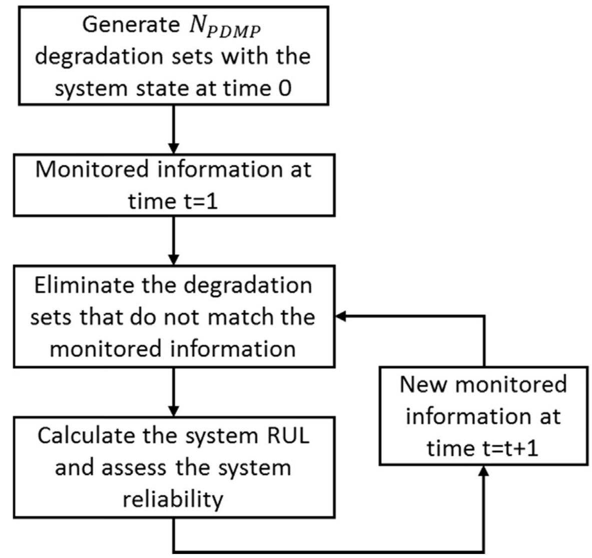

Fig. 18. The flowchart of using PDMP for dynamic reliability assessment and failure prognostics.

$$p\left(\left\{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\right\} X_{1:t}\right) == \frac{p\left(x_t s_{2,t,M_{2,i_{t,2}}}\right) l(\bullet)}{\Delta s_{s,t} \sum_{i_{t,2}=1}^{N_2} p\left(x_t s_{2,t,M_{2,i_{t,2}}}\right) l(\bullet)}. \tag{10}$$

In the parallel Monte Carlo simulation, N_{MC} realizations of the system states are generated for time t. The number of replications of the i-th Monte Carlo simulation for data point $s_{s,t} = \left\{s_{1,t,M_{1,i_{t,1}}}, s_{2,t,M_{2,i_{t,2}}}\right\}$, $i_{t,1}=1, \ldots, N_1 and i_{t,2}=1, \ldots, N_2$, is $N * p(s_{s,t} X_{1:t}) \Delta s_{s,t}$. As $s_{1,t,M_{1,i_{t,1}}}$ and $s_{2,t,M_{2,i_{t,2}}}$ are independent, $p\left(s_{s,t}\left\{s_{1,t-1,M_{1,i_{t-1,1}}}, s_{2,t-1,M_{2,i_{t-1,2}}}\right\}\right)$ can be rewritten as

$$p\left(s_{1,t,M_{1,i_{t,1}}}\left\{s_{1,t-1,M_{1,i_{t-1,1}}}, \quad s_{2,t-1,M_{2,i_{t-1,2}}}\right\}\right) * p\left(s_{2,t,M_{2,i_{t,2}}}\left\{s_{1,t-1,M_{1,i_{t-1,1}}}, s_{2,t-1,M_{2,i_{t-1,2}}}\right\}\right).$$

N_{MC} degradation evolutions of the system are generated for the system at time t. Suppose the estimated RULs of each component in the system degradation evolution set $i, i = 1, \ldots, N_{MC}$ are noted separately $\hat{RUL}_{i,1}, \hat{RUL}_{i,2}$: then, the system RUL, noted as $\hat{RUL}_{i,s}$, is $\hat{RUL}_{i,s} = \min(\hat{RUL}_{i,1}, \hat{RUL}_{i,2})$. The overall reliability and RUL of the system from the N_{MC} replications can, then, be calculated as explained in the previous section.

5. Case study

In this case study, a subsystem of a residual heat removal system of a nuclear power plant is considered [10]. The subsystem is composed of two components, i.e. a pneumatic valve and a centrifugal pump.

The degradation of the centrifugal pump follows a continuous-time homogeneous Markov process with constant transition rates. Fig. 3 shows the degradation process of the pump.

The degradation states of the pump are classified into four states, i.e. state 3, noted as $M_{p,3}$, state 2, noted as $M_{p,2}$, state 1, noted as $M_{p,1}$, state 0, noted as $M_{p,0}$, with state 3 being the perfect functioning state and state 0 the failure state. The transition rates λ_{32}, λ_{21} and λ_{10} are all equal to $\lambda = 0.003/t$.

Under this assumption, if the degradation state of the pump at time t is $M_{p,k}, k \in \{3,2,1\}$, the degradation state of the pump at time $t+1$ can

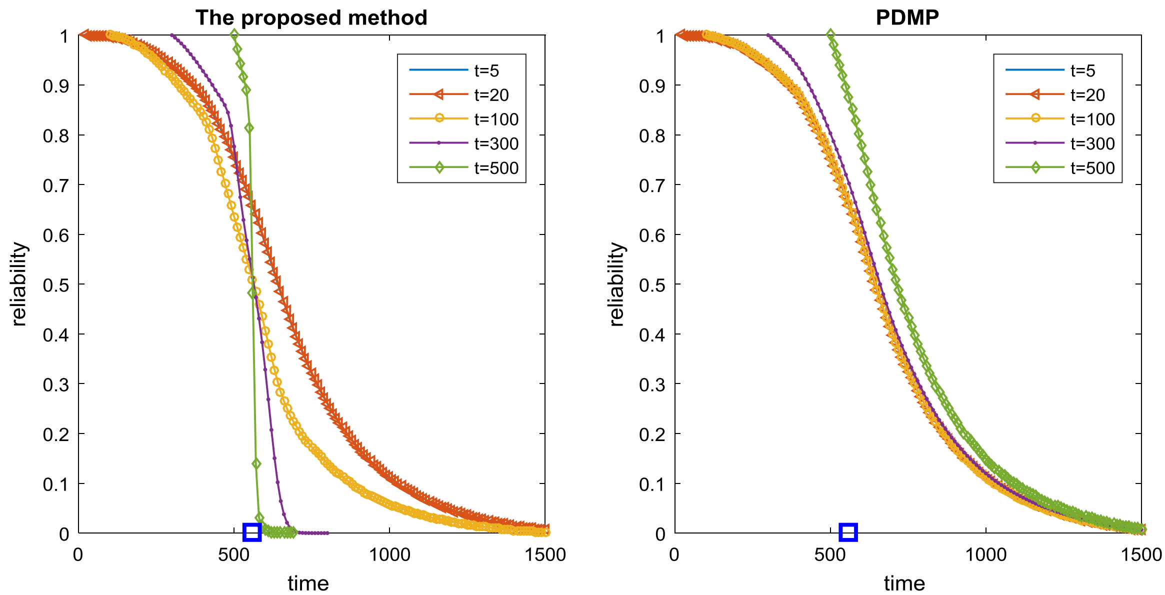

Fig. 19. Reliability of test scenario 1, given the true initial system state and the monitored data until different times$t$, using the proposed method and PDMP. (The true failure time is marked as a square on x axis.).

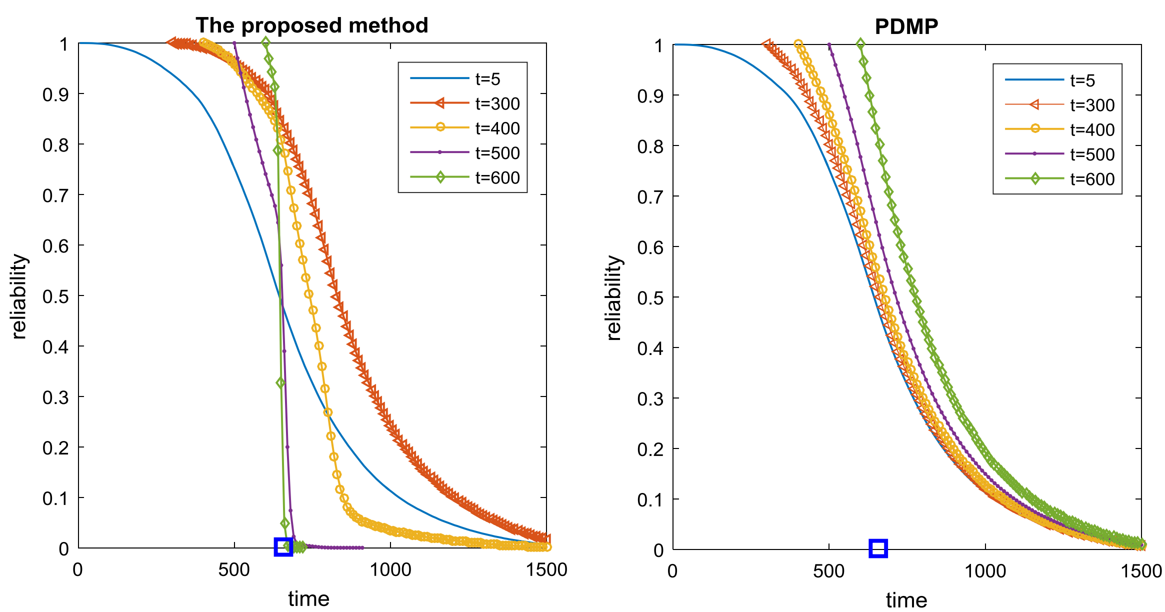

Fig. 20. Reliability of test scenario 2, given the true initial system state and the monitored data until different times$t$, using the proposed method and PDMP. (The true failure time is marked as a square on x axis.).

either be $M_{p,k}$ or$M_{p,k-1}$, and $p(s_{p,t+1,M_{p,k-1}}s_{p,t,M_{p,k}})=1-p(s_{p,t+1,M_{p,k}}s_{p,t,M_{p,k}})=1-e^{-\lambda}$.

The degradation of the pneumatic valve is a continuous degradation process whose degradation rate is dependent on the degradation state of the pump. The relation between the degradation rate of the valve and the degradation state of the pump is $s'_{v,t}=10^{-8}*(4-1.5*(s_{p,t}-1))$, with $s_{p,t}$ the degradation state of the pump and $s'_{v,t}$ the degradation rate of the valve. For example, if the degradation state of the pump at time t is state 2, the degradation rate of the valve is $s'_{v,t}$=2.5*10^{-8}. These relations are simplified by the authors, without loss of generality. As mentioned in the second specification of the system in Section 2, in practice this relation can be obtained according to physical laws or statistically estimated from the historical data. In this paper, we did not focus on how to obtain this relation, but assumed it already available, as the focus was mainly on how to estimate dynamically the RUL and reliability of the system.

The degradation state of the valve is monitored every second with sensor noise, i.e. $x_t=s_{v,t}+\varepsilon$, with x_t the monitored value, $s_{v,t}$ the degradation of the valve and $\varepsilon\in N(0,8*10^{-8})$ the noise. $N(\mu,\sigma)$ represents a Gaussian distribution with a mean of $\mu=0$ and a standard deviation of $\sigma=8*10^{-8}$.

In order to simplify the system analysis, we suppose that the transition of the state of the pump occurs only once between two observations (monitored values) and occurs right before the observation, i.e. the state of the pump is fixed between two observations. Thus, the state of the valve at the next observation time can be expressed as $s_{v,t+1}=s_{v,t}+s'_{v,t}*1+\gamma$, with $\gamma\in N(0,4*10^{-8})$ being the uncertainty related to the operation condition and environment. This assumption is correct under the condition that the transition rate is very low, as expected in practice. Fig. 4 shows the true probabilities of the pump under different states and the estimated probabilities under the previous assumption. One can observe that the difference is very small. Thus, the assumption is reasonable. In fact the assumption is arbitrary,

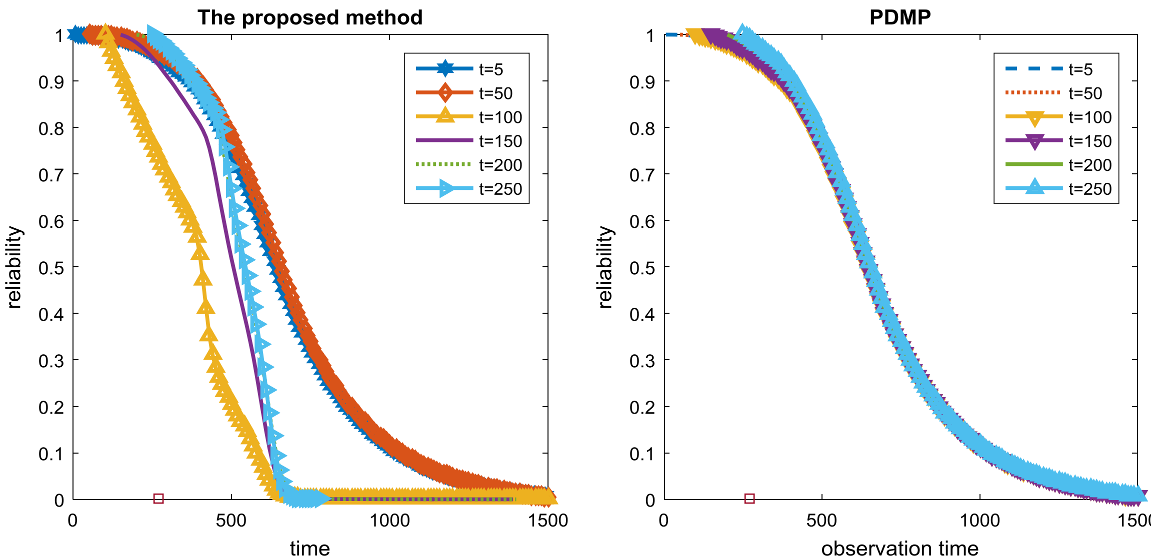

Fig. 21. Reliability of test scenario 3, given the true initial system state and the monitored data until different times t, using the proposed method and PDMP. (The true failure time is marked as a square on x axis.).

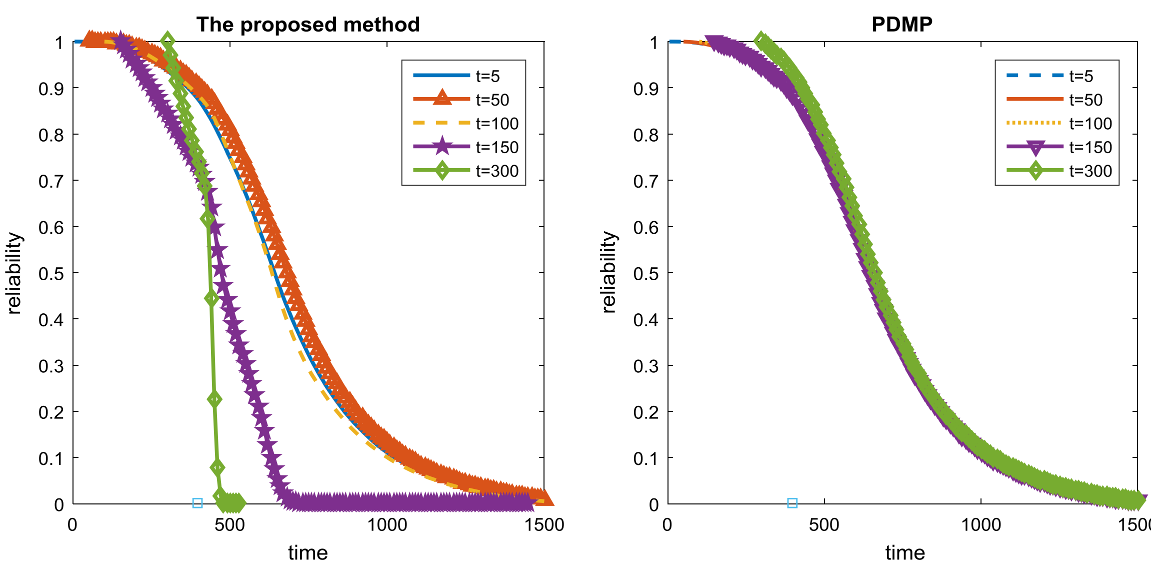

Fig. 22. Reliability of test scenario 4, given the true initial system state and the monitored data until different times t, using the proposed method and PDMP. (The true failure time is marked as a square on x axis.).

we can use a small time unit for the cases of a larger transition rate, e.g. instead of 1 time unit, one can use 1/1000 time unit. This assumption allows simplifying the Monte-Carlo simulation.

The threshold for the failure of the valve is $Th = 1.5*10^{-5}$.

The above values are set by the authors, referring to the previous work of Lin et al. [10].

The system failure can be caused by pump failure or valve failure. In this paper, four failure scenarios are considered with two scenarios with pump failure and the other two with valve failure. The details of the considered failure scenarios are shown in Table 1.

5.1. Case with unknown initial state of the system

We assume that monitored values are available from time 1 to time t and the true degradation state of the system is not known for any time. In this situation, the deterministic method proposed in Lin et al. [10] (briefly introduced in Appendix) does not work, as PDMP needs to be initialized with a precise system state.

The degradation state, $[0, Th]$ of the valve is discretized into 500 states, noted as $M_{v,i}, i = 1,2, \ldots, 500$, with $M_{v,1}$ the failure state of the valve, i.e. the degradation is Th, and $M_{v,500}$ the perfect functioning state of the valve, i.e. the degradation is 0. $s_{v,t,M_{v,i}}$ means that the degradation state of the valve at time t is $M_{v,i}$. And $\Delta s_v = Th/499$.

With the measured value at time 1, the conditional probability $p(s_{v,1,M_{v,i}} \mid x_1)$ at this time is $\frac{p(x_1 s_{v,1,M_{v,i}}) p(s_{v,1,M_{v,i}})}{\Delta s_v \sum_{j=1}^{500} pp(x_1 s_{v,1,M_{v,i}}) p(s_{v,1,M_{v,i}})}$, with $p(s_{v,1,M_{v,i}})=1/500$ and $i, j \in \{1,2, \ldots, 500\}$. As the system is working, the degradation state of the pump is supposed to be $s_{p,1} \in \{M_{p,3}, M_{p,2}, M_{p,1}\}$ and $p(s_{p,1,M_{p,3}}) = p(s_{p,1,M_{p,2}}) = p(s_{p,1,M_{p,1}}) = 1/3$, with $s_{p,t,M_{p,j}} j = 3,2,1$, meaning that the state of the pump at time t is M_j. Thus, the conditional probability of the system state at time 1 is $p(\{s_{v,1}, s_{p,1}\} x_1) = p(s_{v,1} \mid x_1) p(s_{p,1})$. With the recursive Bayesian approach expressed as Eq. (10), we can estimate $p(\{s_{v,t}, s_{p,t}\} \boldsymbol{X}_{1:t})$, with $s_{p,t} \in \{M_{p,3}, M_{p,2}, M_{p,1}\}$ and $s_{v,t} \in [0, Th)$.

The number of replications in the parallel Monte-Carlo simulation is 10^5.

Recursive Bayesian method in Section 3.1 is first used to estimate

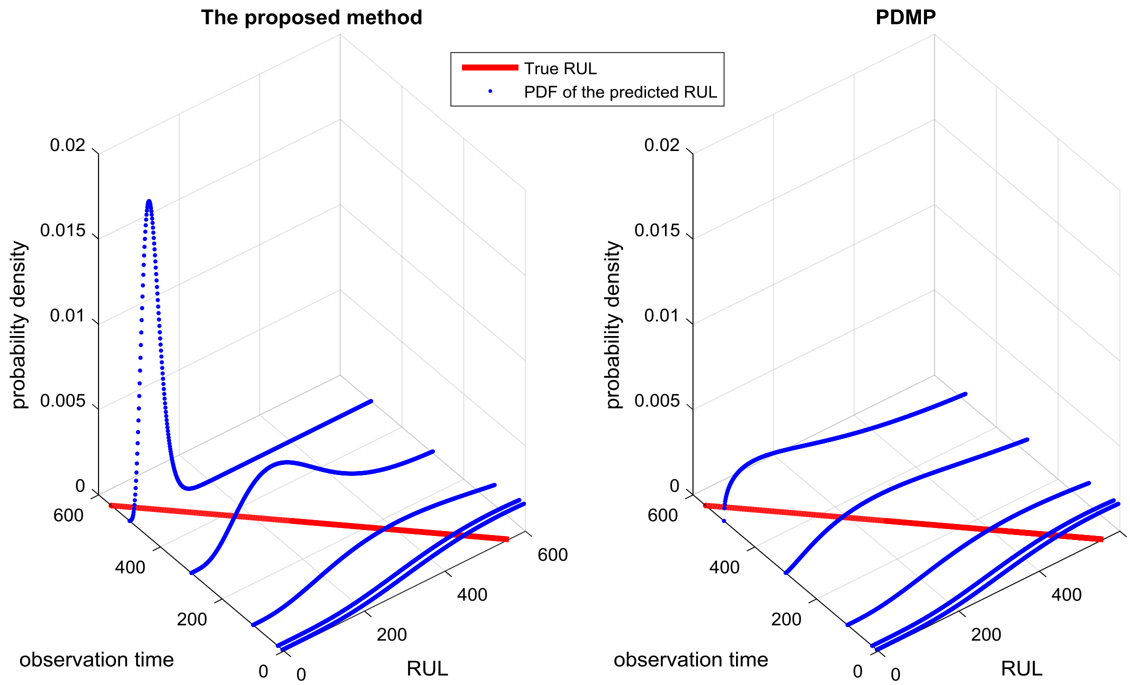

Fig. 23. PDFs of the predicted RUL for test scenario 1, given the true system state at time 0 and the monitored data until different times t, using the proposed method and PDMP.

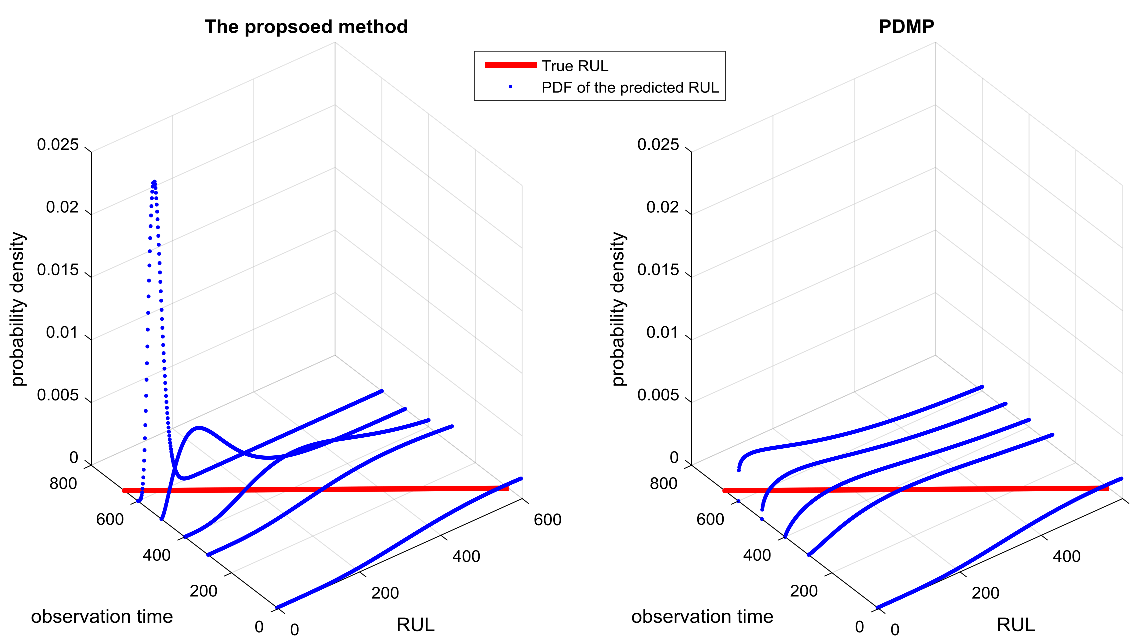

Fig. 24. PDFs of the predicted RUL for test scenario 2, given the true system state at time 0 and the monitored data until different times t, using the proposed method and PDMP.

the system state (pump and valve), then the parallel Monte-Carlo simulation is used to estimate the system RUL and reliability.

Figs. 5–8 show the estimated pump and valve degradation states given the monitored data until different time instances and the true pump and valve degradation state at that time instance.

From these Figures, it is shown that the recursive Bayesian network can effectively estimate the valve degradation state using the monitored data, as it is directly related to the valve degradation.

Given the holding times of the pump under different states in Table 1, one can also observe that the recursive Bayesian method can also estimate properly the degradation state of the pump whose holding time is larger 60. For example, the holding time of the pump on state 3 in Scenario 1 is only 22. Without knowing the initial degradation state of the pump, the probabilities of the estimated pump degradation on states 3 and 2 are comparable (left figure of Fig. 5), i.e. there are not enough measured data for the proposed method to recognize precisely the pump degradation state. This is because the recursive Bayesian method needs a number of data to update the conditional probabilities of the pump degradation state, i.e. there is a delay of the recursive Bayesian method to detect correctly the pump degradation state. However, when the holding time is larger (normally larger than 60), the recursive Bayesian method can estimate correctly the pump degradation state. For example, the holding time of the pump on state 3 in Scenario 2 is 333. After about 50 measure value, the probability of the estimated pump degradation state on State 3 is always larger than 70% until 350 when the pump state transits to State 2 (left figure of

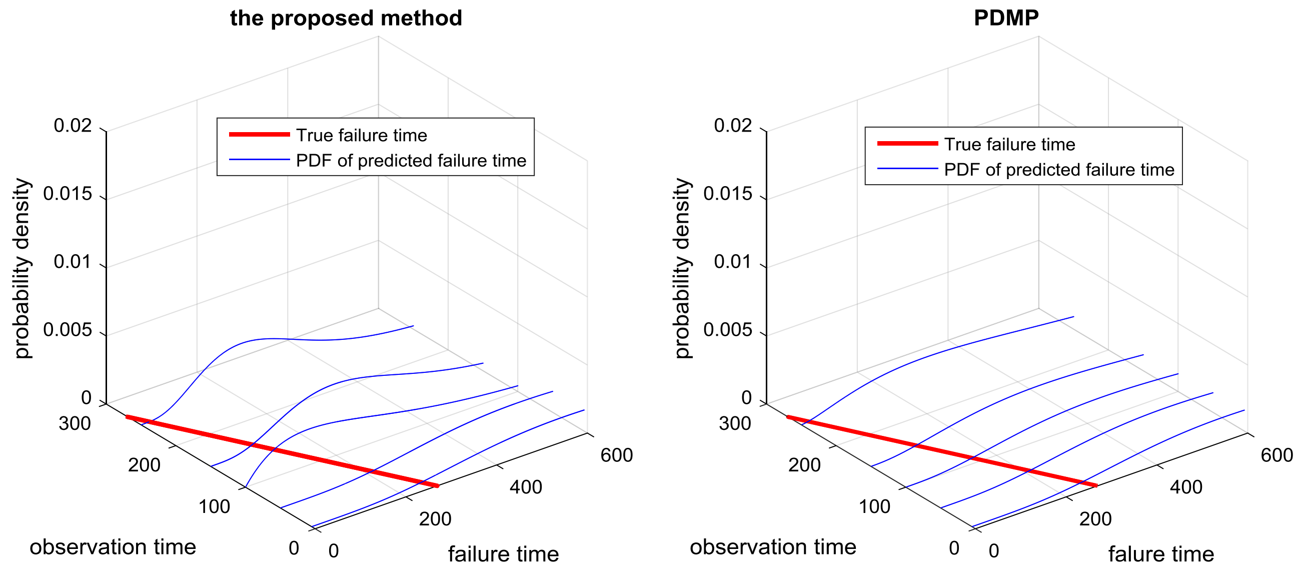

Fig. 25. PDFs of the predicted RUL for test scenario 3, given the true system state at time 0 and the monitored data until different times t, using the proposed method and PDMP.

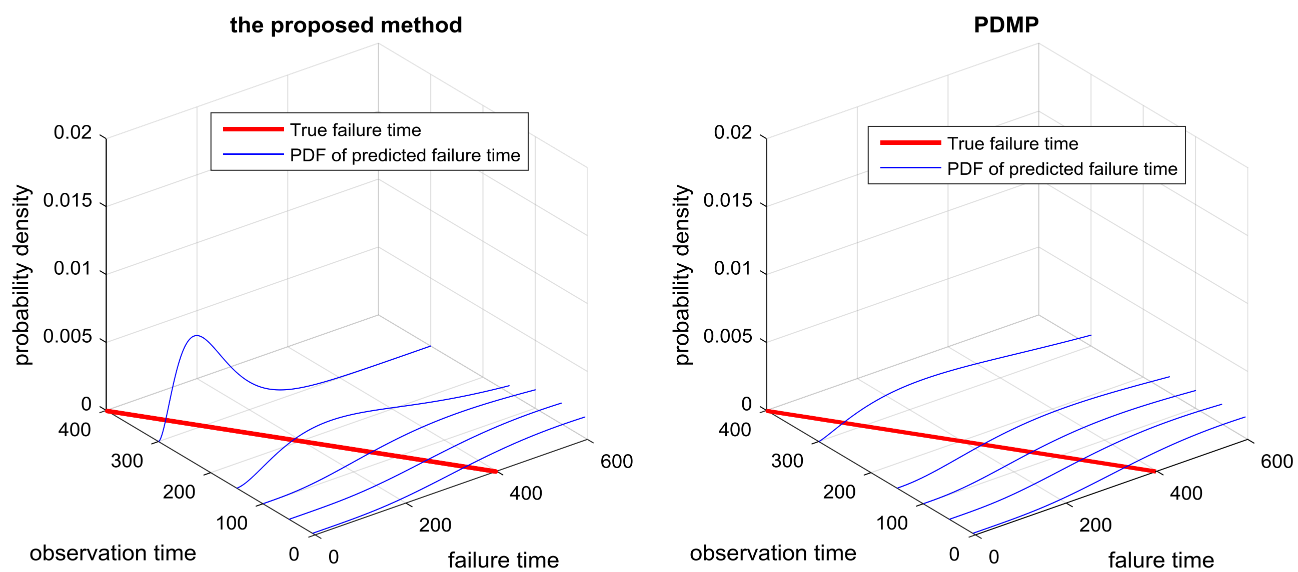

Fig. 26. PDFs of the predicted RUL for test scenario 4, given the true system state at time 0 and the monitored data until different times t, using the proposed method and PDMP.

Fig. 6). The left figure of Fig. 7 shows that the proposed method does not catch effectively the pump degradation state, as the holding times of the pump on States 3 and 1 are too short.

Given the monitored data until different time instances, the reliability at different times $t+t'$ and the RUL of the system can be estimated. The results are shown separately in Figs. 9–16. These figures show that the proposed framework can effectively update the estimated reliability and predicted RUL with the monitored data. It works well especially for the system with valve failure (as shown in Figs. 9–12) and the results are improved with more and more measured data. For example, in Fig. 9 it is shown that the reliability assessment given the monitored data until time t = 500 drops sharply close to the true system failure time, which is 559. On the contrary, with monitored data only available until time t = 100, the reliability of the system is still high. The results in Fig. 9 show that the proposed method, which uses monitored data of a specific system for dynamically assessing its reliability, gives more and more reliable results as more and more monitored data become available with time.

The proposed framework works no so well on the system with pump failure, especially for Scenario 3. With a transition rate of 0.003/t, the mean transition time is around 333. But the holding times of the pump on different states in Scenario 3 are smaller than the mean transition time. The proposed framework does no react quickly enough to capture the true pump degradation state and, thus, the proposed framework does not work for systems with pump failure as well as those with valve failure.

From Fig. 10 and Fig. 12, it is observed that the Probability Density function (PDF) of the RUL can also be updated, based on the monitored data. As more monitored data is available, the uncertainty bounds of the RUL PDF become narrower.

Fig. 17 shows the true system state and estimated probability of the system state at observation time t = 5, 20, 100, 300, using the proposed method for Scenario 1. From Fig. 17, it is observed that the holding time of the pump at state 3 is only 22, with a transition rate λ_{32}=0.003/t, and the proposed method can quickly capture the change of state based on the monitored data (until time t = 100 and t = 300), as shown in the lower two plots in Fig. 17. The Figure shows that only a small part of all the possible system states have a non-zero probability

Table 2
AE of the estimated RUL of the proposed method and PDMP, at different observation times.

	Scenario 1					Scenario 2				
Observation time	5	20	100	300	500	5	300	400	500	600
The proposed method	**114**	**115**	**26**	**9**	**1**	**17**	212	81	**27**	**13**
PDMP	116	116	118	146	216	**17**	**47**	**72**	117	181
	Scenario 3					Scenario 4				
Observation time	5	50	100	150	250	5	50	100	150	300
The proposed method	**404**	420	**111**	**222**	**262**	**274**	314	**266**	**65**	**20**
PDMP	405	**405**	407	411	426	306	**274**	278	282	307

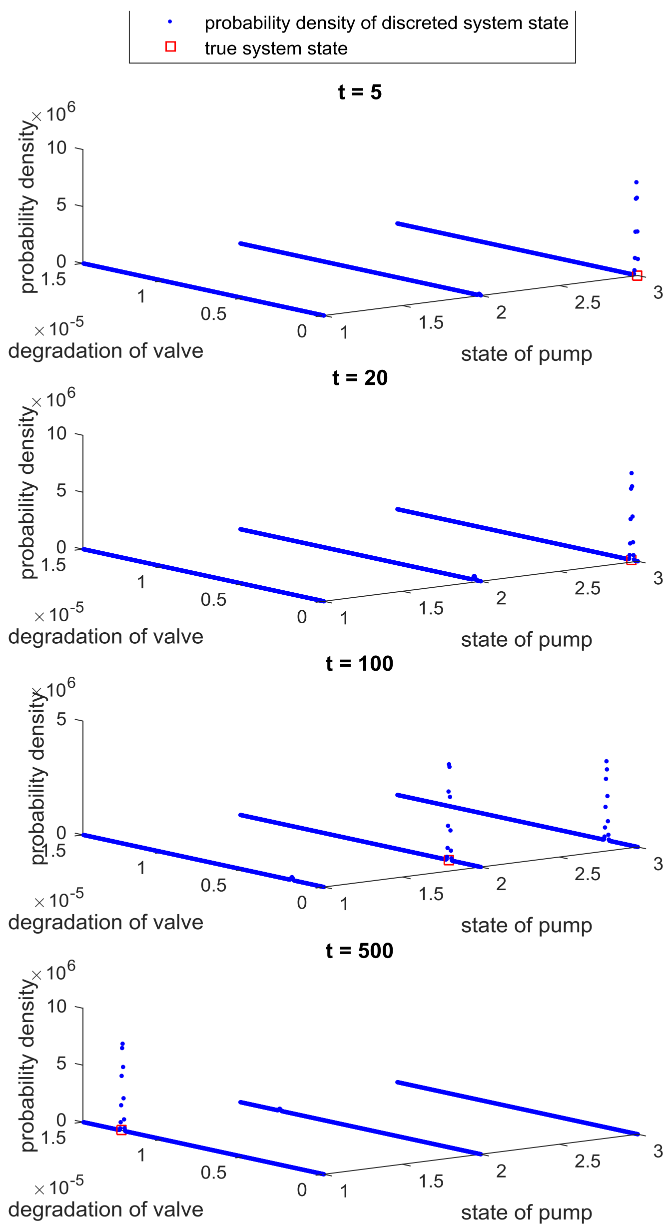

Fig. 27. True and estimated probability of each system state of test scenario 1 at observation time t = 5, 20, 100, 500, using the proposed method.

with the monitored data. Thus, the parallel Monte Carlo simulation is carried out for the system states that have a non-zero probability. Also, advanced filtering methods and Monte Carlo simulation techniques can

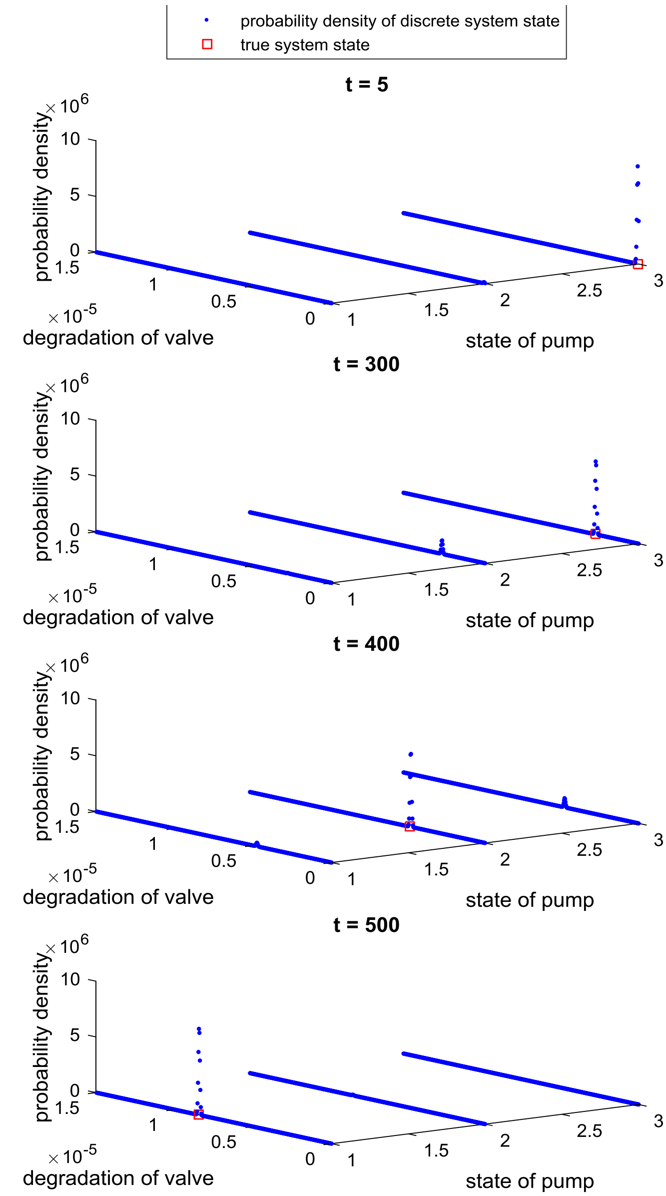

Fig. 28. True and estimated probability of each system state of test scenario 1 at observation time t = 5, 20, 100, 500, using PDMP.

be considered to improve the computational efficiency and reduce the computational burden. For example, particle filtering approaches can be used for estimating a posterior probability distribution of the system states with the state equations and observation equations in the second and third specifications of the system in Section 2. Combined with Monte Carlo simulation, the RUL of the system can be derived.

5.2. Case with known initial state of the system

In this Section, we assume that monitored values are available from time 1 to time t and the system at time 0 is known to be at the perfect state, i.e. $s_{v,0} = M_{v,500}$, $s_{p,0} = M_{p,3}$ and $s_{s,0}=\{M_{v,500}, M_{p,3}\}$. Considering this initial state, the conditional probability of the state of the pump at time 1, is $p\left(s_{p,1,M_{p,3}} \mid s_{p,0,M_{p,3}}\right) = e^{-\lambda}, p\left(s_{p,1,M_{p,2}} \mid s_{p,0,M_{p,3}}\right)=1-e^{-\lambda}, p\left(s_{p,1,M_{p,1}} \mid s_{p,0,M_{p,3}}\right)=0$. And the conditional probability of the valve at state $M_{v,i}, i = 1,2, \ldots, 500$ is $p\left(s_{v,1,M_{v,i}} x_1, s_{s,0}\right) = \frac{p\left(x_1 s_{v,1,M_{v,i}}\right) p\left(s_{v,1,M_{v,i}} s_{s,0}\right)}{\Delta s_v \sum_{j=1}^{500} p\left(x_1 s_{v,1,M_{v,j}}\right) p\left(s_{v,1,M_{v,j}} s_{s,0}\right)}$. Thus, the conditional probability of the system state at time 1 is

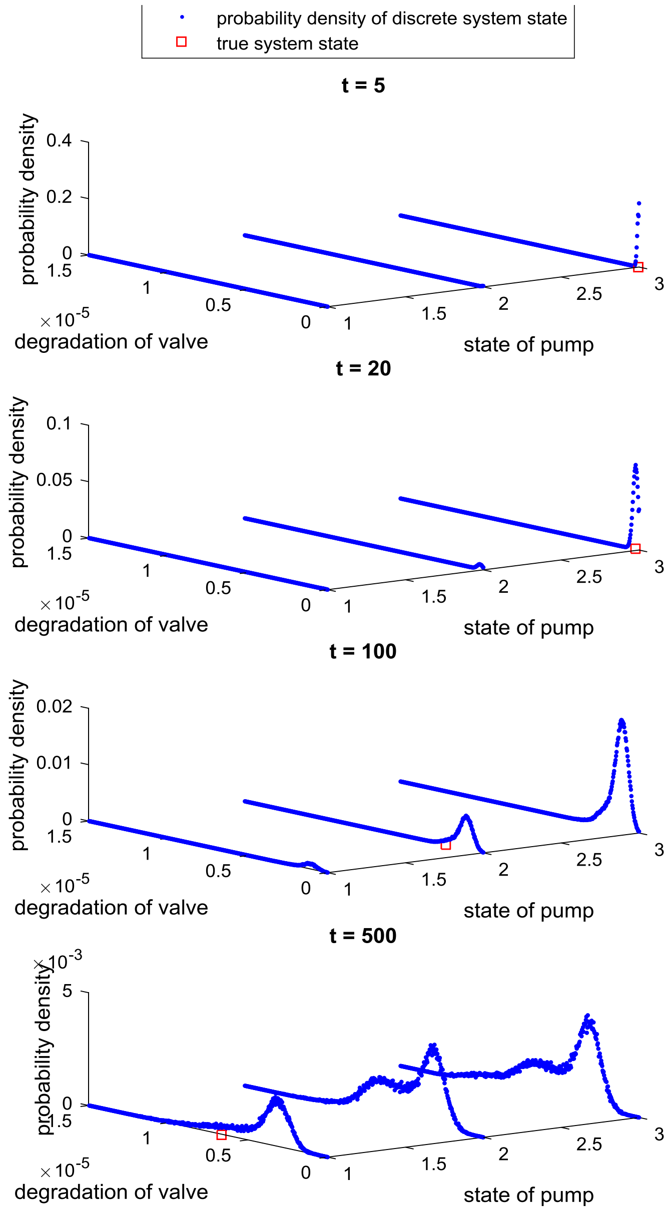

Fig. 29. True and estimated probability of each system state of test scenario 2 at observationtime t = 5, 300, 400, 500, using the proposed method.

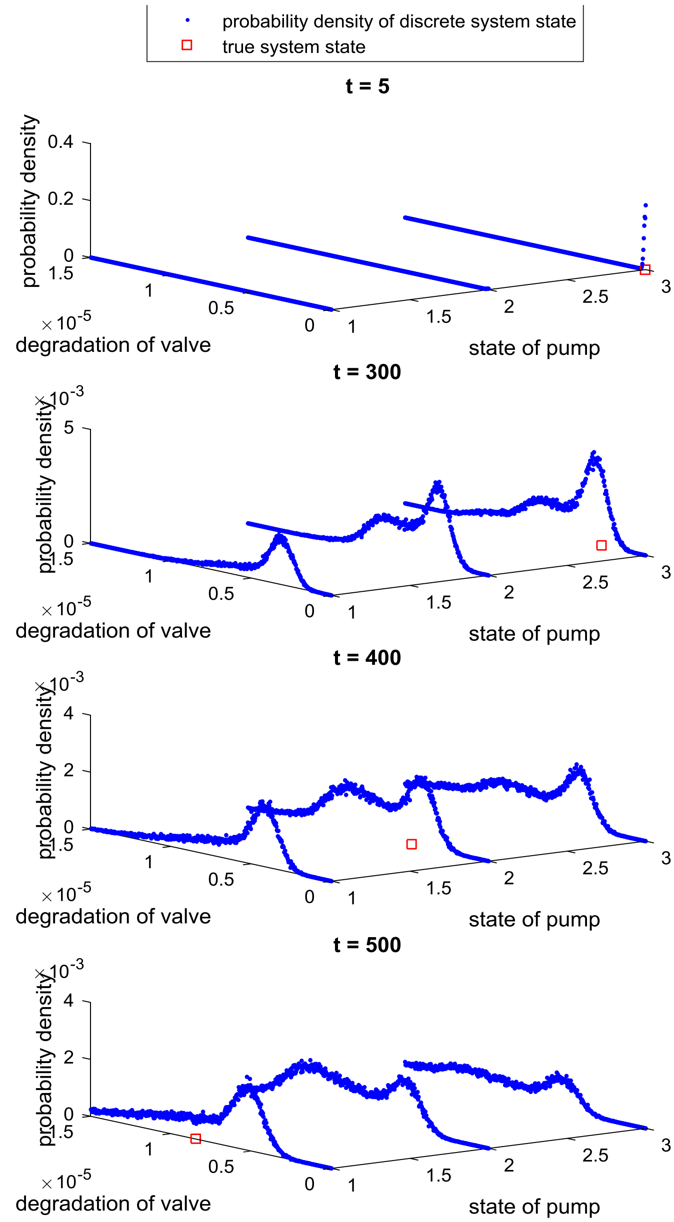

Fig. 30. True and estimated probability of each system state of test scenario 2 at observationtime t = 5, 300, 400, 500, using PDMP.

$p(\{s_{v,1}, s_{p,1}\}x_1, s_{s,0}) = p(s_{v,1} \mid x_1, s_{s,0})p(s_{p,1} \mid s_{p,0,M_{p,3}})$. With the recursive Bayesian approach expressed as Eq. (10), we can estimate $p(\{s_{v,t}, s_{p,t}\}X_{1:t})$, with $s_{p,t}\in\{M_{p,3}, M_{p,2}, M_{p,1}\}$ and $s_{v,t}\in[0, Th)$.

The Piecewise Deterministic Markov Process (PDMP) method proposed in Lin et al. [10] is used as a benchmark method to compare the results of reliability assessment and prognostics. The procedure for updating the results of PDMP is shown in Fig. 18.

The monitored value at each time t shows if the system is functioning or not. If the system is functioning, the degradation evolutions for which the system has already failed at time t are eliminated, and, then, the RUL and the reliability of the system is calculated based on the remaining degradation evolutions, as for the Monte Carlo simulation in Section 4.

In this Section, the four scenarios in Table 1 are considered. The true initial state of the system in the two test scenarios are known, i.e. $\{M_{v,500}, M_{p,3}\}$. Figs. 19–22 show the estimated reliability for these scenarios, with different available monitored data.

It is observed that PDMP gives similar reliability estimations for different test scenarios, while the proposed method can dynamically update the assessed reliability with the monitored data. The proposed method gives relatively better results for scenarios 1, 2 and 4, while worse results for scenario 3. This is caused by the fact that the system in scenario 3 suffers a pump failure and the holding time of the pump on the state 1 is too short (22) and, thus, there are not enough monitored data for the proposed method to catch the transition of the pump state from state 2 to state 1. For scenario 4 with also pump failure, the proposed method gives better results, as the holding time of the pump in scenario 4 is larger and, thus, the proposed method can correctly estimate the pump degradation state before its failure.

Similarly, in the experiments, the PDFs of the predicted RUL given by the proposed method is more accurate and precise than PDMP, as shown in Figs. 23–26.

Table 2 shows the Absolute Error (AE) given by the proposed method and PDMP at different times. The Absolute Error is calculated as $|\hat{RUL}_{s,t} - RUL_{s,t}|$, with $\hat{RUL}_{s,t}$ the predicted RUL of the system at time t and $RUL_{s,t}$ the true RUL of the system at time t.

From Table 2, one can observe that the proposed method and PDMP give comparable results at the beginning, e.g. $t = 5$. But as time goes on, PDMP gives worse predicted RUL for the specific test scenarios. On the contrary, the method proposed in this paper can capture well the specific system state with the monitored data and gives more accurate results on the predicted system RUL. Some clue can be found in Figs. 27–30, which present the PDFs of the estimated system state at different times using the proposed method and PDMP, for the two test scenarios. In comparison with Fig. 17, with a known initial system state, the number of possible system state with non-zero probability is less in Fig. 27. Although the proposed framework gives worse results for scenario 3 than for other scenarios, its AE is still smaller than PDMP in the experiments.

Theoretically, with the same replication times, the computation time of the proposed method should be much longer than that of PDMP, as the calculation of a posterior probability of system states given the monitored data takes considerable time. However in the experiment, the replications is 10^5 for both PDMP and parallel Monte Carlo simulation, and the computation time of PMDP and the proposed method for observation time $t = 200$ are not significantly different, because in the experiment, $p(\boldsymbol{s}_{s,t+1}\boldsymbol{X}_{1:t})$ is calculated as $\sum_{\boldsymbol{s}_{s,t}:\, p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t})\neq 0} p(\boldsymbol{s}_{s,t+1}\boldsymbol{s}_{s,t})*p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t})$ instead of $p(\boldsymbol{s}_{s,t+1}\boldsymbol{X}_{1:t}) = \sum_{all\boldsymbol{s}_{s,t}} p(\boldsymbol{s}_{s,t+1}\boldsymbol{s}_{s,t})*p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t})$. The number of system states with a non-zero probability i.e. $p(\boldsymbol{s}_{s,t}\boldsymbol{X}_{1:t})\neq 0$ until time $t = 50$ has already reduced to 53 among the 1500 possible system states.

6. Conclusions and perspectives

The dynamic reliability assessment and failure prognostics of a system with monitored data is drawing much attention for the great practical opportunity of tailoring the results on the specific system behavior and not on an average behavior. In this paper, a recursive Bayesian method-based framework is proposed to dynamically assess the reliability and predict the RUL of a system with multiple dependent components, whose degradation processes can be continuous. Uncertainties are considered in the degradation model and noise exist in the monitored data. Recursive Bayesian method estimates the system state with the monitored data for the considered system, and, then, a parallel Monte Carlo simulation is carried out to estimate the reliability and RUL of the system. The results can be updated with new monitored data.

The results for a case study show that the proposed method gives reliable and accurate reliability and RUL results. In comparison with PDMP, the proposed method is more capable at capturing the dynamic degradation process of the considered system, even if it is less effective for the scenarios with low possibilities.

The transition of the states for a multi-state component is supposed to occur at most once between two observation time and right before the second observation. If no constraints is posed on the transitions, the estimation of the conditional probability of the system state will be much more complicated. But the proposed framework still works.

In this paper, the continuous state has been discretized and, in order to be precise, a larger number of states has been created. In the experiment, it is shown that only a small number of all the possible system state are with non-zero probability given the monitored data. Advanced filtering methods should be integrated for reducing the number of possible system states.

Appendix

Brief introduction of PDMP method proposed in Lin et al. [10].

The considered system is composed of dependent degrading component. $\overrightarrow{Z}' = \begin{pmatrix} \overrightarrow{X}(t) \\ \overrightarrow{Y}_t \end{pmatrix}$ represents the system degradation state at time t, where $\overrightarrow{Y}_t$ is the state of the components following a stochastic multiple state degradation process and $\overrightarrow{X}(t)$ is the state of the components following a continuous degradation process which is dependent on $\overrightarrow{Y}_t$. These relations are known. $\mathcal{F}_X$ is the failure threshold of the components in $\overrightarrow{X}(t)$.

Below is the pseudo-code of PDMP:

SetN_{max} (the maximum number of replications) and k=0 (index of replication)
Set$k' = 0$ (number of trials that end in the failure state)
While$k < N_{max}$
Initialize the system by setting $\overrightarrow{Z}' = \begin{pmatrix} \overrightarrow{X}(0) \\ \overrightarrow{Y}_0 \end{pmatrix}$ (initial state), and the time T=0 (initial system time)
Sett'=0 (state holding time)
While$T < T_{miss}$
Sample a holding time t' by using the probability density function of the holding time at current degradation state given the external influencing factors
Sample an arrival state $\overrightarrow{Y}'$ for stochastic process $\overrightarrow{Y}(t)$ from all the possible states by using the conditional probability function of component state given the holding time
Set $T = T + t'$
Calculate $\overrightarrow{X}(T)$ by using the physics equations
Set $\overrightarrow{Z}' = \begin{pmatrix} \overrightarrow{X}(T) \\ \overrightarrow{Y}' \end{pmatrix}$
If$T \leq T_{miss}$
Calculate all the extreme values $(\overrightarrow{X}_n^{\,o}, n\in\mathbb{N})$ of $\overrightarrow{X}(t)$ in the interval $[T - t', T]$ by using the physics equations
If $(\exists\, m\in\mathbb{N}, \overrightarrow{X}_m^{\,o} \in \mathcal{F}_X) \quad \vee (\overrightarrow{Z}' \quad \in \mathcal{F})$
Set$k' = k'+1$

Break
End if
Else (when $T > T_{miss}$)
Calculate all the extreme values ($\vec{X}_n^{\,o}$, $n \in \mathbb{N}$) of $\vec{X}(t)$ in the interval $[T - t', T_{miss}]$ by using the physics equations
If $\exists\ m \in \mathbb{N}, \vec{X}_m^{\,o} \in \mathcal{F}_X$
Set $k' = k' + 1$
Break
End if
End if
End While
Set $k = k + 1$
End While

The estimated reliability of component at time T_{miss} can be obtained by

$$\hat{R}(T_{miss}) = 1 - k'/N_{max}$$

where k' is the number of trials that the system fails, and the sample variance is:

$$var_{\hat{R}(T_{miss})} = \hat{R}(T_{miss})(1 - \hat{R}(T_{miss}))/(N_{max} - 1)$$

References

[1] Cocozza-Thivent C, Eymard R, Mercier S. A finite-volume scheme for dynamic reliability models. IMA J Numer Anal 2006;26(3):446–71.
[2] Dong M, He D. A segmental hidden semi-Markov model (HSMM)-based diagnostics and prognostics framework and methodology. Mech Syst Signal Process 2007;21(5):2248–66.
[4] Ghasemi A, Yacout S, Ouali MS. Evaluating the reliability function and the mean residual life for equipment with unobservable states. Reliab IEEE Trans on 2010;59(1):45–54.
[5] Heng A, Zhang S, Tan AC, Mathew J. Rotating machinery prognostics: state of the art, challenges and opportunities. Mech Syst Signal Process 2009;23(3):724–39.
[6] Jardine AK, Lin D, Banjevic D. A review on machinery diagnostics and prognostics implementing condition-based maintenance. Mech Syst Signal Process 2006;20(7):1483–510.
[7] Jiang, L., Feng, Q., Coit, D.W. Reliability analysis for dependent failure processes and dependent failure threshold. In: Proceedings of the IEEE international conference on quality, reliability, risk, maintenance, and safety engineering (ICQR2MSE); 2011. pp. 30–34.
[8] Keedy E, Feng Q. A physics-of-failure based reliability and maintenance modeling framework for stent deployment and operation. Reliab Eng Syst Saf 2012;103:94–101.
[9] Li W, Pham H. Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks. Reliab IEEE Trans on 2005;54(2):297–303.
[10] Lin YH, Li YF, Zio E. Fuzzy reliability assessment of systems with multiple-dependent competing degradation processes. Fuzzy Syst IEEE Trans on 2015;23(5):1428–38.
[11] Liu B, Xu Z, Xie M, Kuo W. A value-based preventive maintenance policy for multi-component system with continuously degrading components. Reliab Eng Syst Saf 2014;132:83–9.
[12] Liu Y, Zuo MJ, Li YF, Huang HZ. Dynamic reliability assessment for multi-state systems utilizing System-level inspection data. Reliab IEEE Trans on 2015;64(4):1287–99.
[13] Liu J, Zio E. A SVR-based ensemble approach for drifting data streams with recurring patterns. Appl Soft Comput 2016;47:553–64.
[14] Moghaddass R, Zuo MJ, Liu Y, Huang HZ. Predictive analytics using a nonhomogeneous semi-Markov model and inspection data. IIE Trans 2015;47(5):505–20.
[15] Orchard ME. A particle filtering-based framework for on-line fault diagnosis and failure prognosis [Doctoral dissertation]. Atlanta, USA: Georgia Institute of Technology; 2007.
[17] Ramirez-Marquez JE, Coit DW. A Monte-Carlo simulation approach for approximating multi-state two-terminal reliability. Reliab Eng Syst Saf 2005;87(2):253–64.
[18] Reggiani S, Poli S, Denison M, Gnani E, Gnudi A, Baccarani G, Wise R. Physics-based analytical model for HCS degradation in STI-LDMOS transistors. Electron Devices IEEE Trans on 2011;58(9):3072–80.
[19] Shin J, Son H, Heo G. Development of a cyber security risk model using Bayesian networks. Reliab Eng Syst Saf 2015;134:208–17.
[20] Si XS, Wang W, Chen MY, Hu CH, Zhou DH. A degradation path-dependent approach for remaining useful life estimation with an exact and closed-form solution. Eur J Oper Res 2013;226(1):53–66.
[21] Tombuyses B, Aldemir T. Computational efficiency of the continuous cell-to-cell mapping technique as a function of integration schemes. Reliab Eng Syst Saf 1997;58(3):215–23.
[22] Vitanza C. RIA failure threshold and LOCA limit at high burn-up. J Nucl Sci Technol 2006;43(9):1074–9.
[23] Ye ZS, Wang Y, Tsui KL, Pecht M. Degradation data analysis using Wiener processes with measurement errors. Reliab, IEEE Trans on 2013;62(4):772–80.