POLITECNICO DI MILANO

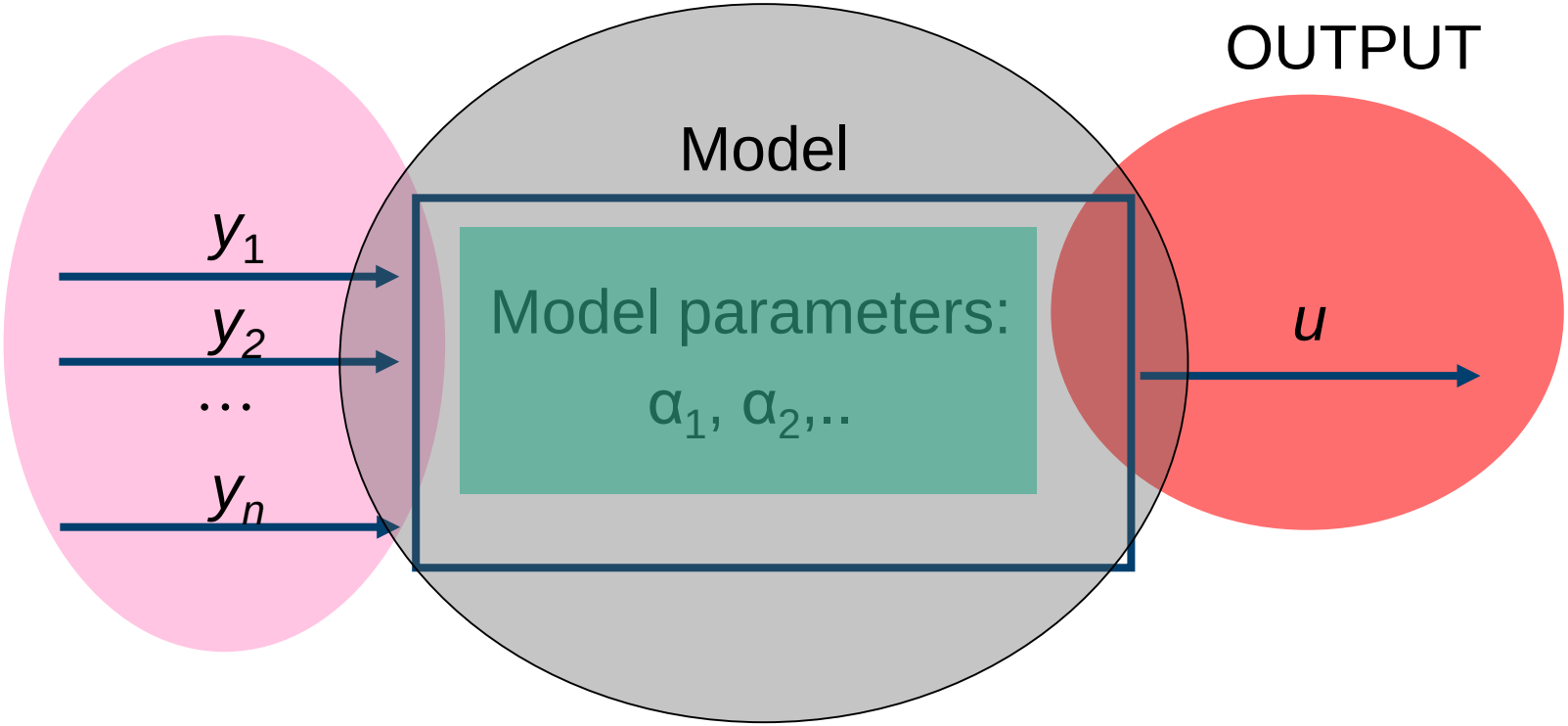




Monte Carlo and Possibilistic Methods for Uncertainty Analysis

Dr. P. Baraldi

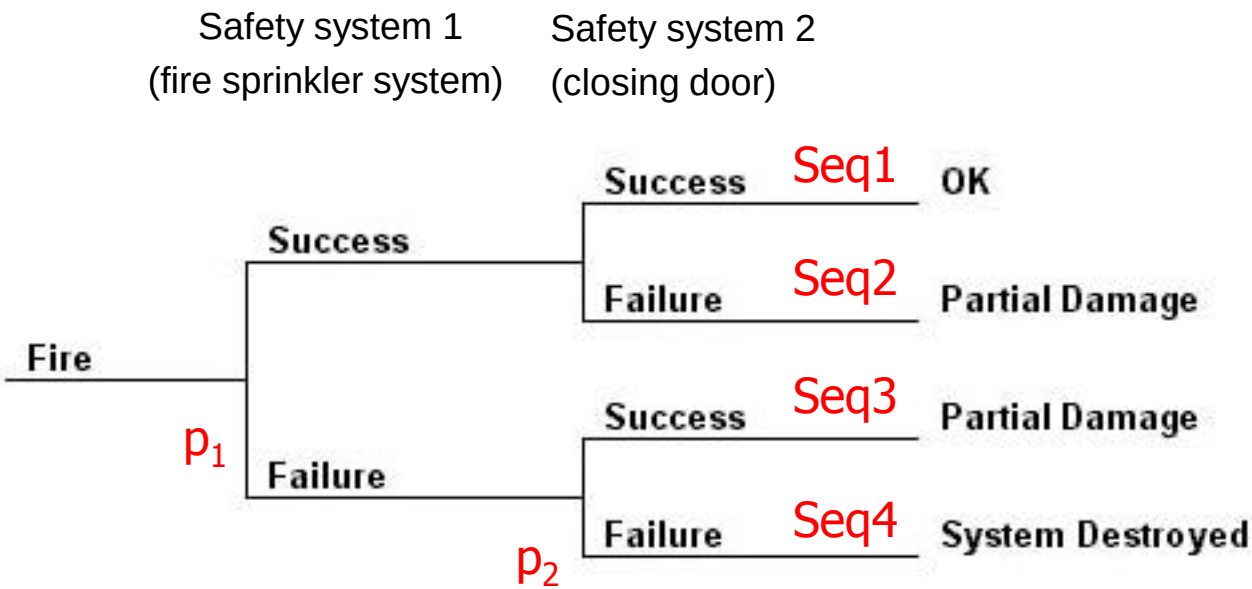
‘The Uncertainty Factor’

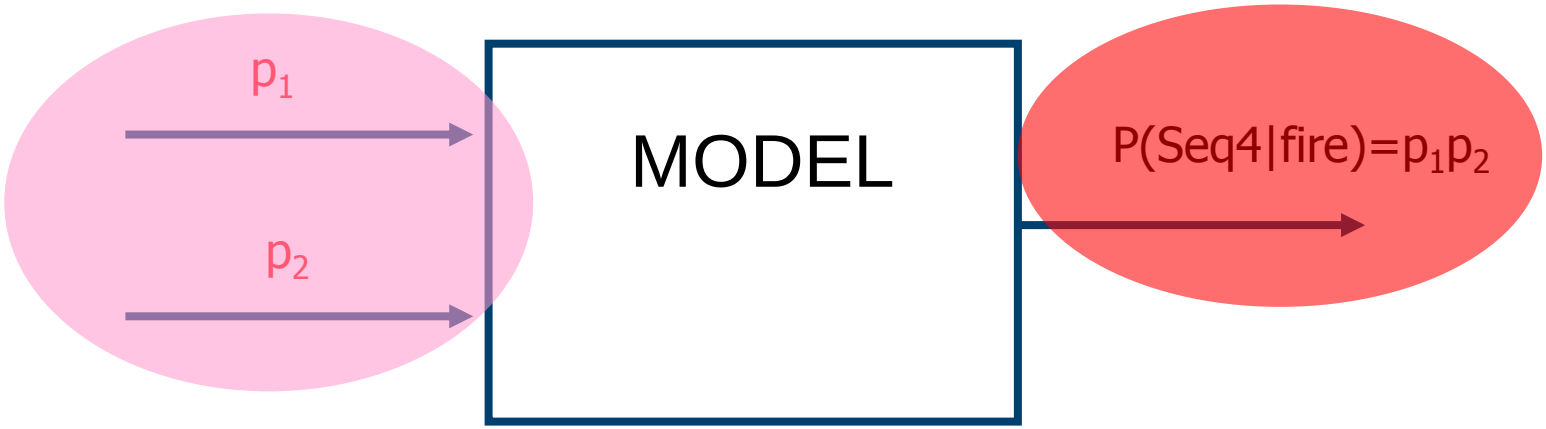


Uncertainty representation

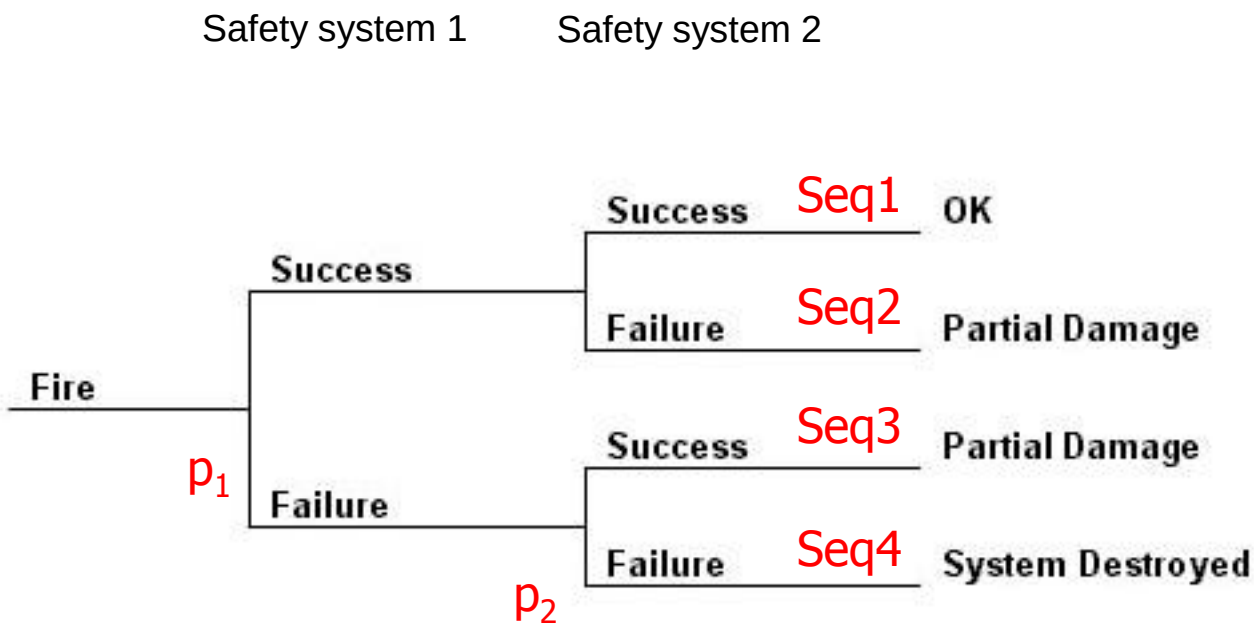
Uncertainty propagation

Example 1: Uncertainty in Event Tree analysis



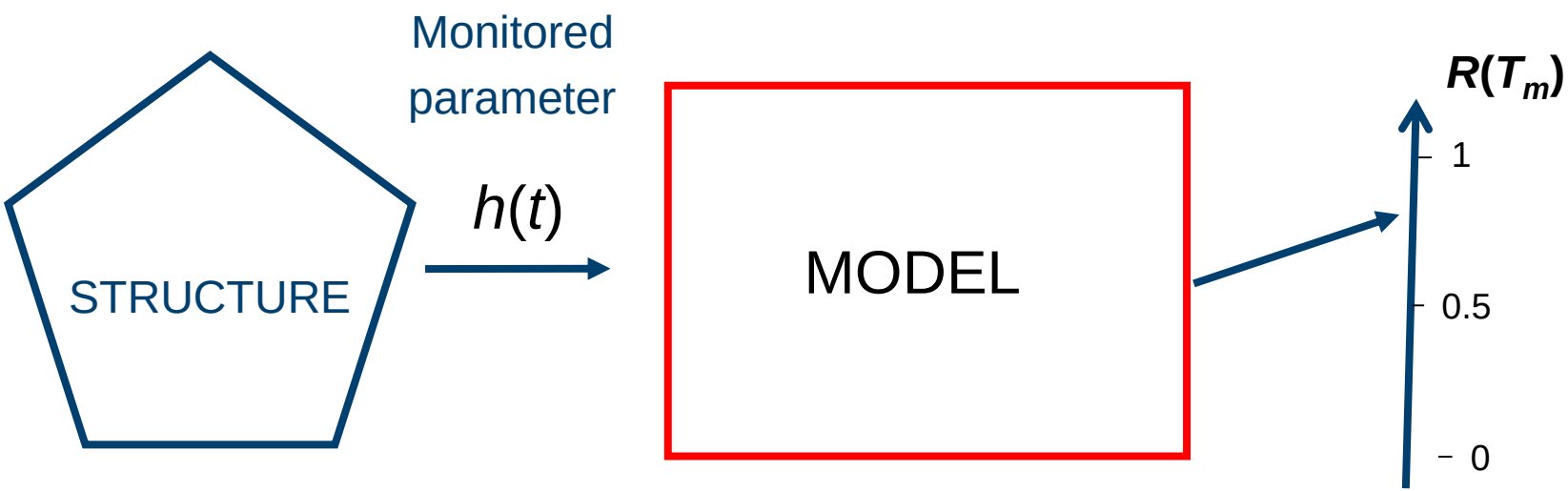


Example 1: Uncertainty in Event Tree Analysis



- How to characterize the uncertainty on the probability of occurrence of the events (p_1, p_2)?
- How to propagate the uncertainties from the single events to the sequences ($p(\text{seq}_4|\text{fire})$)?

Example 2: Reliability Assessment of a Degrading Structure



$h(t)$ = actual crack depth

$h(t) > h_{max}$ → structure fails

MODEL:

$$\frac{dh}{dt} = e^{\omega} C (\beta \sqrt{h})^{\eta}$$

h = actual crack depth

ω = random variable that describes the stochasticity of the degradation process

C, β, η = constants related to the material properties

t =time

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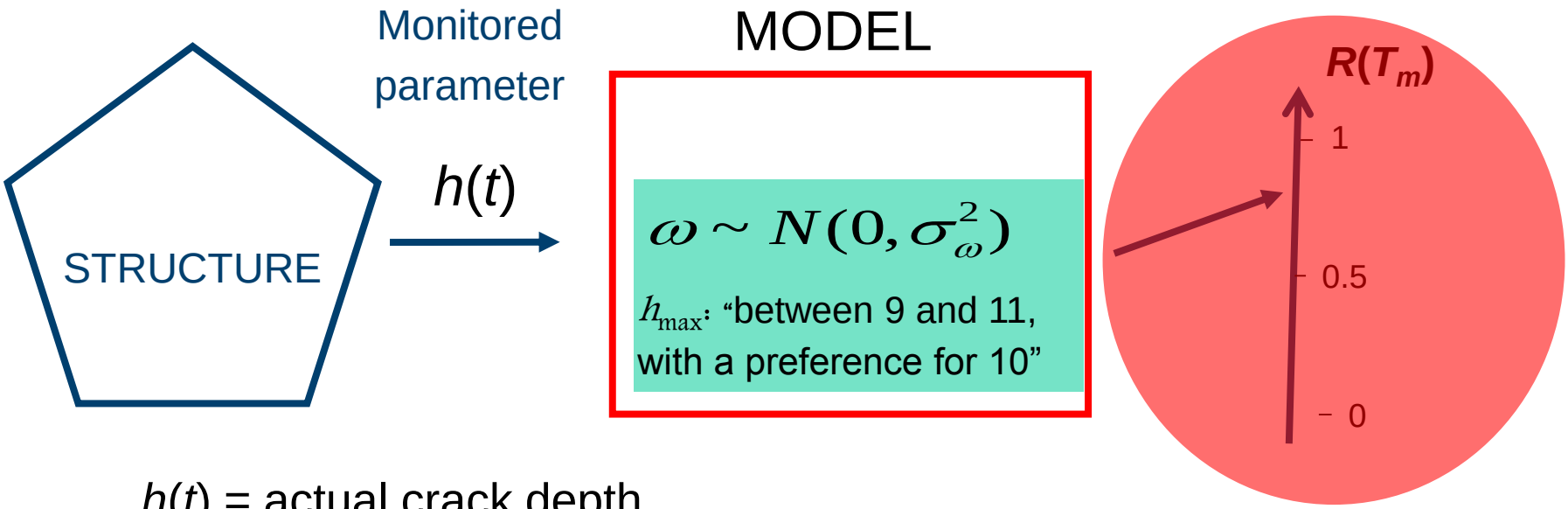

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h = actual crack depth

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C, β, η = constants related to the material properties

t =time

In This Lecture

Uncertainty representation

- Probability distributions
- Possibility distributions

Uncertainty propagation

- Level 1:
  - Purely probabilistic method
  - Purely possibilistic
  - Hybrid probabilistic and possibilistic method
- Level 2
  - Purely probabilistic method

Applications

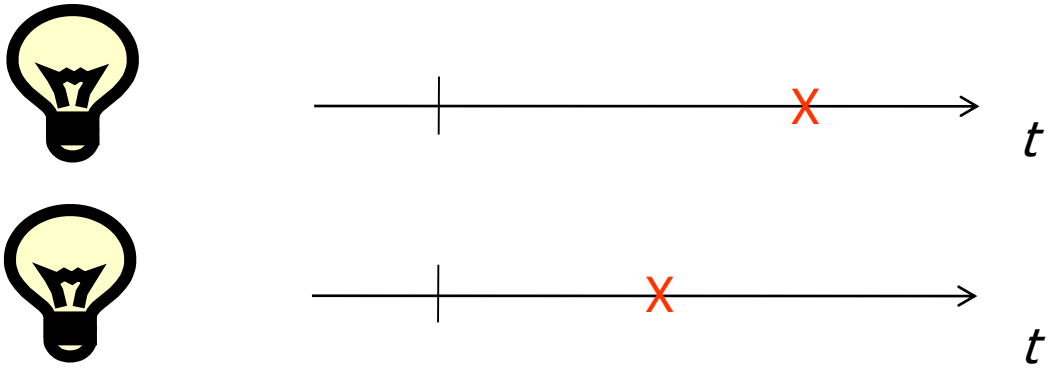
- Event tree uncertainty analysis
- Reliability assessment of a degrading component

PART I: Uncertainty Representation

Uncertainty classification

Uncertain quantity: we do not known its value!

Aleatory uncertainty: randomness due to inherent variability in the system



Epistemic uncertainty: imprecision due to lack of knowledge and information on the system

Uncertainty representation

- Aleatory uncertainty

probability distributions

- Epistemic uncertainty

Probability distributions

Possibility distributions

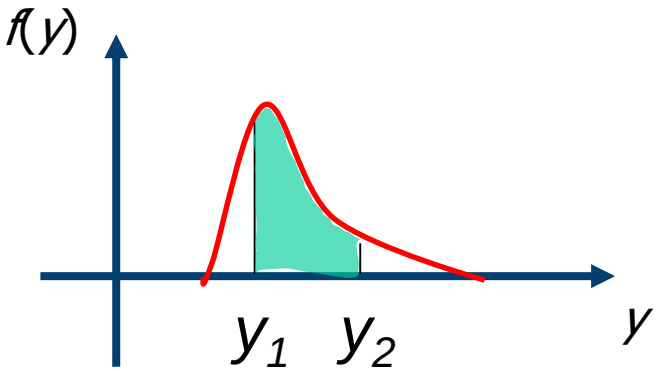
Epistemic Uncertainty Representation

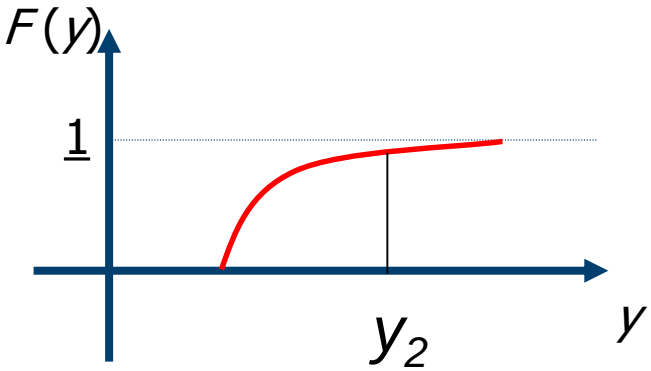
Epistemic Uncertainty Representation (Sufficient Informative Data)

Sufficient informative data:

e.g. N experiments → $(y_1, y_2, \ldots, y_N)$

Probability distributions





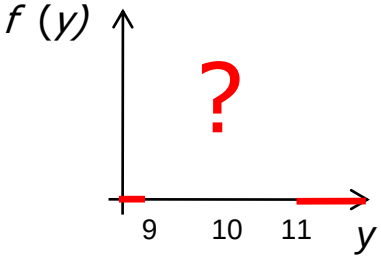
$$P\{y \in [y_1, y_2)\} = \int_{y_1}^{y_2} f(y)\,dy$$

$$F(y_2) = P\{y < y_2\}$$

Epistemic Uncertainty Representation (Scarce Information)

Scarce information and of qualitative nature on the parameter value.

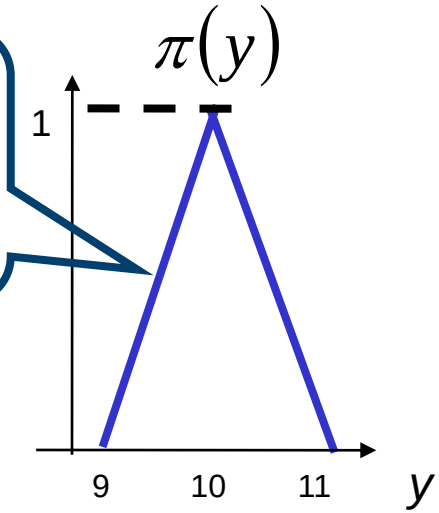
Expert: "the value of Y is between 9 and 11, with a preference for 10"



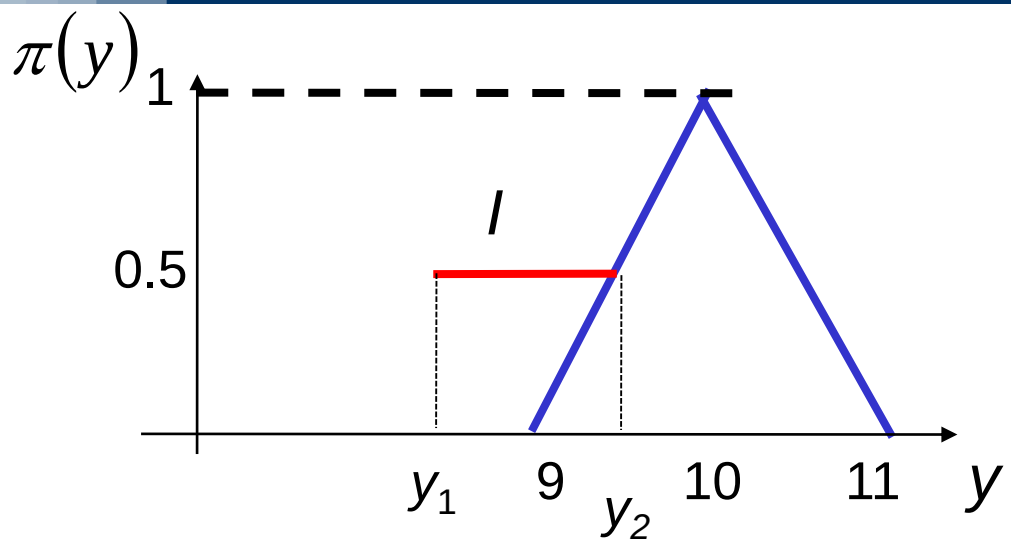
Information difficult to catch with a probability distribution

Possibility representation of the uncertainty

$\pi(y)$ = possibility distribution = Degree of possibility that the uncertain variable Y be equal to y



Possibility Distribution: definitions



For any interval $I=[y_1,y_2]$

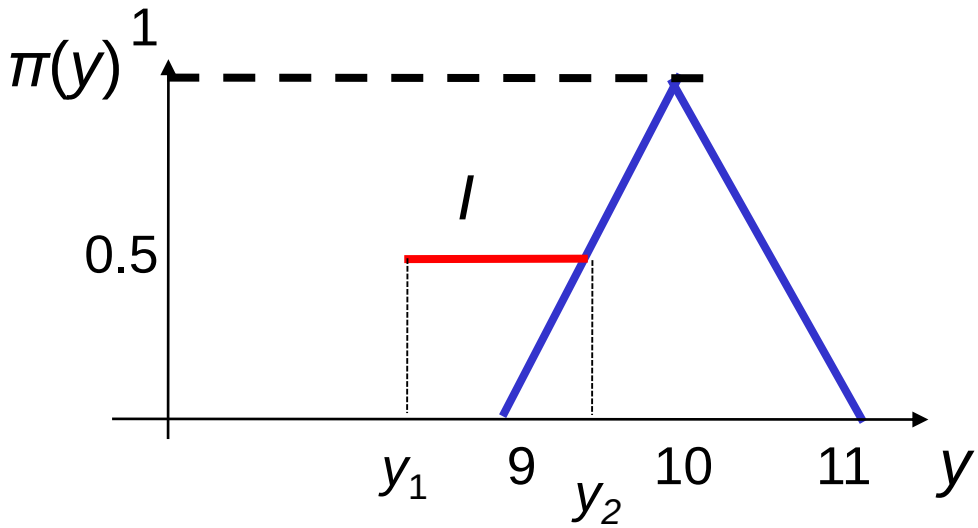
Possibility Measure: $\Pi(I)=\max_{y\in I}\pi(y)$

« to what extent I is consistent with the knowledge π? »

Necessity Measure $N(I)=1-\Pi(\bar{I})=1-\max_{y\notin I}\pi(y)$

«to what extent I is certainly implied by the knowledge π?»

Example 1

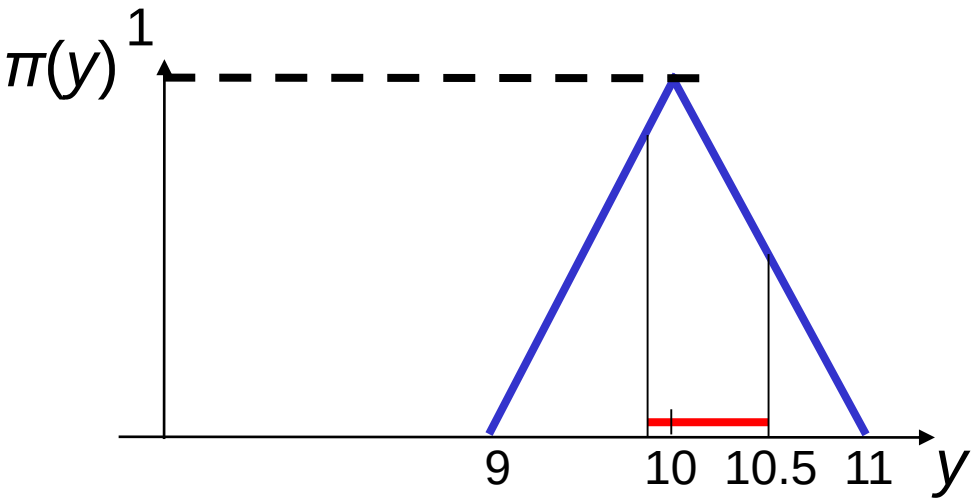


For any interval $I=[y_1,y_2]$

Possibility Measure: $\Pi(I)=?$

Necessity Measure $N(I)=?$

Example 2

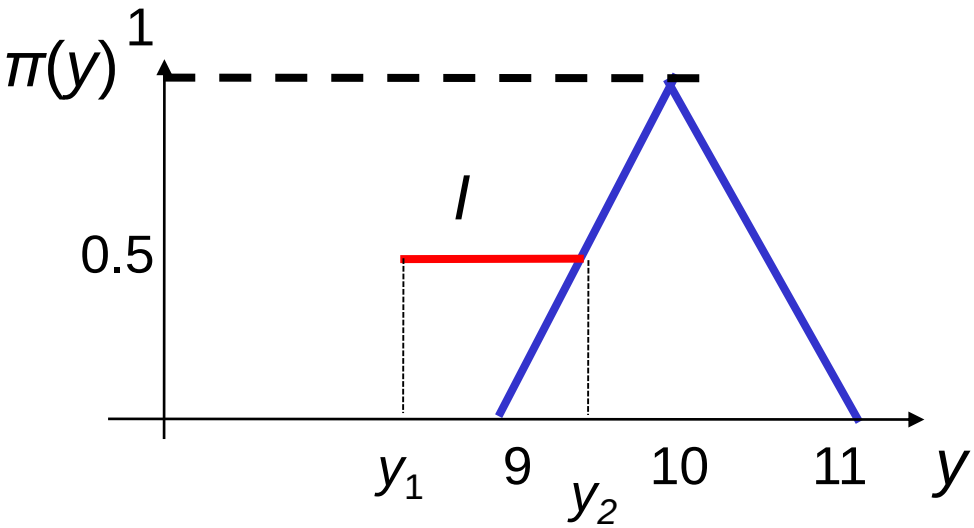


I=[9.8, 10.5]

Possibility Measure: $\Pi(I) = ?$

Necessity Measure $N(I) = ?$

Interpretation of the Possibility Distribution



For any interval $I=[y_1,y_2]$

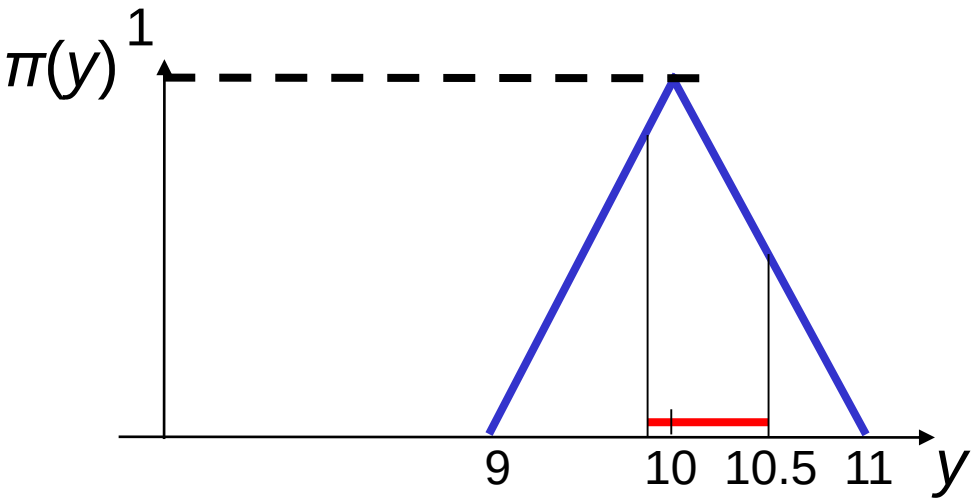
Necessity Measure, $N(I)$

Possibility Measure, $\Pi(I)$

Lower limiting probability value ≤ $P\{Y \in I\}$ ≤ Upper limiting probability value

$$1-\max_{y\notin I}\{\pi(y)\} \le P\{Y \in I\} \le \max_{y\in I}\{\pi(y)\}$$

Example 2: Interpretation

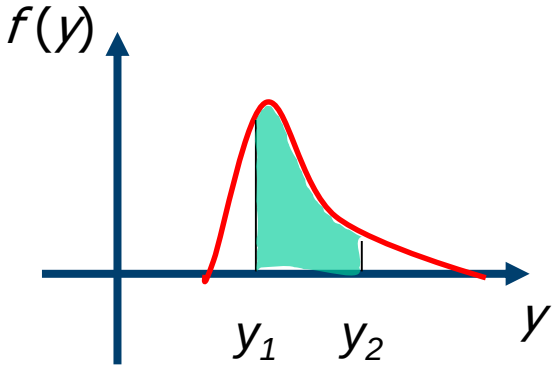
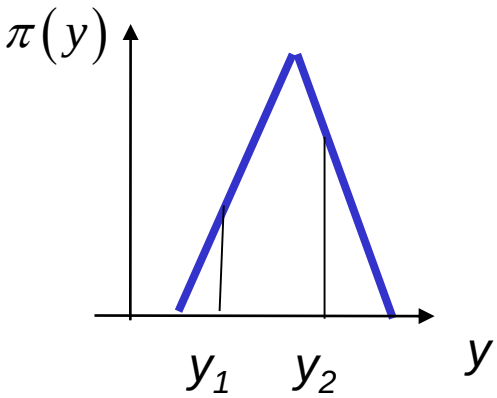


I=[9.8, 10.5]

$$1-\max_{y\notin I}\{\pi(y)\}\le P\{Y\in I\}\le \max_{y\in I}\{\pi(y)\}$$

$$1-0.8\le P\{Y\in I\}\le 1$$

Probability and Possibility: Comparison

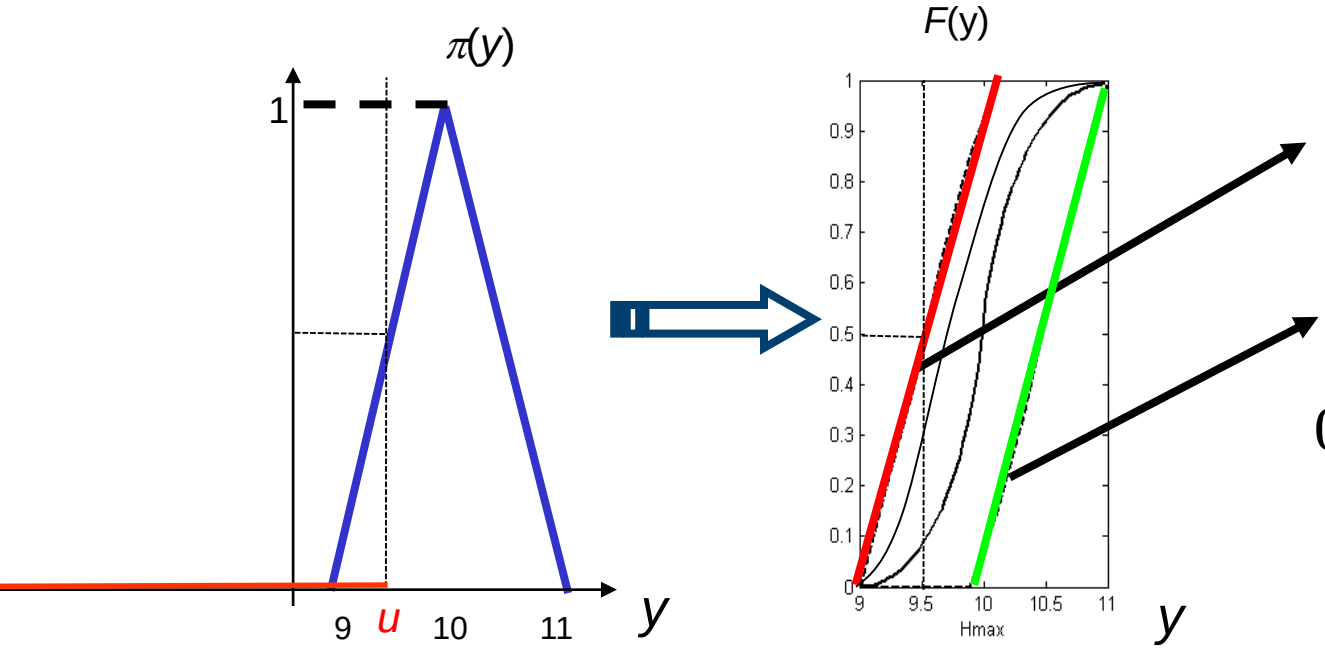
	Probability	Possibility
		
$P\{Y \in [y_1, y_2)\}$	$\int_{y_1}^{y_2} f(y)\,dy$	$\left[1 - \max_{y \notin I}\{\pi(y)\},\ \max_{y \in I}\{\pi(y)\}\right]$
Normalization	$\int_{-\infty}^{+\infty} f(y)\,dy = 1$	$\max\{\pi(y)\} = 1$

Possibility Distribution → Limiting Cumulative Distributions

$$I = (-\infty, u]$$

$$1 - \max_{y \notin I}\{\pi(y)\} \le P\{Y \le u\} \le \max_{y \in I}\{\pi(y)\}$$

$$1 - \max_{y \notin I}\{\pi(y)\} \le F(u) \le \max_{y \in I}\{\pi(y)\}$$



Upper cumulative distribution

Lower cumulative distribution

$$0 \le F(9.5) \le 0.5$$

Possibility Distribution → Limiting Cumulative Distributions

$$I = (-\infty, u]$$

$$1 - \max_{y \notin I}\{\pi(y)\} \le P\{Y \le u\} \le \max_{y \in I}\{\pi(y)\}$$

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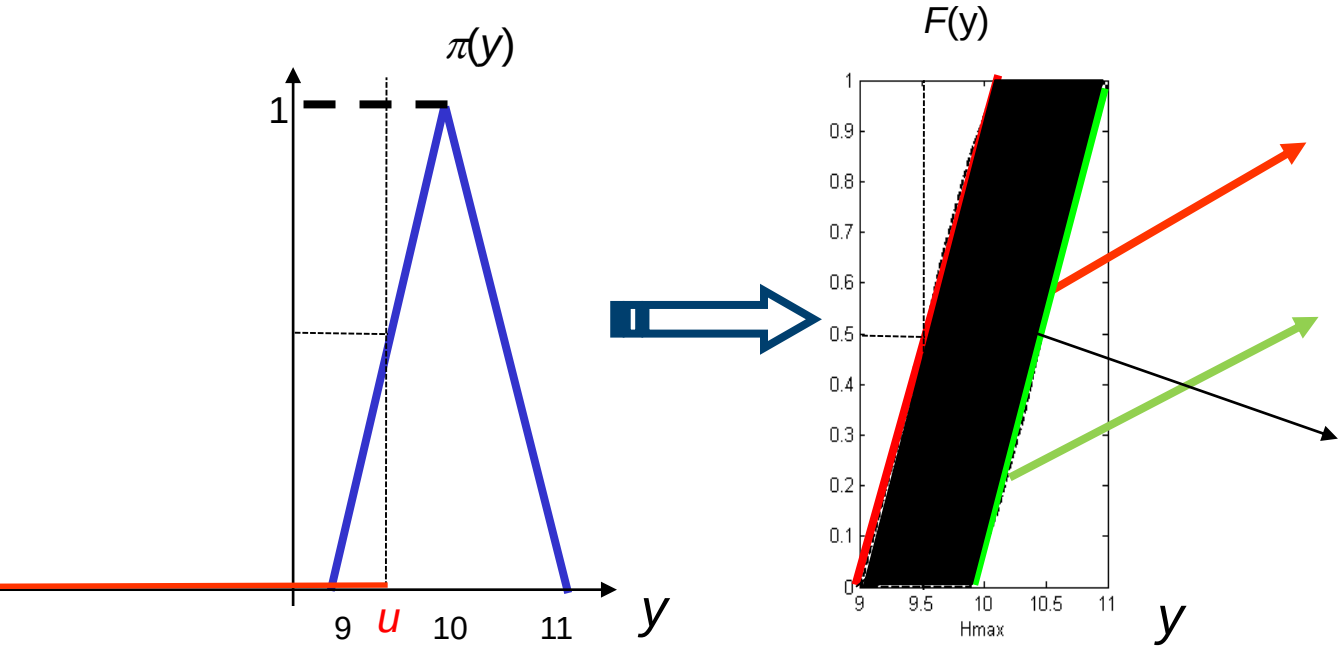


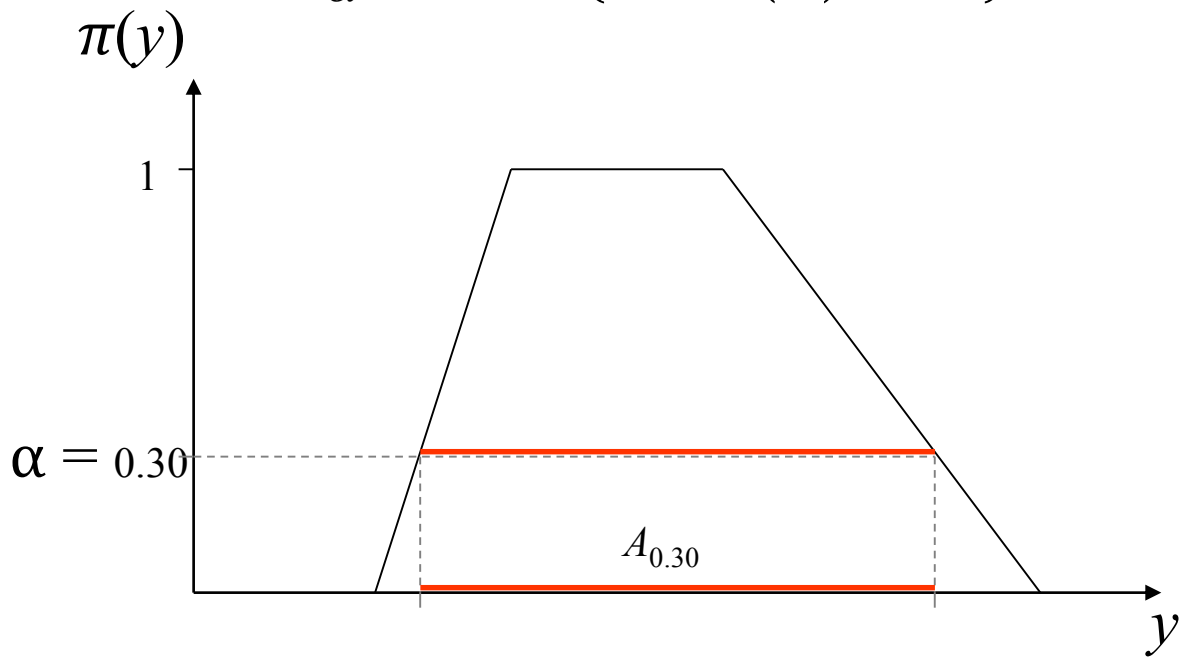
Upper cumulative distribution

Lower cumulative distribution

Family of probability distributions

Definition of an α-cut

$$A_\alpha = I = \{y : \pi(y) \geq \alpha\}$$



$$1 - \max_{y \notin A_\alpha}\{\pi(y)\} \leq P\{Y \in A_\alpha\} \leq \max_{y \in A_\alpha}\{\pi(y)\}$$

$$1 - \max_{y \notin A_\alpha}\{\pi(y)\} \leq P\{Y \in A_\alpha\} \leq 1$$

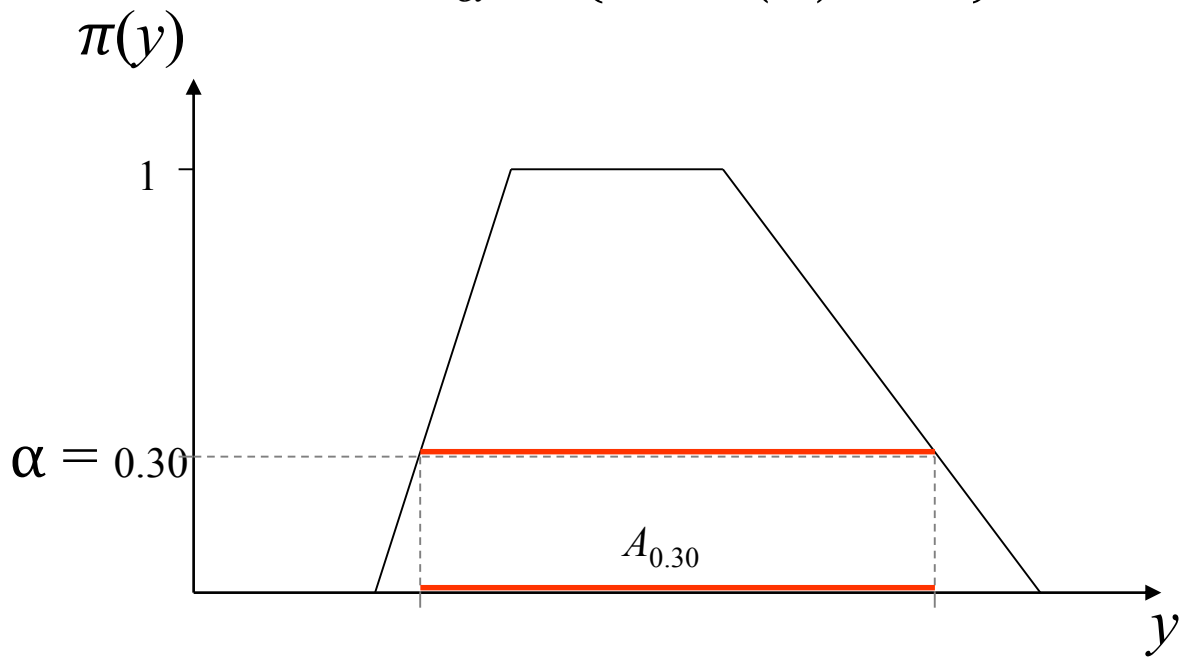
$$1 - \alpha \leq P\{Y \in A_\alpha\} \leq 1$$

Interpretation:

"A_α is certain to degree 1-α"

Example

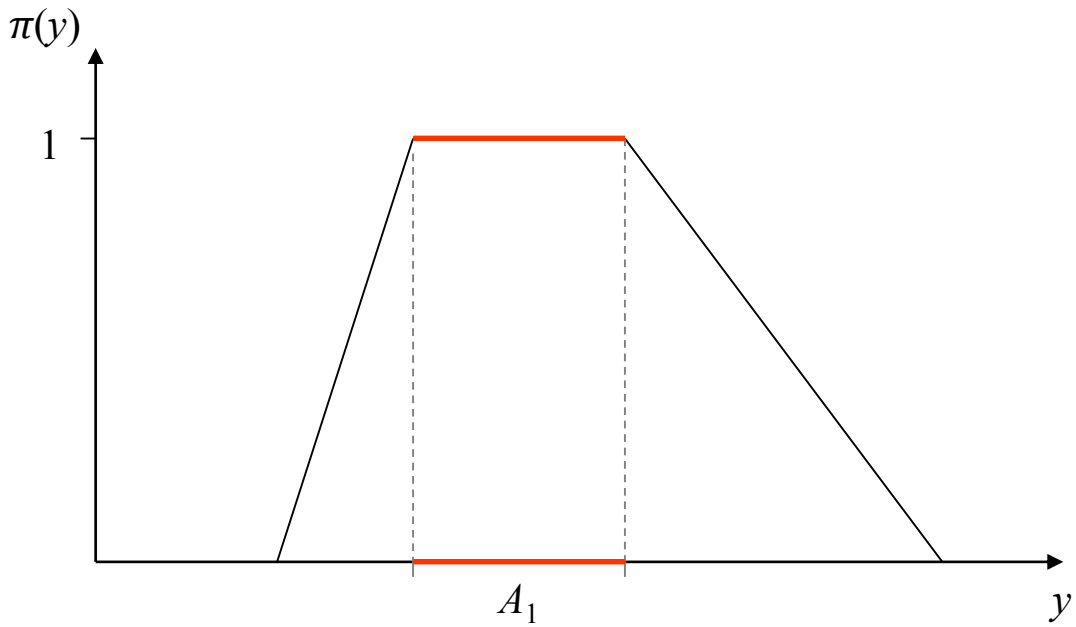
$$I = A_\alpha = \{y : \pi(y) \geq \alpha\}$$



$$1 - \max_{y \notin A_{0.30}} \{\pi(y)\} < P\{Y \in A_{0.30}\} \leq \max_{y \in A_{0.30}} \{\pi(y)\}$$

$$0.7 < P(Y \in A_{0.3})$$

Possibility distribution: Core

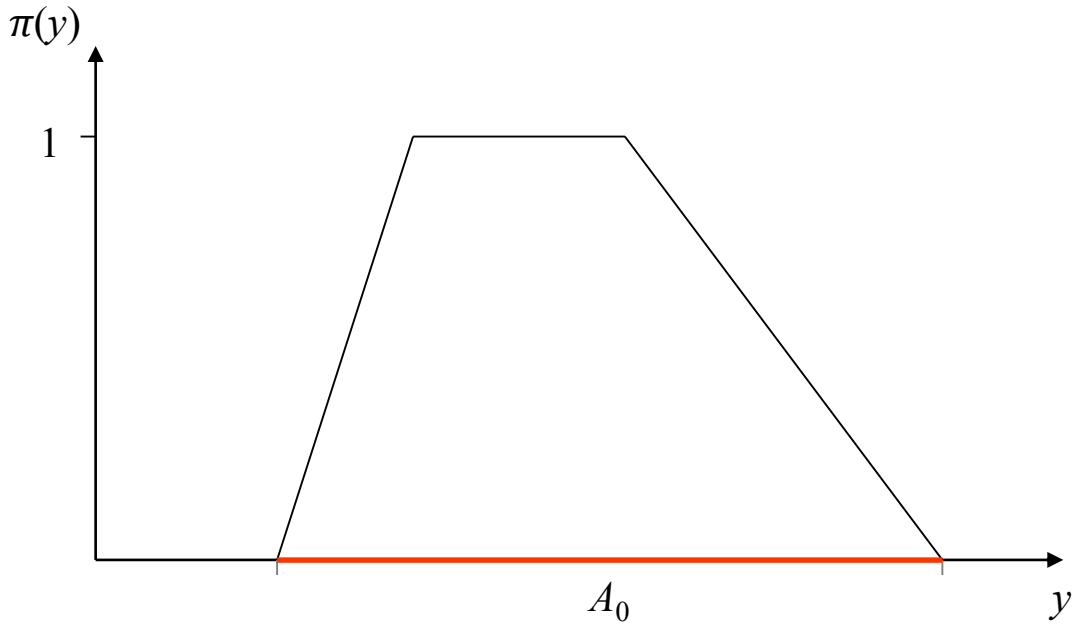


$$0 \le P(Y \in A_1) \le 1$$

π possibility distribution

A_1 core

Possibility Distribution: Support



$$P(Y \in A_0) = 1$$

π possibility distribution

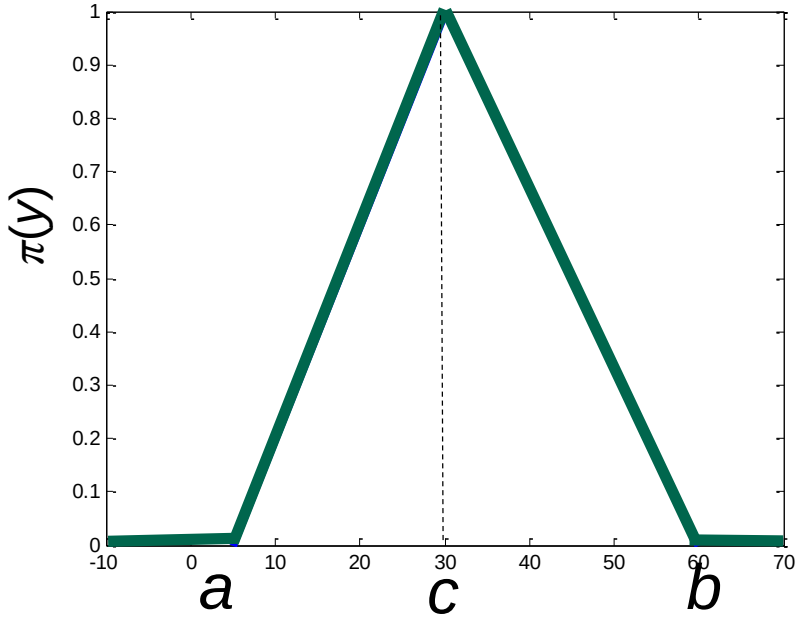
A_0 support

Constructing a possibility distribution

AVAILABLE INFORMATION

- **physical limits [*a*, *b*]**
- **most likely value *c***

(E.g., expert: “the true value of *Y* is between *a* and *b*, with a preference for *c*”)



= family of all the probability distributions with support [*a*, *b*] and mode *c*

Constructing a possibility distribution: Chebyshev inequality

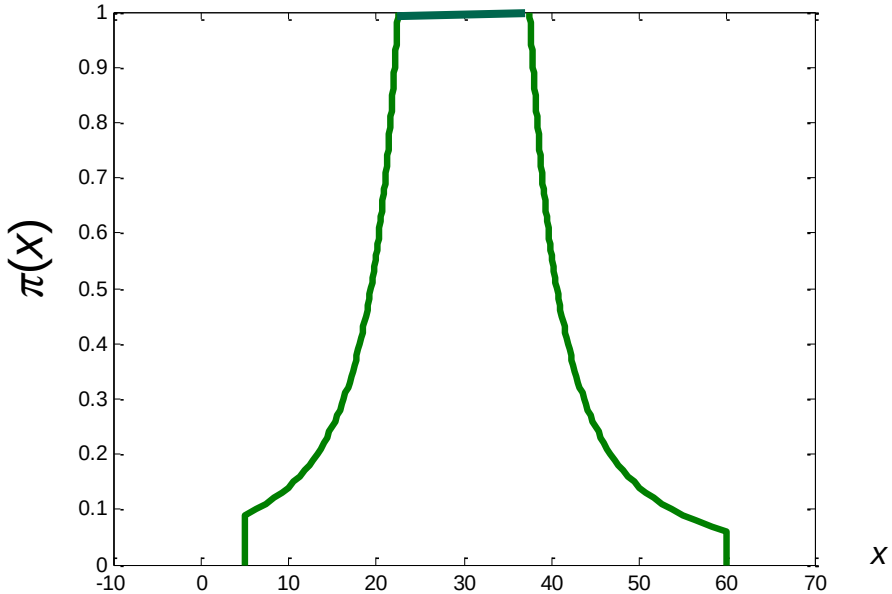
AVAILABLE INFORMATION

- **mean value μ_x**
- **standard deviation σ_x**

Chebyshev inequality

$$P(\mu_x - k\sigma_x \le X \le \mu_x + k\sigma_x) \ge 1 - \frac{1}{k^2}, \qquad \text{for } k \ge 1$$

$$A_{\frac{1}{k^2}} = [\mu_x - k\sigma_x, \mu_x + k\sigma_x] \rightarrow P\left\{X \in A_{\frac{1}{k^2}}\right\} \ge 1 - \frac{1}{k^2} \qquad \text{for } k \ge 1$$

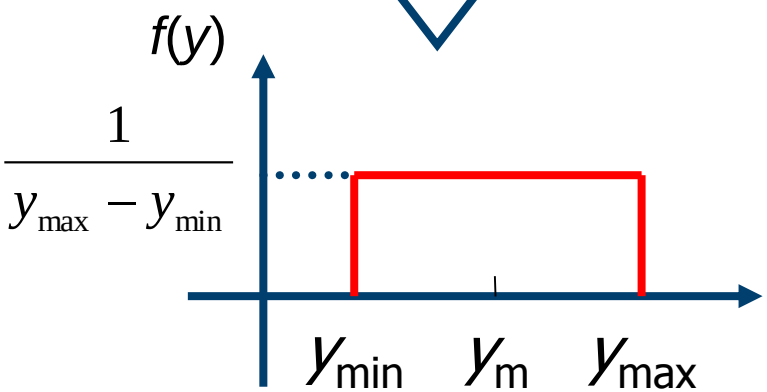


= family of all the probability distributions with mean μ_x and standard deviation σ_x

Comparison of Probabilistic/Possibilistic Approach in Case of Scarce Information (1)

LIMIT CASE: No available information on a model parameter *y* except that its value is located somewhere between a value y_{min} and y_{max}

Laplace principle of insufficient reason:
"all that is equally plausible is equally probable"



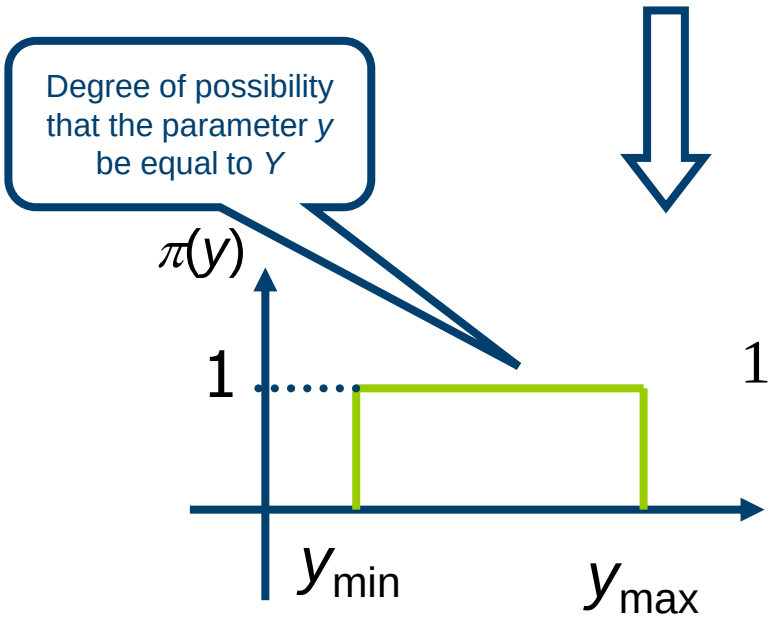
$$y_m = \frac{y_{max} - y_{min}}{2} \Rightarrow P(y \in [y_{min}, y_m]) = P(y \in [y_m, y_{max}])$$

This is a paradox!!!

No information available →No relation between $P(Y \in [y_{min}, y_m])$ and $P(Y \in [y_m, y_{max}])$

Comparison Probabilistic/Possibilistic Approach in Case of Scarce Information (2)

LIMIT CASE: No available information on a model parameter *y* except that its value is located somewhere between a value y_{min} and y_{max}



For any Interval $B \subset [y_{min}, y_{max}]$

$$1 - \max_{y \notin B}\{\pi(y)\} < P(Y \in B) \leq \max_{y \in B}\{\pi(y)\}$$

$$0 < P(Y \in B) \leq 1$$

the only information available is that: $0 \leq P(B) \leq 1$

Uncertainty representation: conclusions

UNCERTAINTY

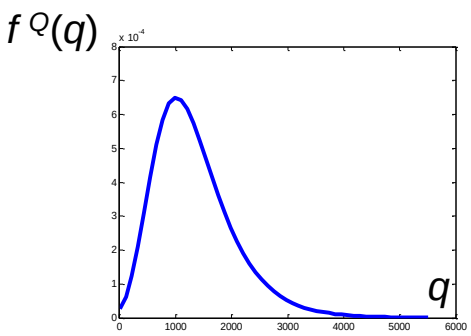
Types:

Randomness (aleatory) → **Representation:** Probability distributions

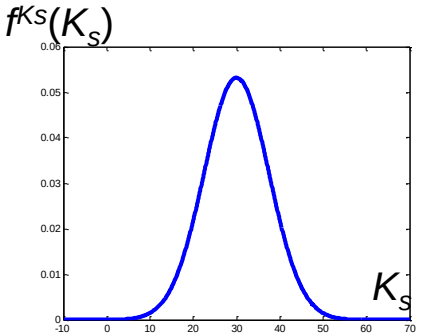
Lack of knowledge (epistemic) →

- **Representation:** Probability distributions (Sufficient statistical information, i.e., data)
- **Representation:** Possibility distributions (Scarce information and/or of qualitative nature, e.g., expert opinion)

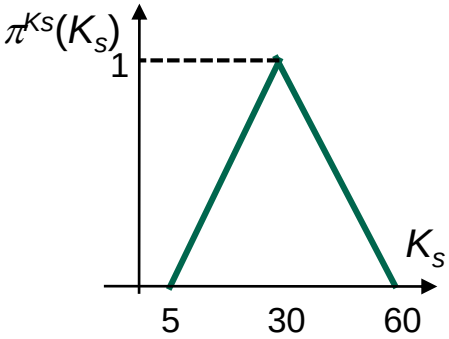
Example:



Gumbel pdf



Gaussian pdf

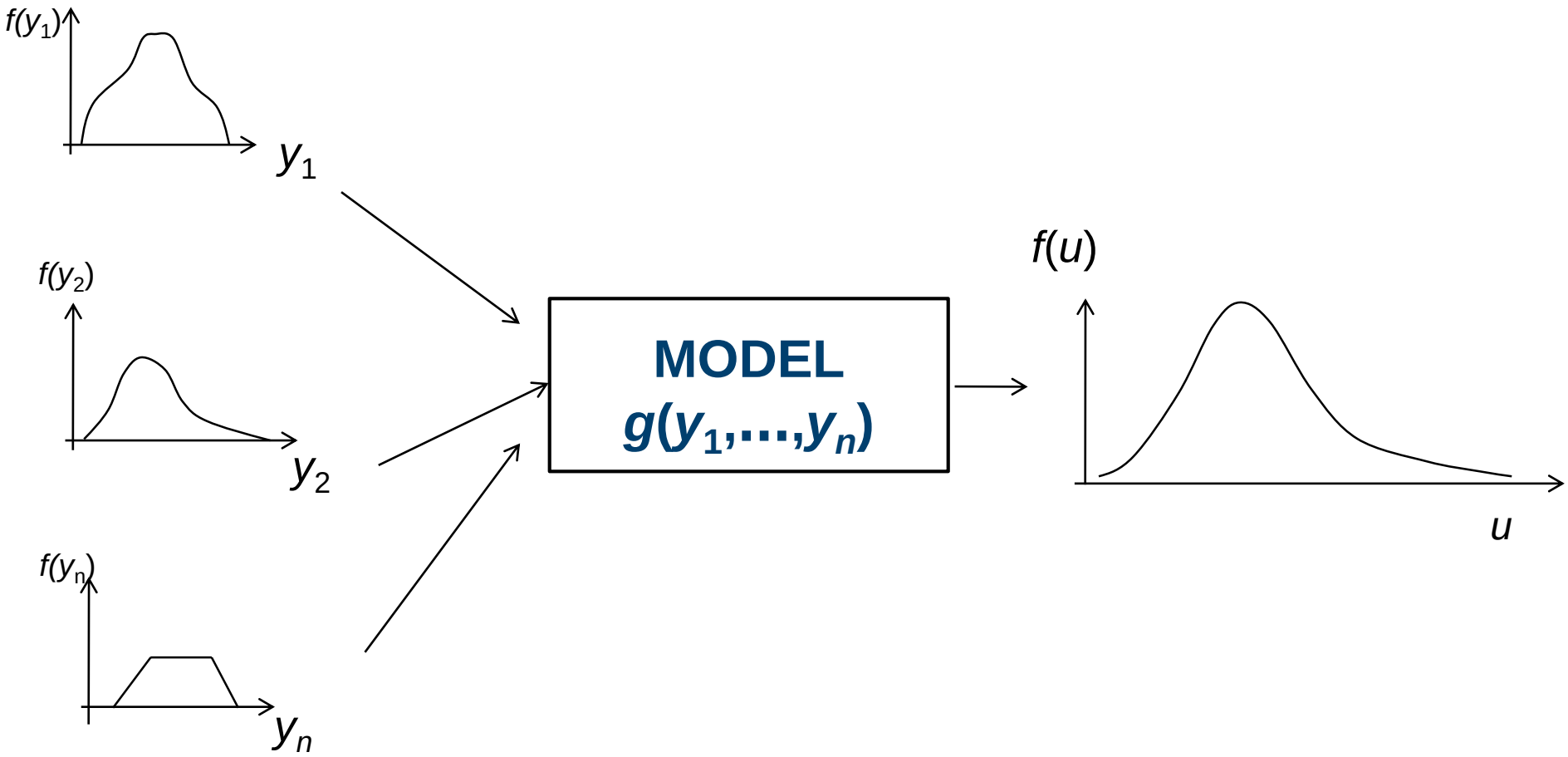


"The value of K_s is between 5 and 60, with a preference for 30"

PART II: Uncertainty propagation

$$u = g(y_1, \ldots, y_n)$$

Uncertainty propagation: objective

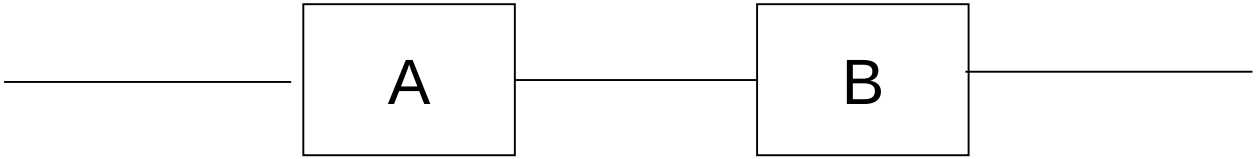


Level 1 uncertainty propagation

$$u = g\,(y_1,\ldots,y_n)$$

- **Level 1**:
  - $y_1,\ldots,y_n$ are uncertain parameters described by the probability distributions $f_{\boldsymbol{\theta i}}(y_i)$
  - the parameters $\boldsymbol{\theta}_i$ of the probability distributions $f_{\boldsymbol{\theta i}}(y_i)$ are known

Level 1 uncertainty propagation: Example



Model Input

$y_1 = T_A$ = failure time of A

•$y_2 = T_B$ = failure time of B

MODEL

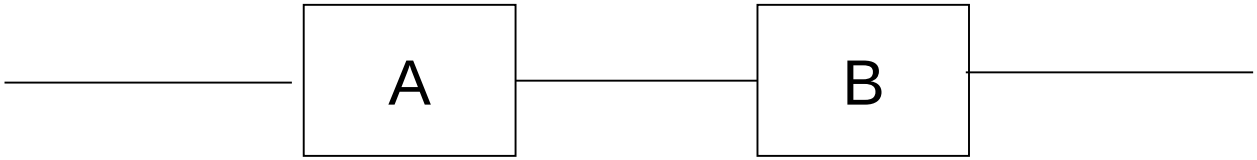
Model output:

$u = T_{sys}$ = failure time of the system $= g(T_A, T_B)$

Level 1 uncertainty propagation

$$u = g(y_1, \ldots, y_n)$$

- **Level 1**:
  - $y_1, \ldots, y_n$ are uncertain parameters described by the probability distributions $f_{\boldsymbol{\theta}_i}(y_i)$
  - the parameters $\boldsymbol{\theta}_i$ of the probability distributions $f_{\boldsymbol{\theta}_i}(y_i)$ are known



Model Input:

- $y_1 = t_A$ = failure time of A
- $y_2 = t_B$ = failure time of B

Model output:

- $u = t_{sys}$ = failure time of the system $= g(t_A, t_B)$

input uncertainty (aleatory)

$$T_A \sim f_{\lambda_A}(t_A) = \lambda_A e^{-\lambda_A t_A}$$

$$T_B \sim f_{\lambda_B}(t_B) = \lambda_B e^{-\lambda_B t_B}$$

LEVEL 1 uncertainty

Parameter of f

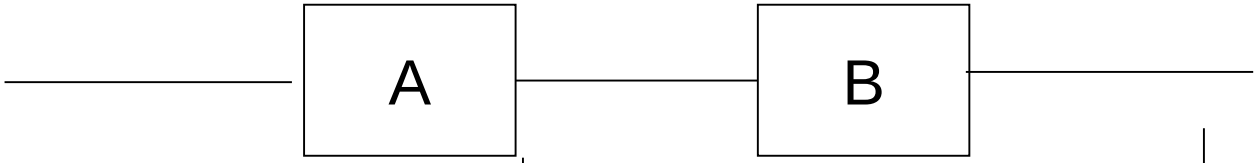
$$\lambda_A = 0.001$$

$$\lambda_B = 0.0008$$

Level 2 uncertainty propagation

$$u = g(y_1, \ldots, y_n)$$

- **Level 2**:
  - $y_1, \ldots, y_n$ are uncertain parameters described by the probability distributions $f_{\boldsymbol{\theta i}}(y_1)$
  - the parameters $\boldsymbol{\theta}_i$ of the probability distributions $f_{\boldsymbol{\theta i}}(y_1)$ are uncertain (*epistemic uncertainty*)



Model Input:

- $y_1 = t_A$ = failure time of A
- $y_2 = t_B$ = failure time of B

Model output:

- $u = t_{sys}$ = failure time of the system $= g(t_A, t_B)$

input uncertainty (aleatory)

$$T_A \sim f_{\lambda_A}(t_A) = \lambda_A e^{-\lambda_A t_A}$$

$$T_B \sim f_{\lambda_B}(t_B) = \lambda_B e^{-\lambda_B t_B}$$

LEVEL 1 uncertainty

Parameter of f

$$\Lambda_A \sim N(\mu_A, \sigma_A)$$

$$\Lambda_B \sim N(\mu_B, \sigma_B)$$

LEVEL 2 uncertainty

Uncertainty propagation – Level 1

$$u = g(y_1,\ldots,y_k, y_k+1,\ldots,y_n)$$

1. Purely probabilistic method
   - $y_1,\ldots,y_n$ = ***probability distributions***

2. Purely possibilistic method
   - $y_1,\ldots,y_n$ = ***possibility distributions***

3. Hybrid Monte Carlo and Possibilistic method
   - $y_1,\ldots,y_k$ = ***probability distribution***
   - $y_{k+1},\ldots,y_n$ = ***possibility distribution***

Uncertainty propagation – Level 1

$$u = g(y_1,\ldots,y_n)$$

1. Purely probabilistic method
   - ***$y_1,\ldots,y_n$ are uncertain parameters described by the probability distributions $f(y_1),\ldots, f(y_n)$***

Purely probabilistic uncertainty propagation

$$u = g(y_1, y_2)$$
$$Y_1 \approx f_{Y_1}(y_1)$$
$$Y_2 \approx f_{Y_1}(y_1)$$

$$F_U(u)???$$

$$F_U(u) = P\{U \le u\} = P\{g(Y_1, Y_2) \le u\} = \int_{(y_1, y_2): g(y_1, y_2) \le u} f_{Y_1}(y_1) f_{Y_2}(y_2) dy_1 dy_2 =$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I_g(y_1, y_2) f_{Y_1}(y_1) f_{Y_2}(y_2) dy_1 dy_2$$

with $I_g(y_1, y_2) = \begin{cases} 1 & if \quad f(y_1, y_2) \le u \\ 0 & otherwise \end{cases}$

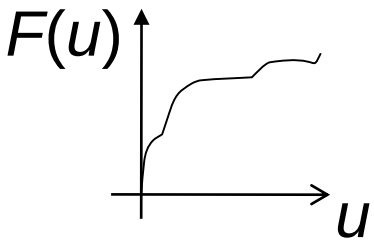
MC analog dart game:

- sample (y_1^i, y_2^i) from $f_{Y_1}(y_1), f_{Y_1}(y_1)$
- corresponding award is: $I_g(y_1^i, y_2^i)$

Consider *N* trials $\{(y_1^1, y_2^1), (y_1^2, y_2^2), \ldots, (y_1^N, y_2^N)\}$

$$F_U(u) = \frac{\sum_{i=1,\ldots,N} I_g(y_1^i, y_2^i)}{N}$$

Purely probabilistic uncertainty propagation: the procedure



Estimate the cumulative distribution $F(u)$ from: $u^1,\ldots,u^N$

i=0

i=i+1

NO

i=N?

YES

Sample a realization $y_1^i, y_2^i,\ldots, y_n^i$ of the uncertain parameters from their probability distributions

$$f_{Y_1}(y_1), f_{Y_2}(y_2),\ldots, f_{Y_n}(y_n)$$

Compute:

$$u^i = g(y_1^i, y_2^i,\ldots, y_n^i)$$

Uncertainty propagation – Level 1

$$u = g(y_1,\ldots,y_n)$$

1. Purely possibilistic method
   - ***$y_1,\ldots,y_n$ are uncertain parameters described by the possibility distributions $\pi(y_1),\ldots,\pi(y_n)$***

Purely Possibilistic method: Uncertainty Propagation through *U*=*g*(*Y*)

Extension principle of fuzzy set theory

- Simple case:
  - 1 input quantity, $Y \in \Re$,whose uncertainty is described by the possibility distribution $\pi(y)$
  - 1 output quantity, $U = g(Y), U \in \Re$ whose uncertainty is described by the possibility distribution $\pi_U(u)$:

$$\pi_U(u) = \sup_{y, g(y)=u} \left(\pi_Y(y)\right)$$

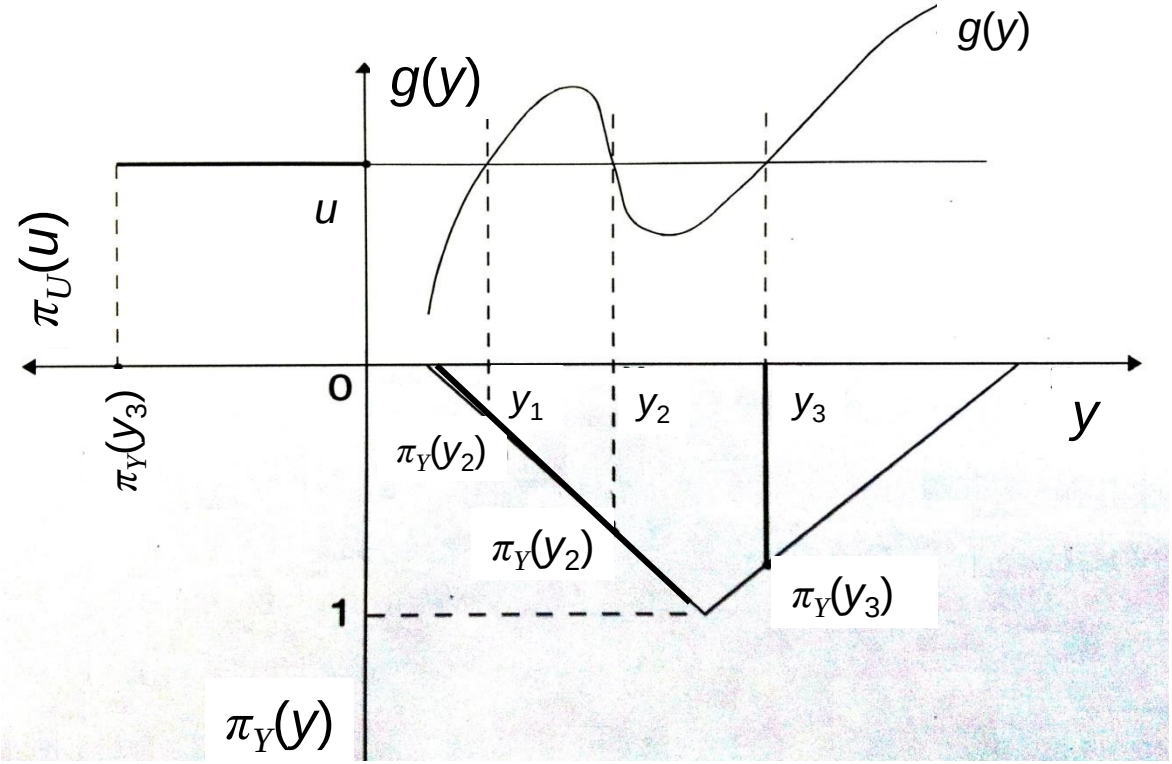
Basic idea: extend the function g

- From a function from $\Re$ to $\Re$
- to a function from and to the class of all the possibility distributions defined on $\Re$

Extension principle of fuzzy set theory

$$\pi_U(u) = \sup_{y, g(y)=u}(\pi_Y(y))$$

- For a given *u*, we proceed in three steps:
  - Select the set of y values such that $g(y)=u$
  - Compute the corresponding set of $\pi_Y(y)$
  - Choose for $\pi_U(u)$ the maximum among $\pi_Y(y)$



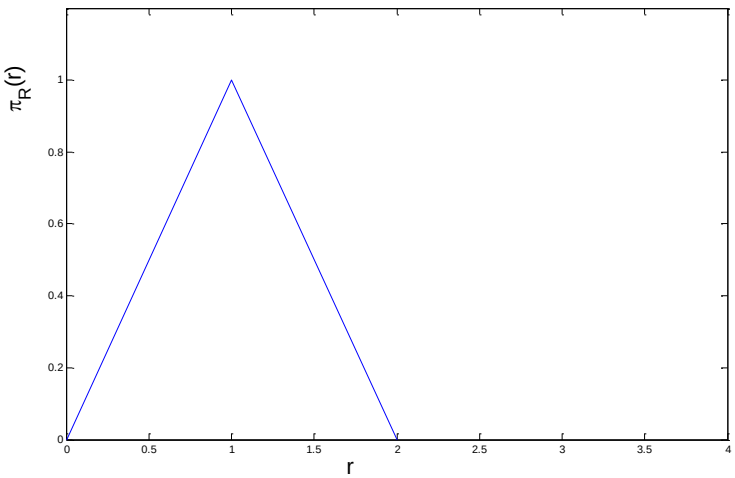
Extension Principle: Example

For a chemical reactor, we consider a physical model g used for evaluating the consequence of a catastrophic failure event. We assume that the model specifies that the consequences C (in arbitrary units) of a catastrophic failure are a quadratic function of the quantity of toxic gas release R (in arbitrary units), i.e.

$$C = g(R) = R^2$$

The epistemic uncertainty related to R is described by the triangular possibility distribution:

$$\pi_R(r) = \begin{cases} 0 & \text{if } r < 0 \text{ or } r > 2 \\ r & \text{if } 0 \le r \le 1 \\ 2 - r & \text{if } 1 < r \le 2 \end{cases}$$



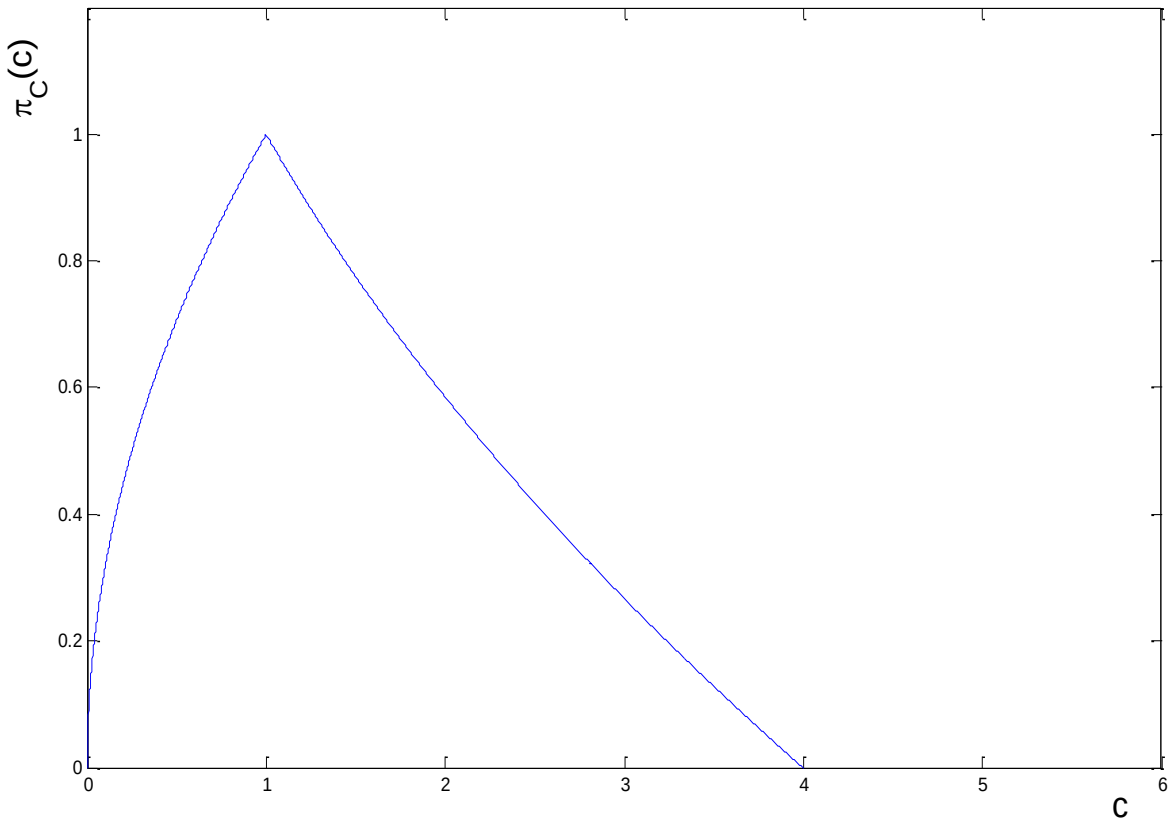
Extension Principle: Example

For a fixed value $c > 0$ of the consequences:

- find the quantity of toxic gas release r such that $c = r^2$. In this particular case, the solutions are $r_1 = -\sqrt{c}$ and $r_2 = \sqrt{c}$.
- evaluate the possibility distribution of the quantity of toxic gas release, π_R, at the identified r values, i.e. $\pi_R(r_1)$ and $\pi_R(r_2)$,
- assign to the possibility distribution of the consequence, π_C, the maximum of $\pi_R(r_1)$ and $\pi_R(r_2)$. Since, in this case, $\pi_R(r_1) = \pi_R(-\sqrt{c})$ is equal to 0, the maximum value is obtained as $\pi_R(r_1) = \pi_R(\sqrt{c})$ which, according to the definition of $\pi_R(r)$ is given by:

$$\pi_C(c) = \pi_R(r_2) = \pi_R(\sqrt{c}) = \begin{cases} \sqrt{c} & if \quad 0 < c \leq 1 \\ 2 - \sqrt{c} & if \quad 1 < c \leq 4 \\ 0 & if \quad c > 4. \end{cases}$$

Extension principle of fuzzy set theory



Extension principle of fuzzy set theory

Multi-input case: $U=g(Y_1,Y_2 \ldots,Y_n)$

- input quantities: $Y_1,Y_2 \ldots,Y_n$, whose uncertainty is described by the possibility distribution $\pi(y_1)$, $\pi(y_2),\ldots,$ $\pi(y_n)$
- 1 output quantity, $U=g(Y_1,Y_2 \ldots,Y_n)$ whose uncertainty is described by the possibility distribution $\pi_U(u)$:

$$\pi_U(u) = \sup_{y_1,y_2,\ldots,y_n,\, g(y_1,y_2,\ldots,y_n)=u} \min\{\pi_{Y_1}(y_1), \pi_{Y_1}(y_1),\ldots,\pi_{Y_n}(y_n)\}$$

Basic idea: extend the function g

- The use of the minimum operator is justified by:

$$\pi_{Y_1,Y_1,\ldots,Y_n}(y_1,y_2,\ldots,y_n) = \min\{\pi_{Y_1}(y_1), \pi_{Y_1}(y_1),\ldots,\pi_{Y_n}(y_n)\}$$

Extension Principle: alternative formulation

- Output possibility distribution is represented in the form of a nested set of intervals (alpha-cut)

$$A_\alpha = \left[\underline{u}_\alpha, \overline{u}_\alpha\right] = \left\{u : \pi_U(u) \geq \alpha\right\}$$

- Indicating as $Y_{1\alpha}, \dots, Y_{n\alpha}$ the α-cuts of the input quantities $Y_1, \dots, Y_N$, the extension principle, for a given value of α in $[0,1]$, becomes:

$$\underline{u}_\alpha = \inf(g(y_1, \dots, y_n), y_1 \in Y_{1\alpha} \dots., y_n \in Y_{n\alpha}),$$
$$\overline{u}_\alpha = \sup(g(y_1, \dots, y_n), y_1 \in Y_{1\alpha} \dots., y_n \in Y_{n\alpha}).$$

Uncertainty propagation – Level 1

$$u = g(y_1,\ldots,y_k, y_k+1,\ldots,y_n)$$

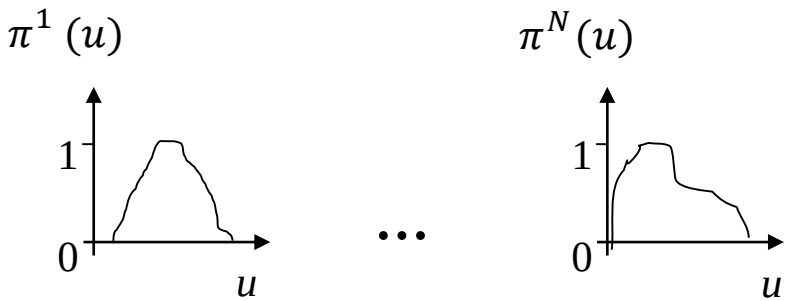
2. Hybrid Monte Carlo and Possibilistic method
   - $y_1,\ldots,y_k$ = ***probability distributions:*** $p(y_1), \ldots, p(y_k)$
   - $y_{k+1},\ldots,y_n$ = ***possibility distributions:*** $\pi(y_{k+1}), \ldots, \pi(y_n)$

Hybrid Monte Carlo and Possibilistic method

Repeat *N* simulations

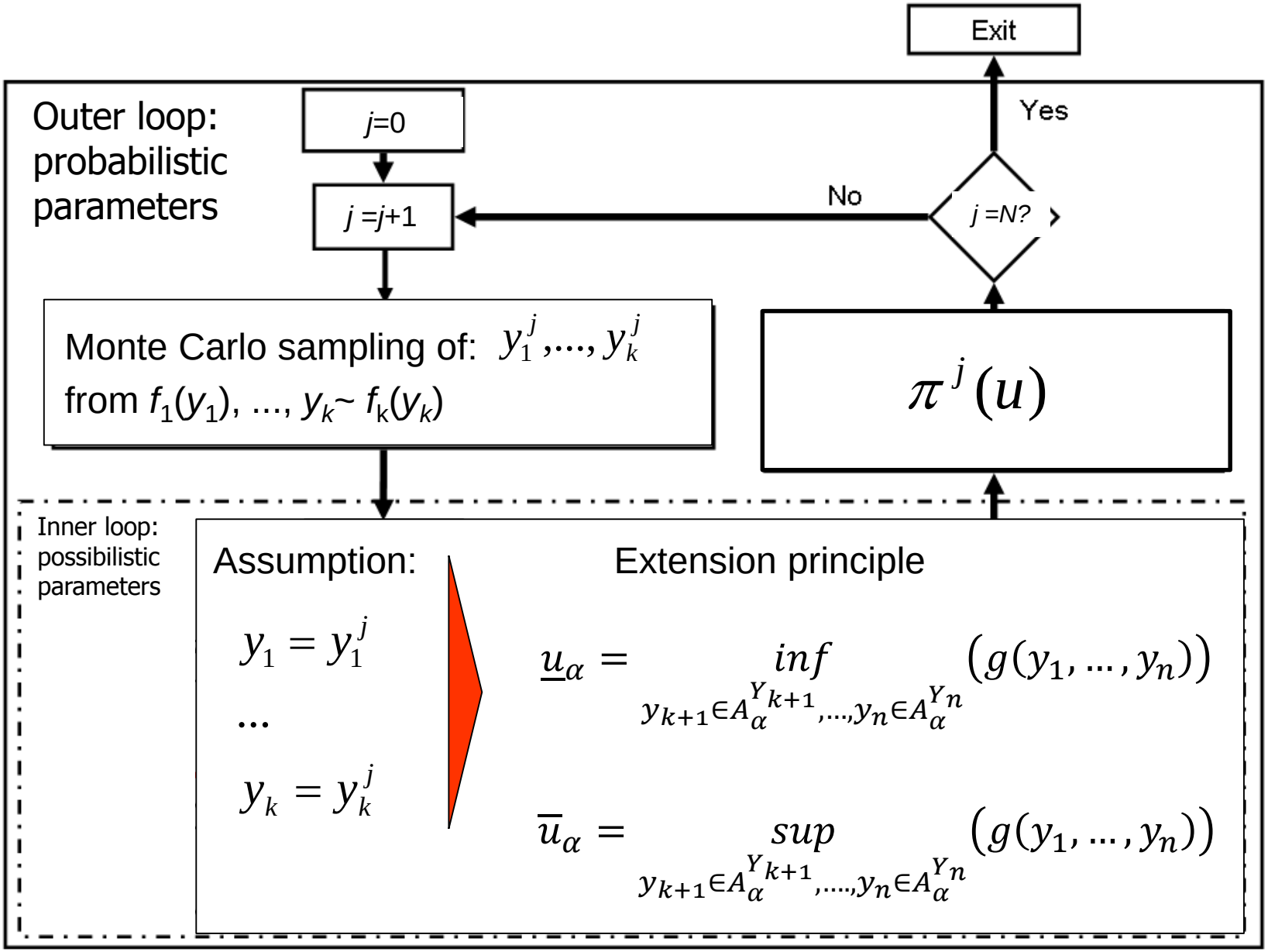
- Monte Carlo sampling of the variables described by probability distributions $y_1^j \sim f_1(y_1), \ldots, y_k^j \sim f_k(y_k)$
- extension principle to process uncertainty described by possibility distributions

Possibility distribution $\pi^j(u)$ of $u = g(y_1,\ldots,y_k, y_{k+1},\ldots,y_n)$

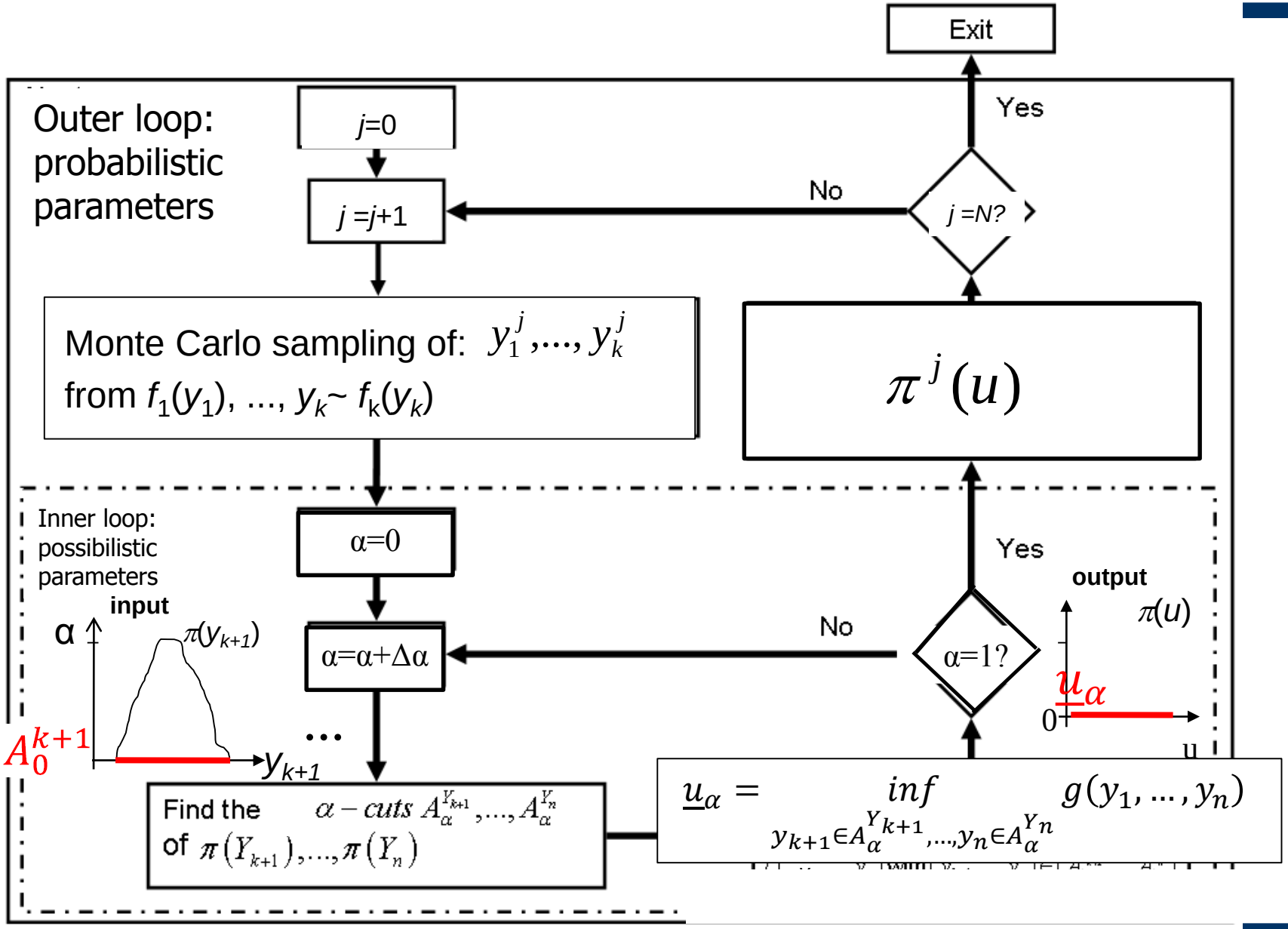


Simulation 1 Simulation *N*

Details of the Hybrid Monte Carlo and Possibilistic method



Details of the Hybrid Monte Carlo and Possibilistic method



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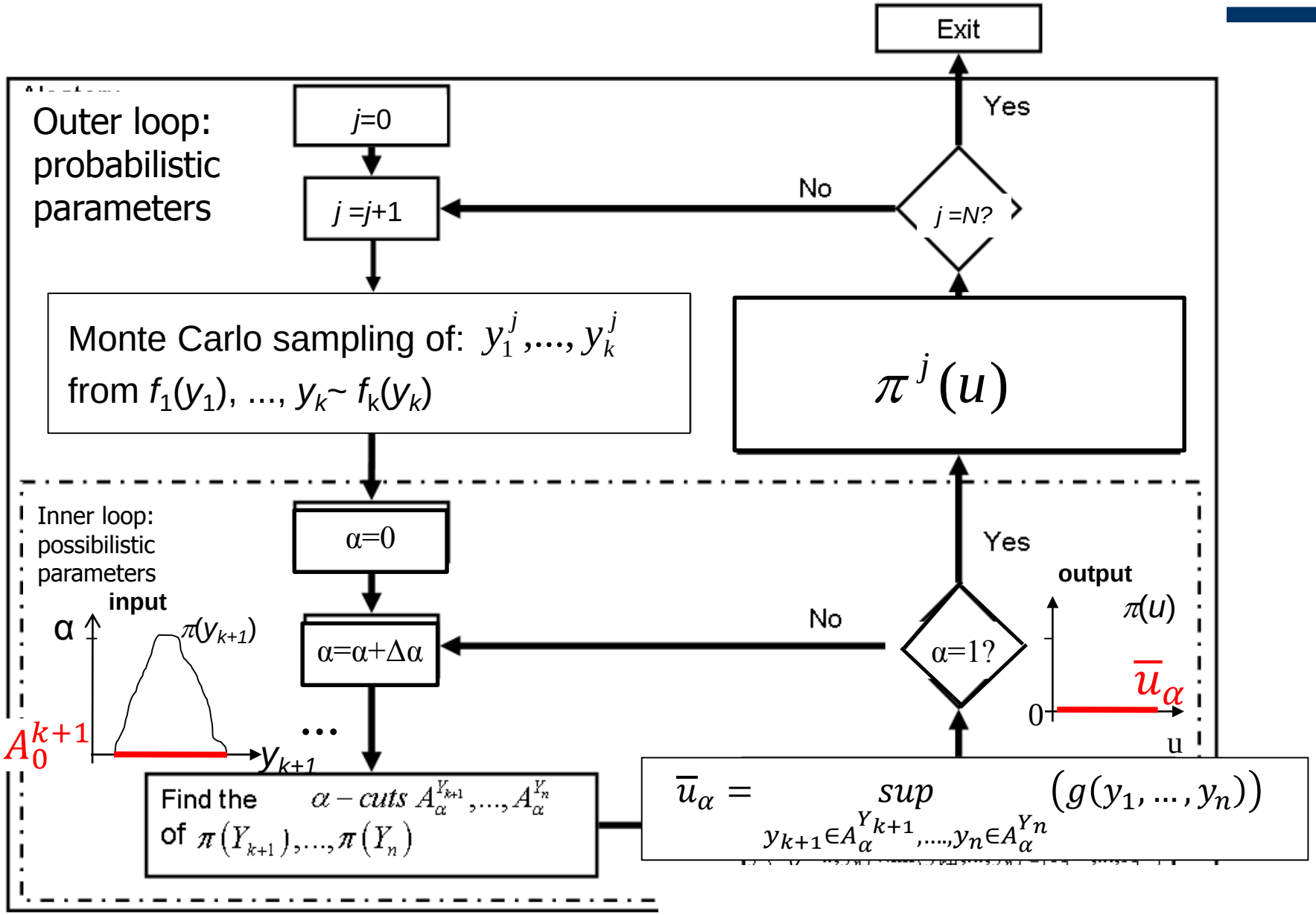

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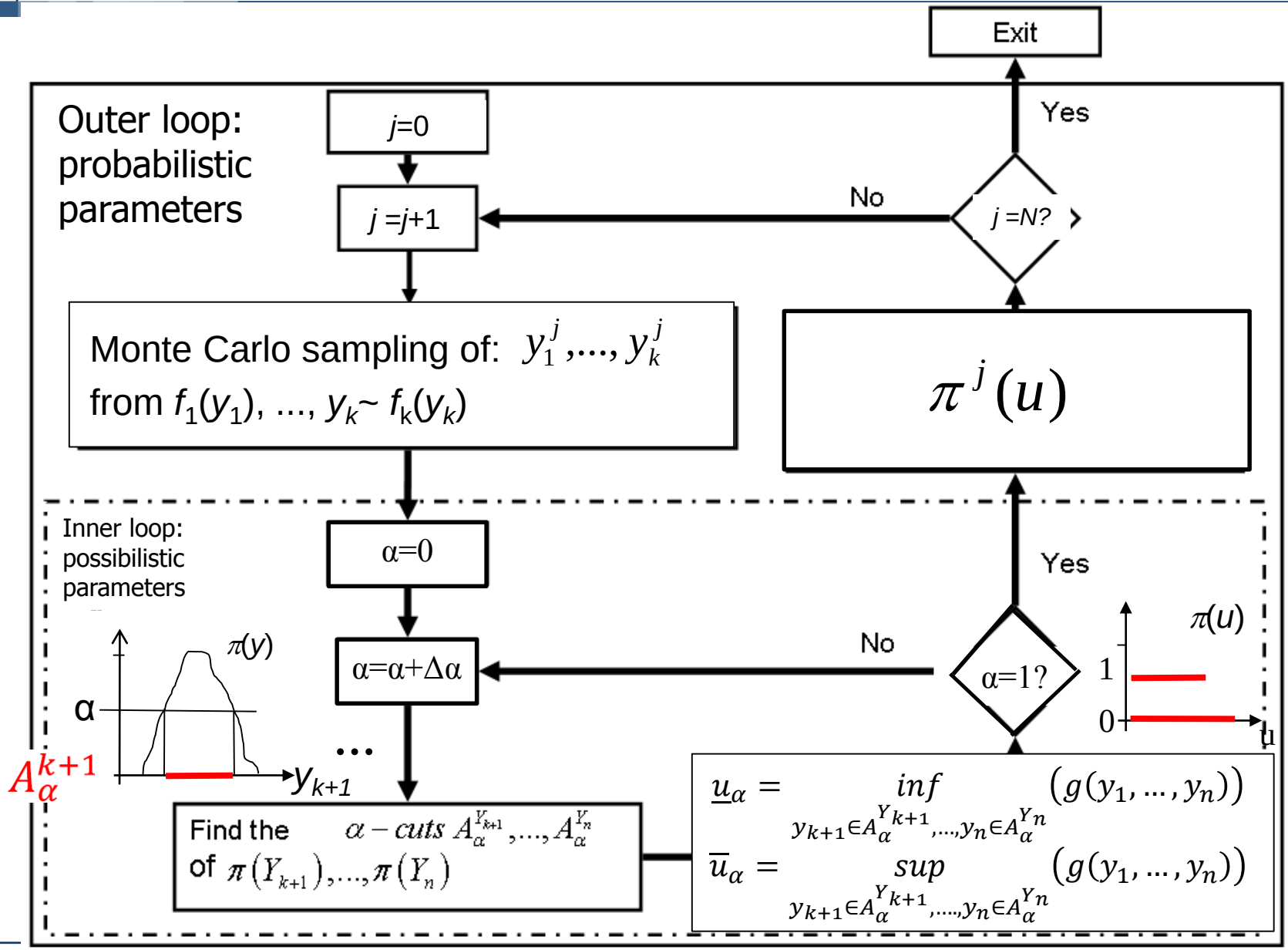

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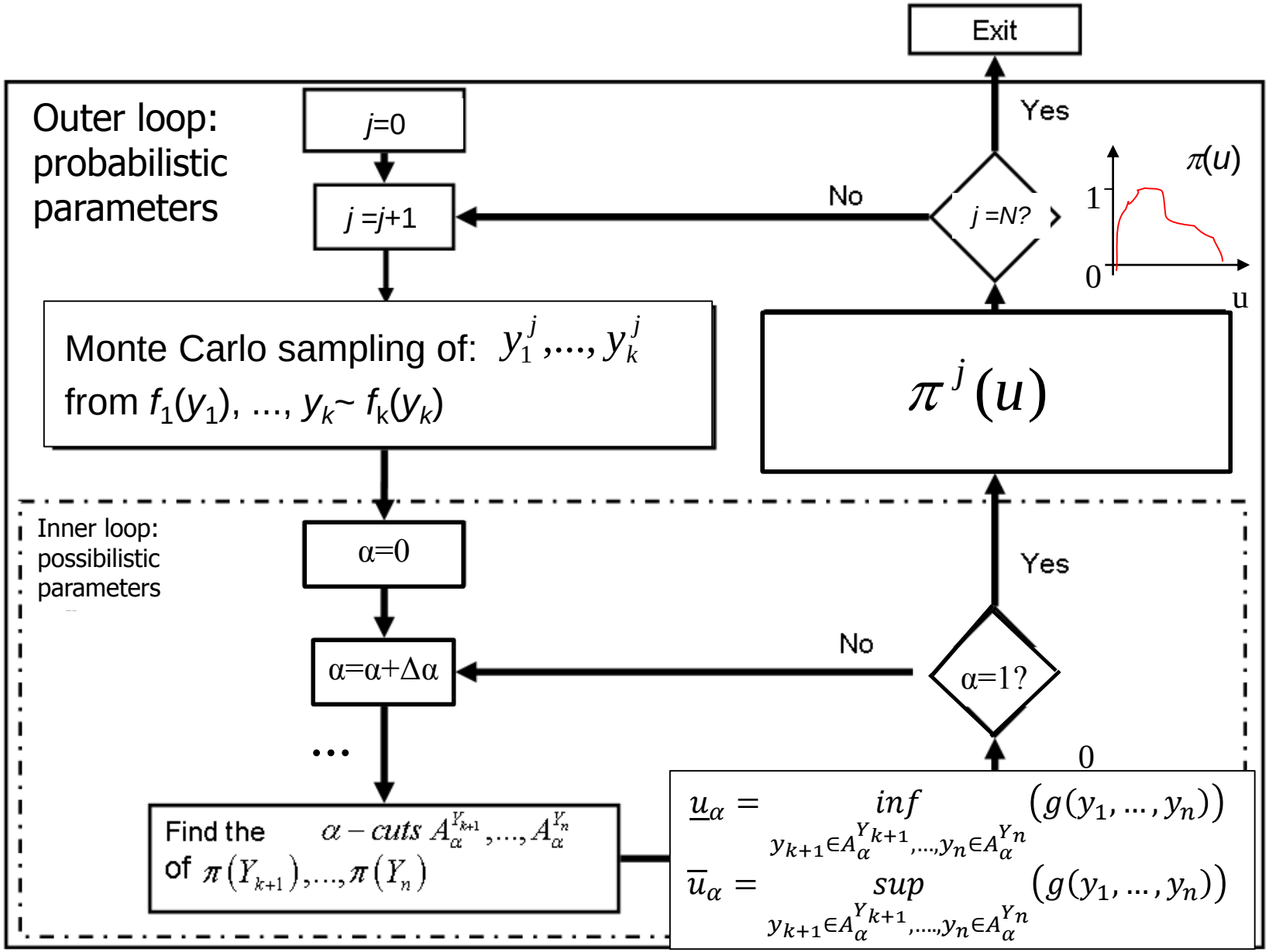
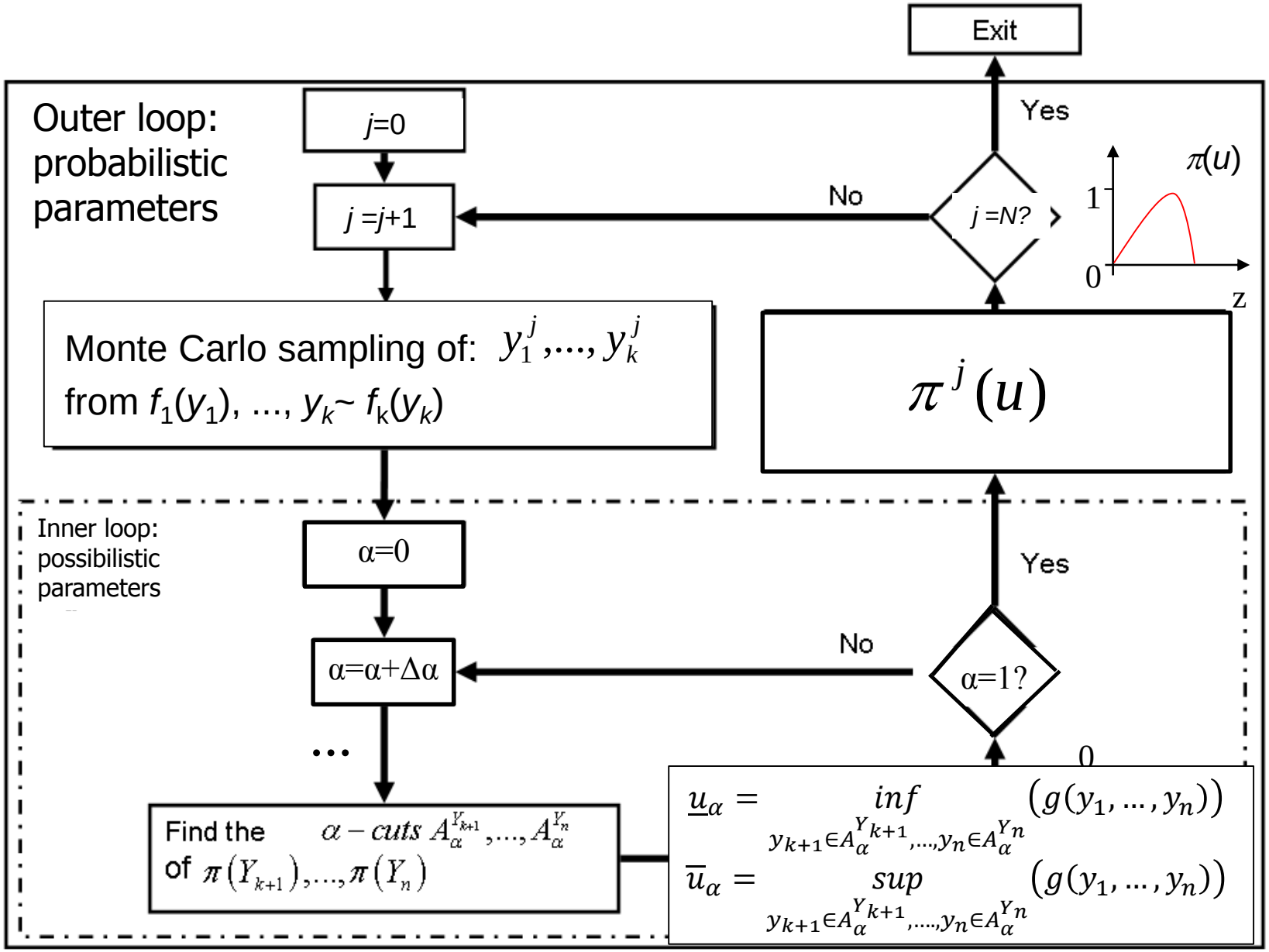
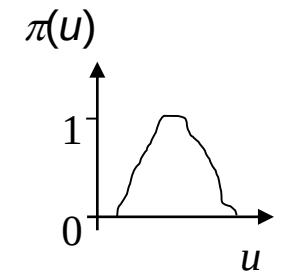


Details of the Hybrid Monte Carlo and Possibilistic method

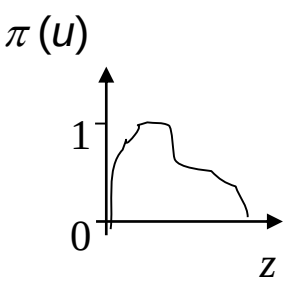


Details of the aggregation (1)



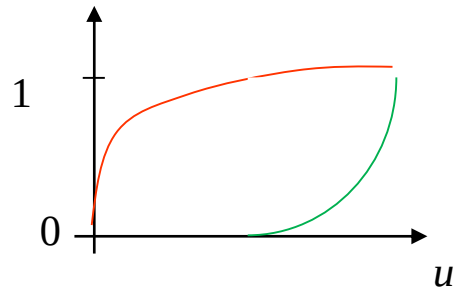
Realization 1

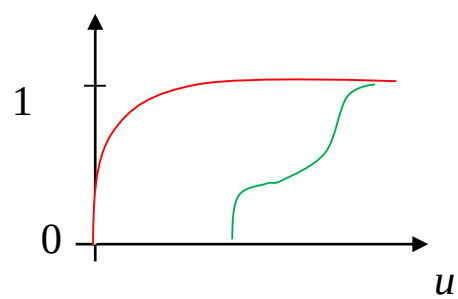
…



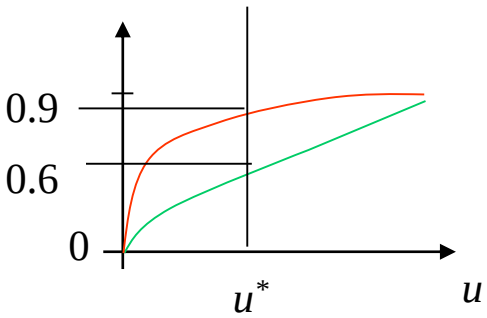
Realization m

Find associated limiting Cumulative distributions





Upper and lower cumulative distributions of u : for any u compute the average of the obtained cumulative distributions



Level 1 uncertainty propagation - applications:

1. Event tree analysis
2. Reliability Assessment of a Degrading Component

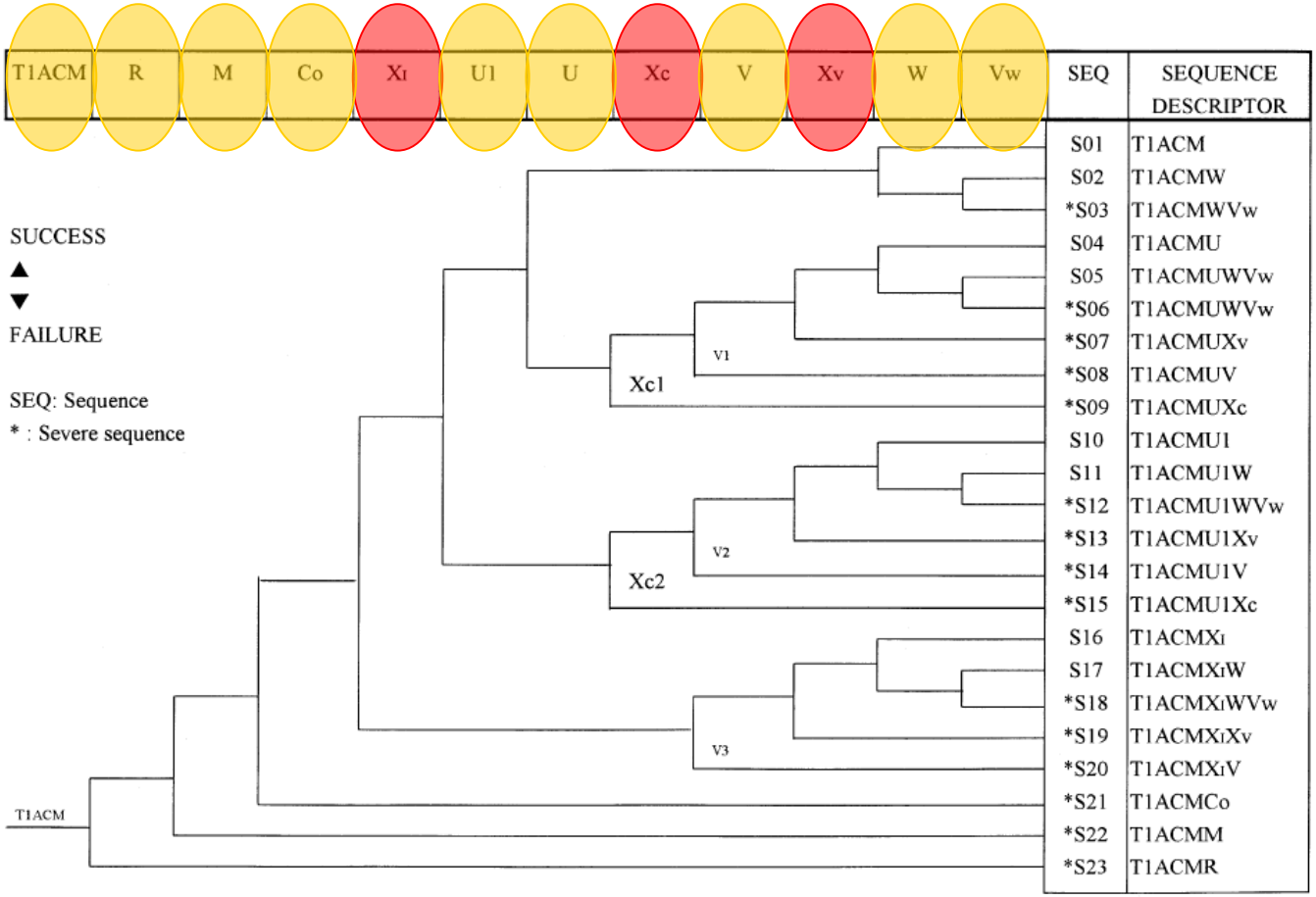
Application 1)

Event tree analysis

(only epistemic uncertainty is considered)

Case study

Anticipated Transient Without Scram (ATWS) in a PWR nuclear reactor



Hardware-Failure-Dominated (HFD) Events

Human-Error-Dominated (HED) Events

Objective: compute the probability of occurrence of a severe sequence

Example of HED event

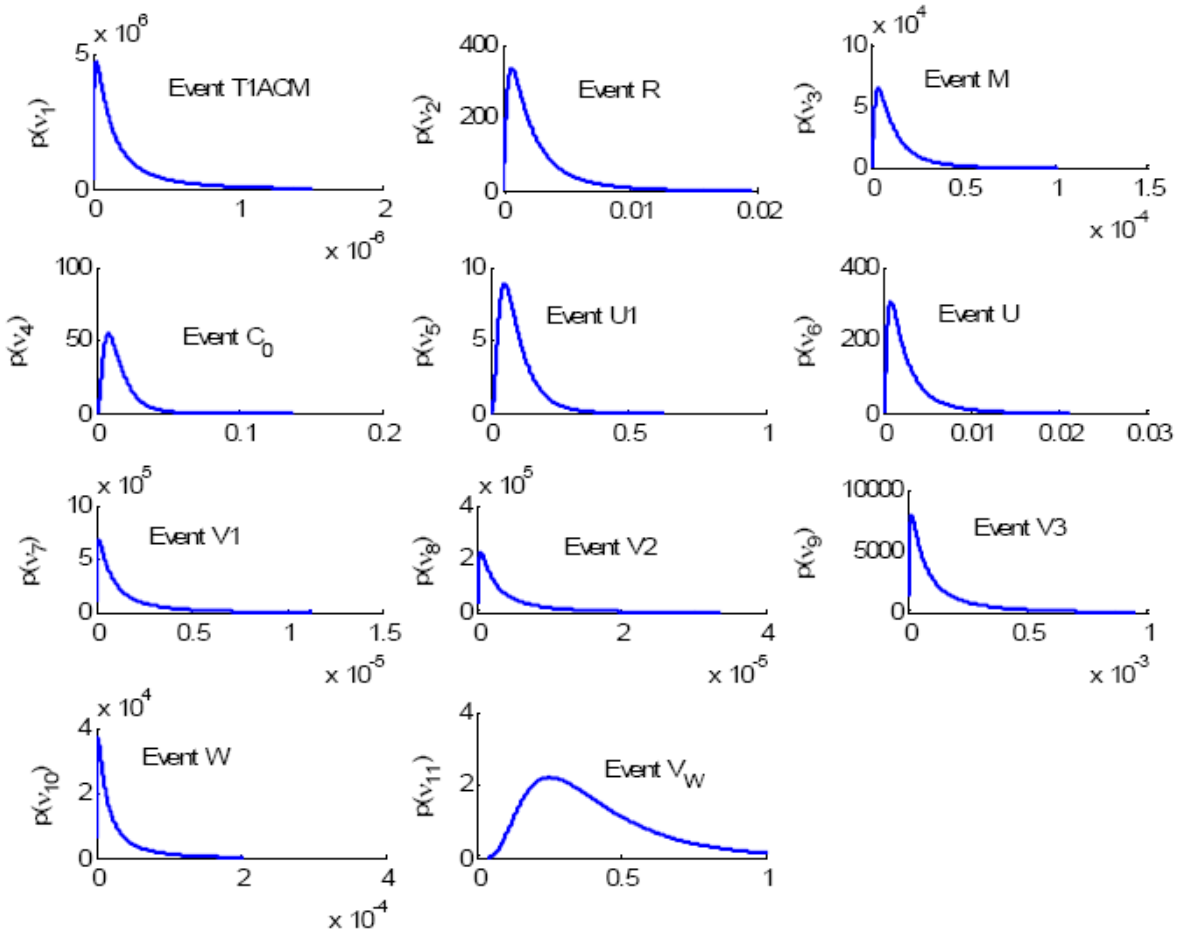
ADS inhibit	X_I	*HED*	ν_{12}	*Automatic depressurization system (ADS) is designed to decrease the pressure of the reactor in order to start the low-pressure system. The low-pressure system will inject water into the reactor vessel to protect the fuel. When an ATWS event happens, the reactor power is controlled by the level of water in core. Since high-level water will cause high power, the operator should inhibit all ADS valves manually. If the operator fails to do so, event X_I will occur.*

Epistemic uncertainty representation (mixed information)

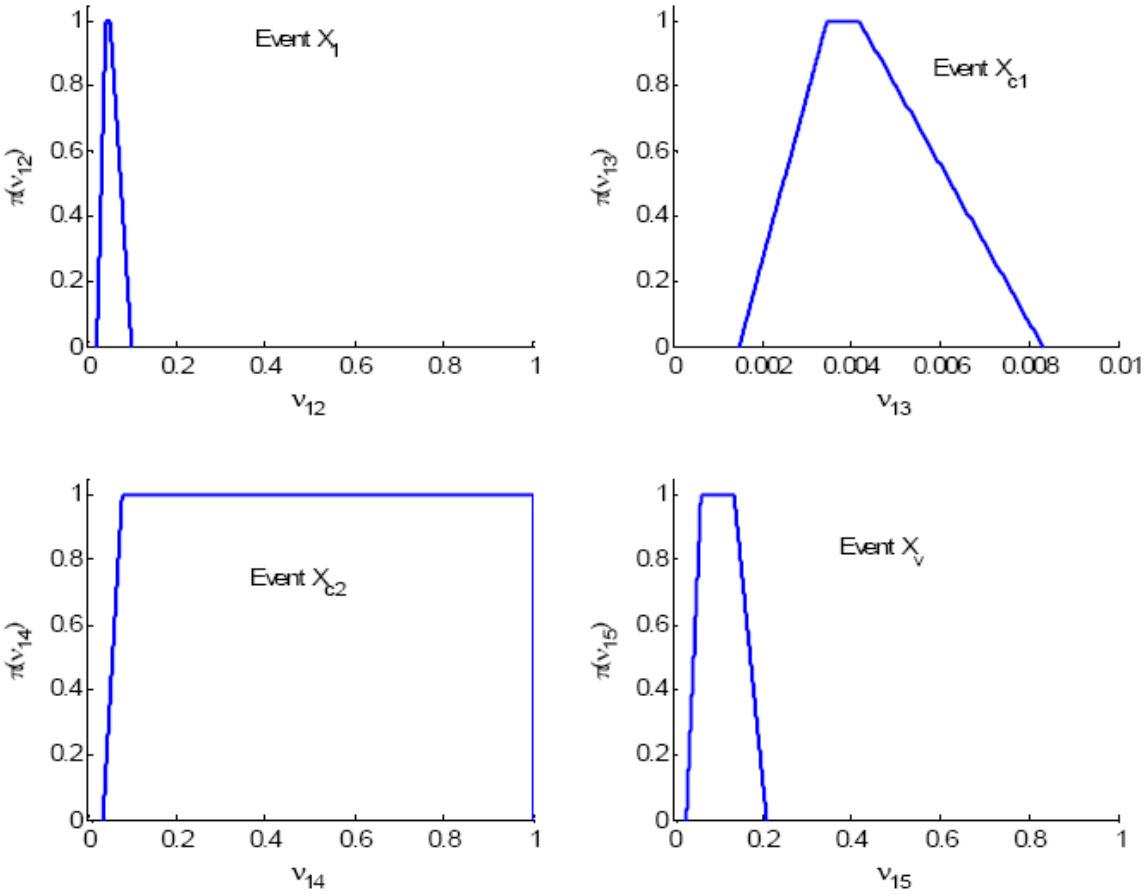
Uncertainty representation:

- Probability of Hardware-Failure-Dominated (HFD) event → p.d.f
- Probability of Human-Error-Dominated event (HED) → possibility distributions

Hardware-Failure-Dominated events



Human-Error-Dominated events



Epistemic uncertainty representation (mixed information)

Uncertainty representation:

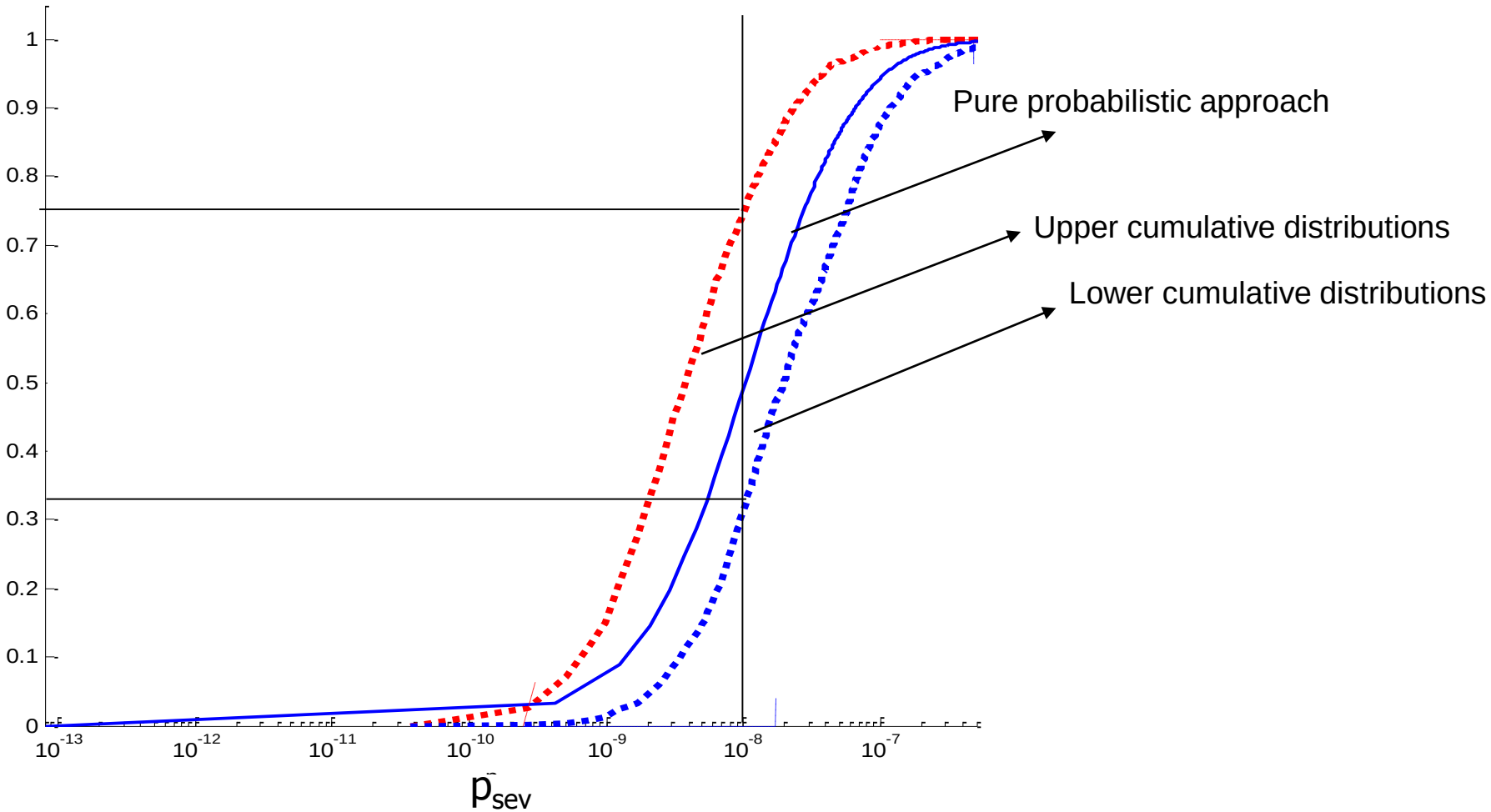
- Probability of Hardware-Failure-Dominated (HFD) event → p.d.f
- Probability of Human-Error-Dominated event (HED) → possibility distributions

Uncertainty propagation: Hybrid Monte Carlo and Possibilistic method

Event sequence probability → limiting cumulative distributions

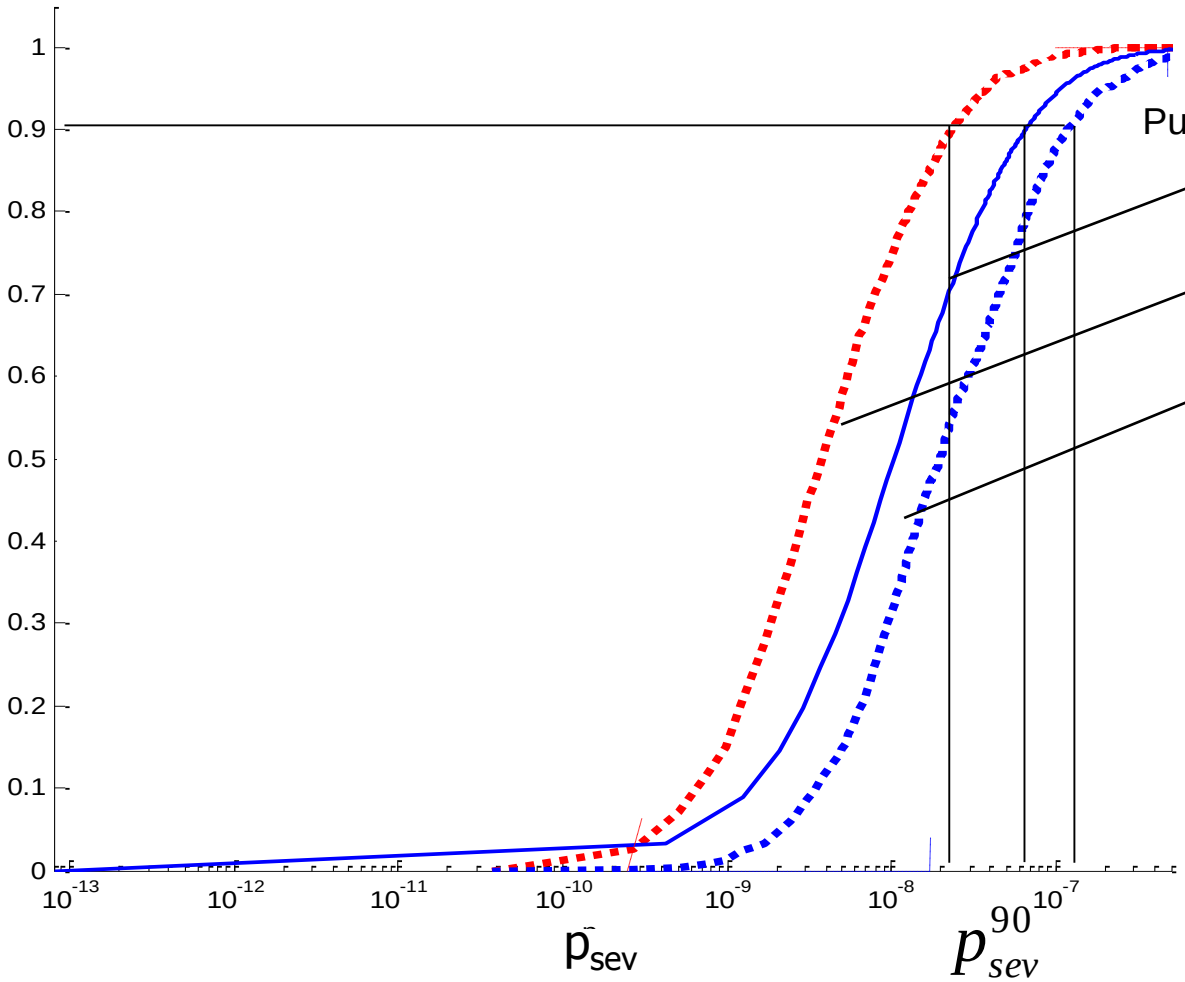
Results

Severe consequence accident



Results: 90% percentile

Severe consequence accident



Pure probabilistic approach

Upper cumulative distributions

Lower cumulative distributions

Pure probabilistic approach:

With probability 90% we can say that the probability of having a severe accident is lower than 7 10^-7

Results

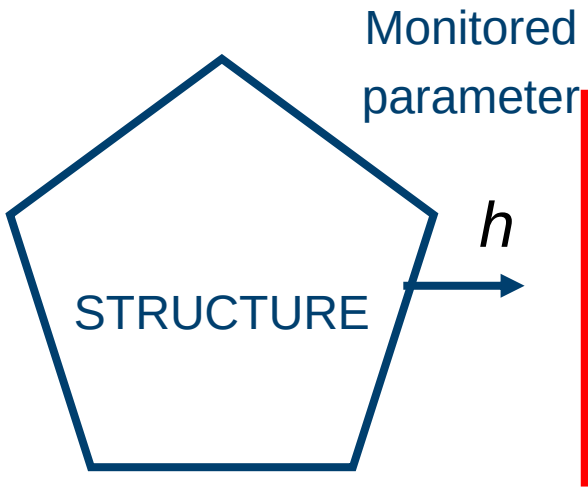
95% percentile for the probability of occurrence of a severe consequence accident

Hybrid approach	$(3.78 \cdot 10^{-8}, 1.95 \cdot 10^{-7})$
Probabilistic approach	$1.09 \cdot 10^{-7}$

Application 2:

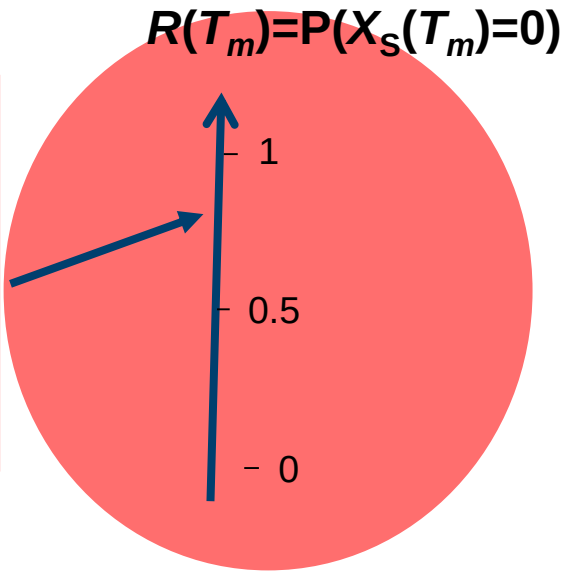
Reliability Assessment of a Degrading Structure

Example 3: Reliability Assessment of a Degrading Structure



$$\omega \sim N(0, \sigma_\omega^2)$$

H_{max}: "between 9 and 11, with a preference for 10"



$$\frac{dh}{dt} = e^{\omega} C(\beta\sqrt{h})^{\eta}$$

h = actual crack depth

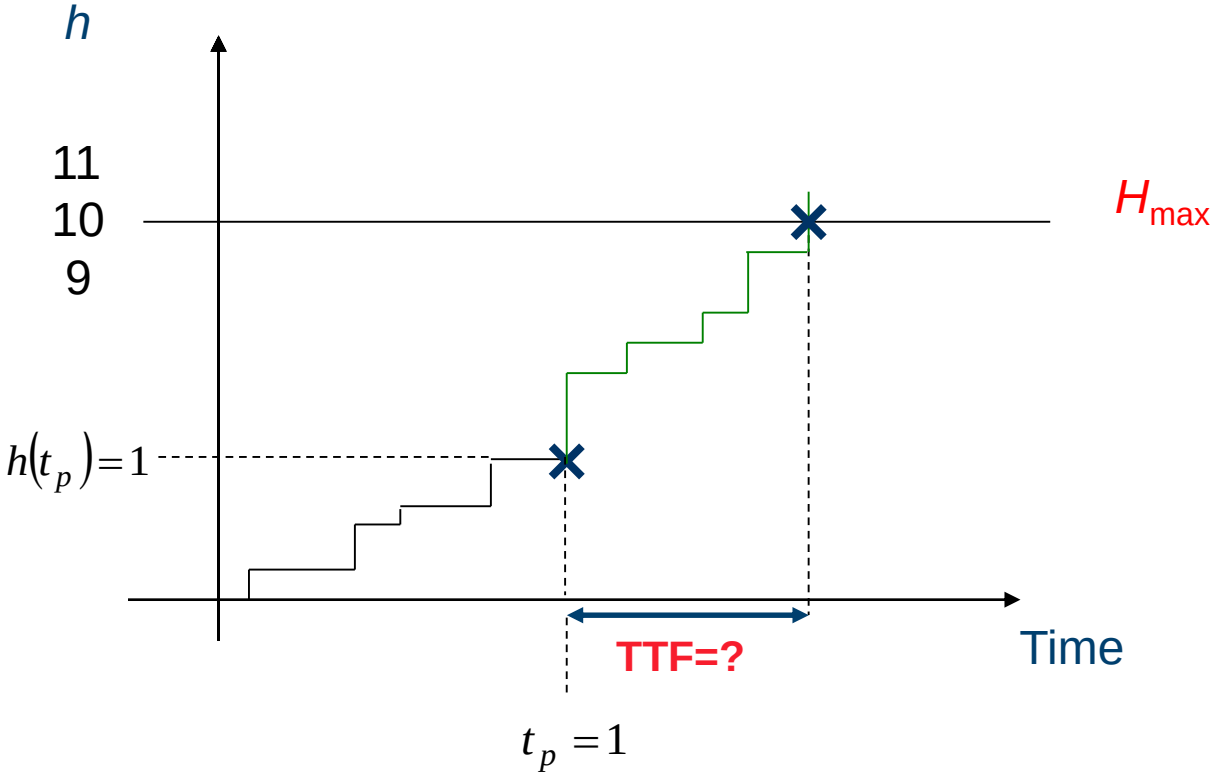
C, β, η = constants related to the material properties

ASSUMPTIONS:

- h monitored parameter
- $h > H_{max}$ → structure fails

Uncertainty in the case study

- aleatory → $\frac{dh}{dL} = e^{\omega} C(\beta\sqrt{h})^{\eta}$ with $\omega \sim N(0, \sigma_{\omega}^{2})$
- epistemic H_{max}: between 9 and 11, with preference for 10



Uncertainty in the case study

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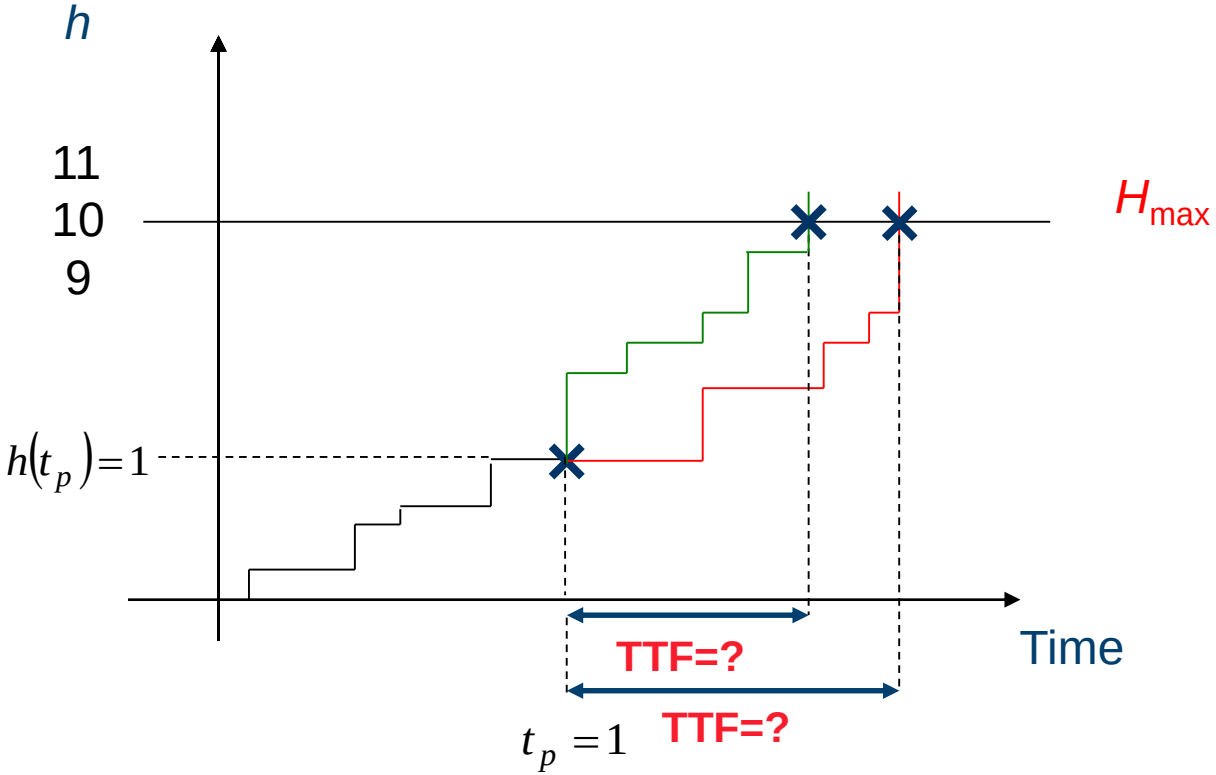
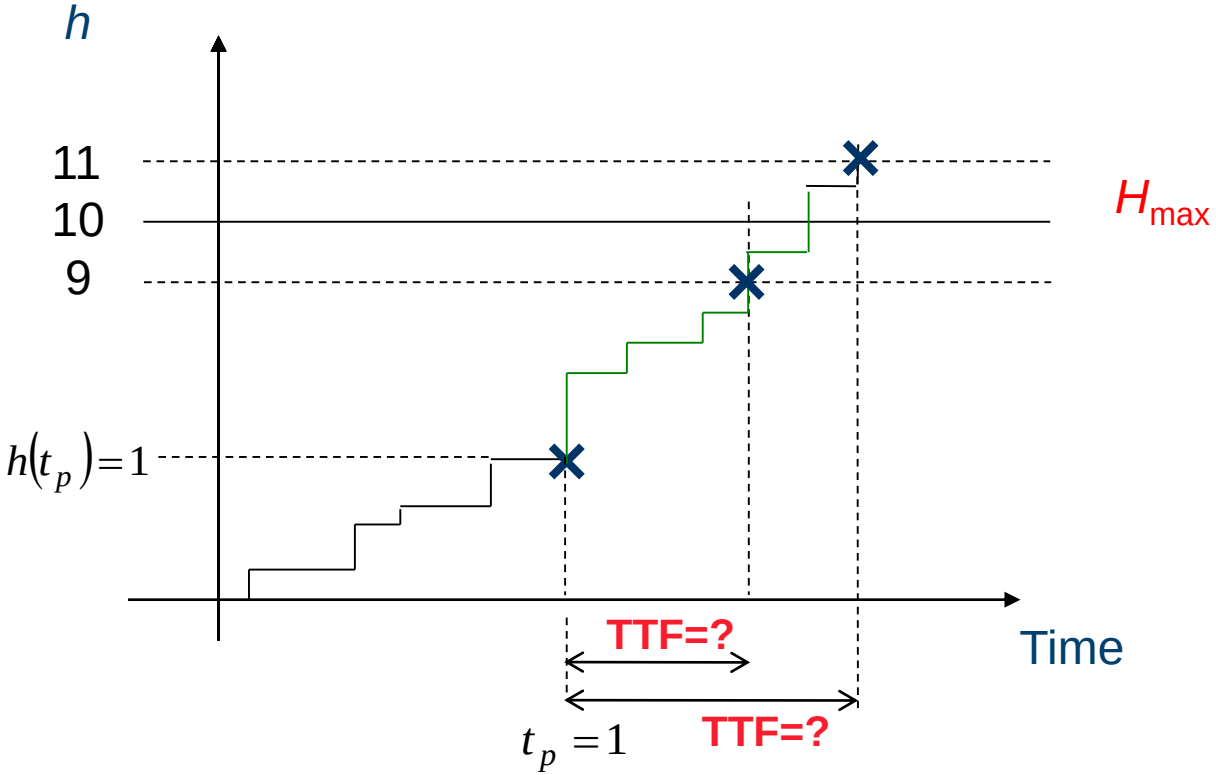


Epistemic uncertainty representation in the case study

- aleatory → $\frac{dh}{dL} = e^{\omega} C(\beta\sqrt{h})^{\eta}$ with $\omega \sim N(0, \sigma_{\omega}^{2})$
- epistemic H_{max}: between 9 and 11, with preference for 10

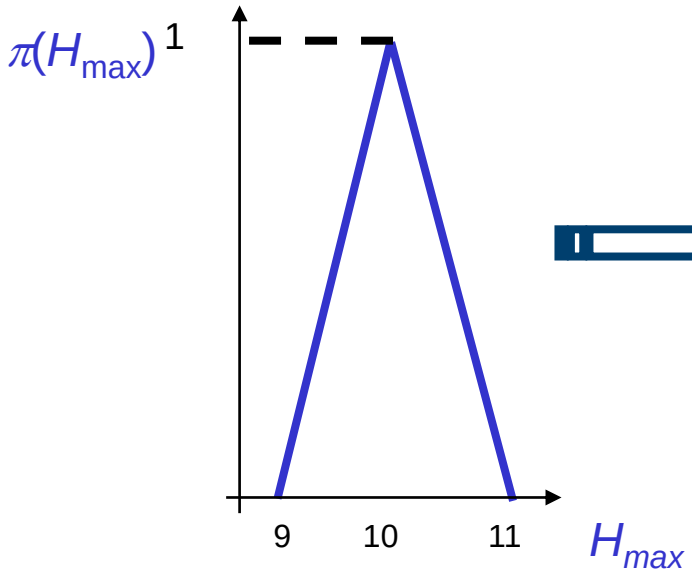


Application of the Hybrid Monte Carlo and possibilistic method

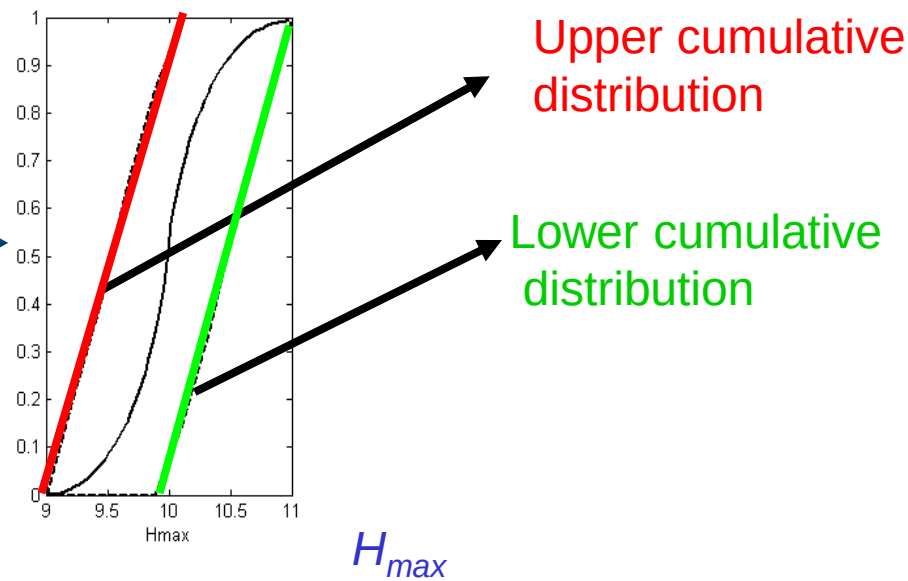
Epistemic uncertainty representation in the case study

Available information on H_{max} : $\begin{cases} \in [9, 11] \\ \text{most likely value is 10} \end{cases}$

Possibility distribution



$F(H_{max})$



Uncertainty propagation

- $$X_S(T_{miss}) = f(\underbrace{h(T_{miss})}_{\omega}, H_{\max}) = \begin{cases} 0 & \text{if} \quad h(T_{miss}) < H_{\max} \\ 1 & \text{otherwise} \end{cases}$$

  working

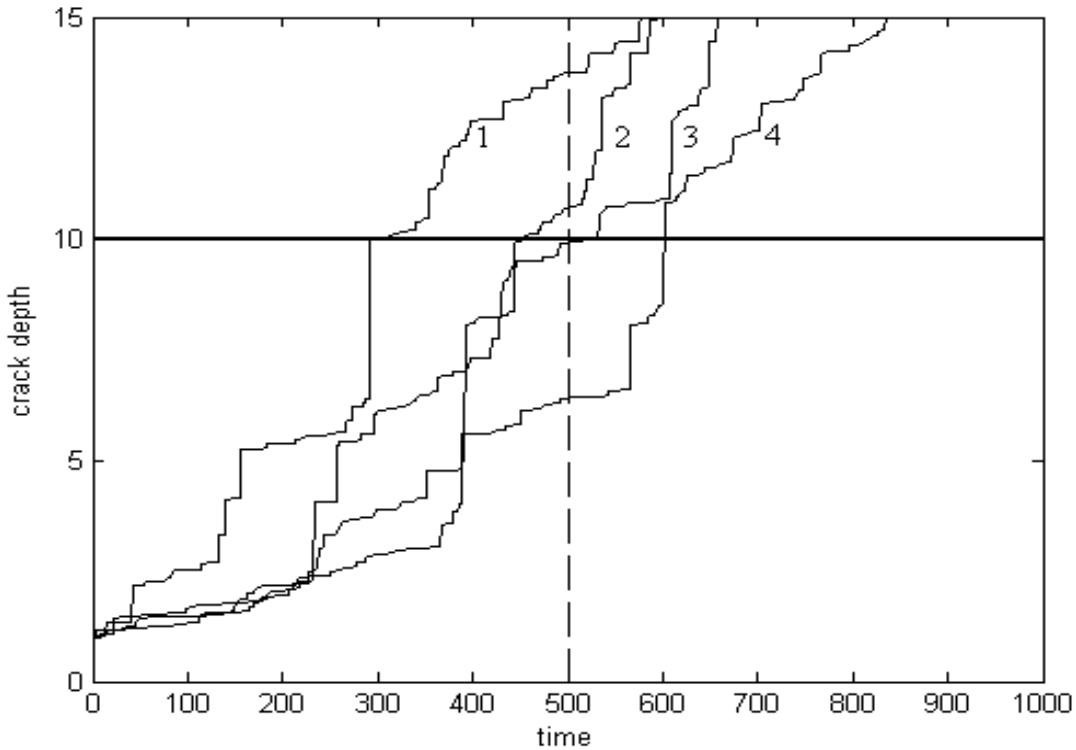
  failed

  ω → Probabilistic distributions

  $H_{\max}$ → Possibilistic distributions

- $$R(T_{miss}) = P(X_S(T_{miss}) = 0)$$

MC simulations of the degradation evolution: $h(T_{miss})$



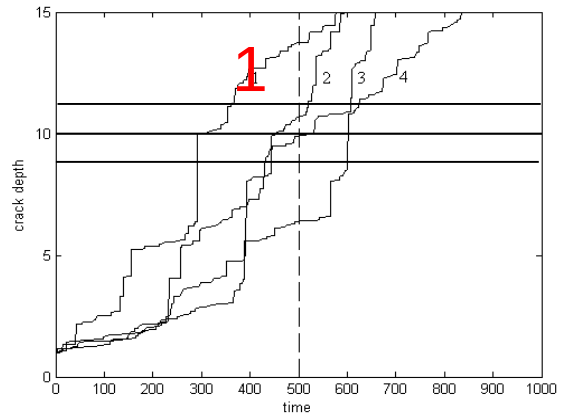
$T_{miss} = 500$

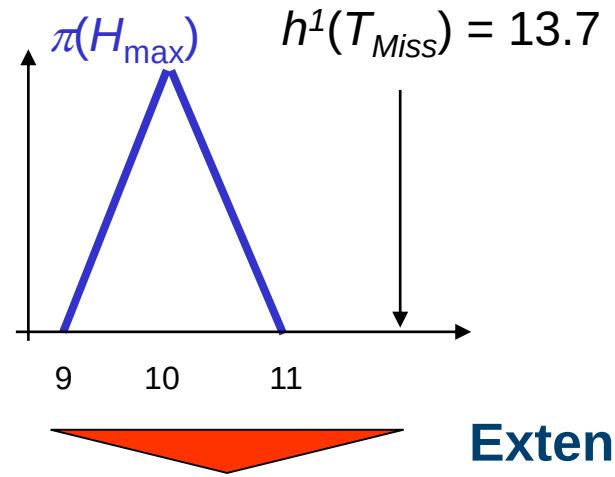
$$\omega_t \approx N\left(0, \sigma_\omega^2\right)$$

$$h(t) = h(t-1) + e^{\omega_t} C(\Delta K)^\eta$$

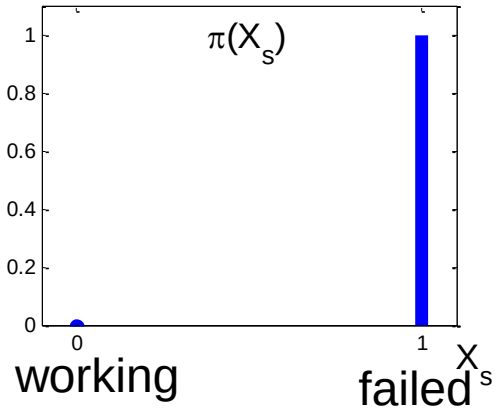
Possibility distributions of the component state: MC simulation 1

$$X_S(T_{miss}) = f(h(T_{miss}), H_{\max}) = \begin{cases} 0 & \text{if} \quad h(T_{miss}) < H_{\max} \\ 1 & \text{otherwise} \end{cases}$$





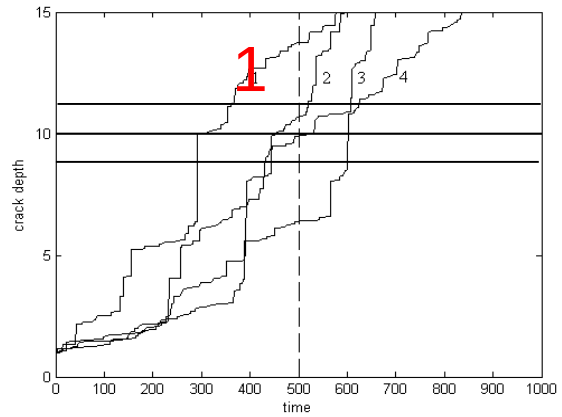
inputs

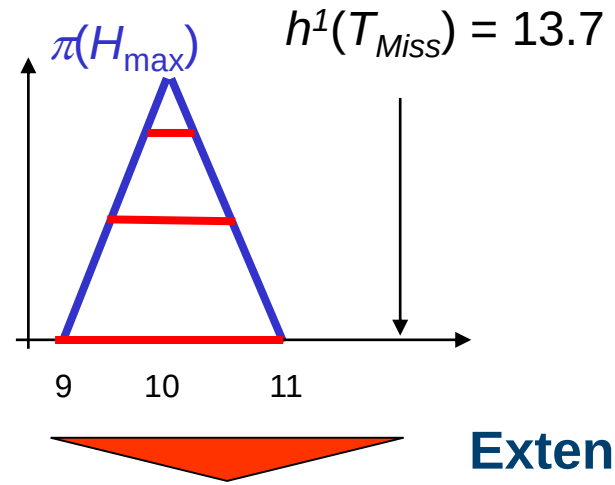


output

Possibility distributions of the component state: MC simulation 1

$$X_S(T_{miss}) = f(h(T_{miss}), H_{\max}) = \begin{cases} 0 & \text{if} \quad h(T_{miss}) < H_{\max} \\ 1 & \text{otherwise} \end{cases}$$





inputs

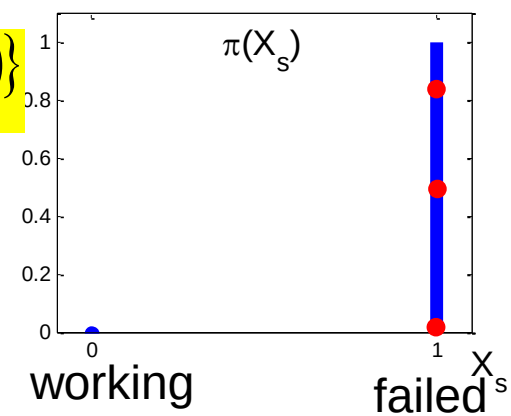
Extension principle

Interpretation

$$1 - \max_{X_s \neq 0}\{\pi(X_s)\} \leq P\{X_s = 0\} \leq \max_{X_s = 0}\{\pi(X_s)\}$$

$$0 \leq P\{X_S(T_{miss}) = 0\} \leq 0$$

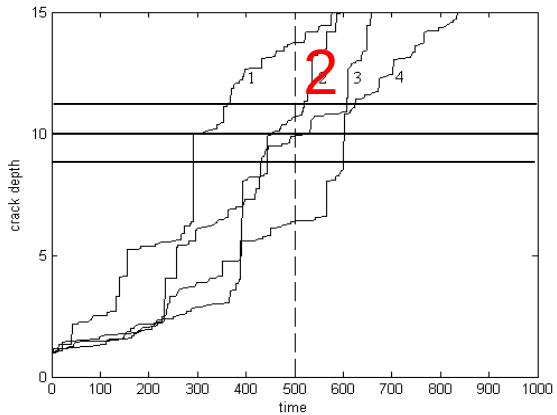
$$R(T_{miss}) = P\{X_S(T_{miss}) = 0\} = 0$$

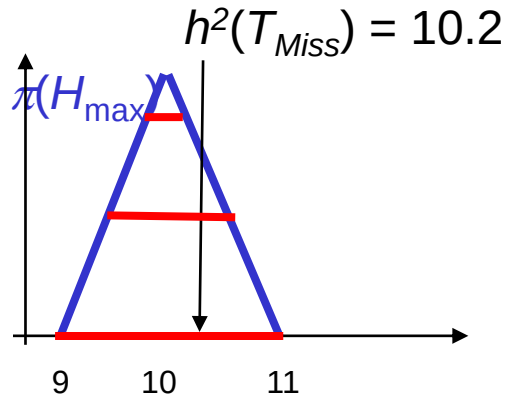


output

Possibility distributions of the component state: MC simulation 2

$$X_S(T_{miss}) = f(h(T_{miss}), H_{\max}) = \begin{cases} 0 & \text{if} \quad h(T_{miss}) < H_{\max} \\ 1 & \text{otherwise} \end{cases}$$





inputs

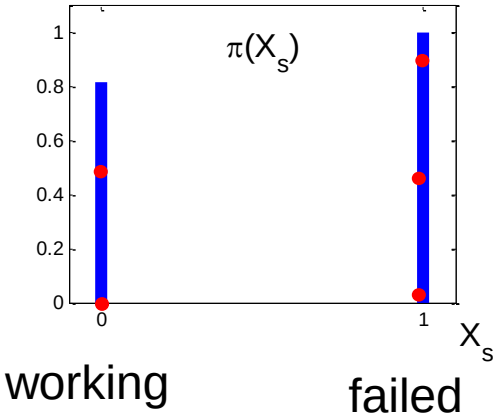
Extension principle

Interpretation

$$1 - \max_{X_s \neq 0}\{\pi(X_s)\} \leq P\{X_s = 0\} \leq \max_{X_s = 0}\{\pi(X_s)\}$$

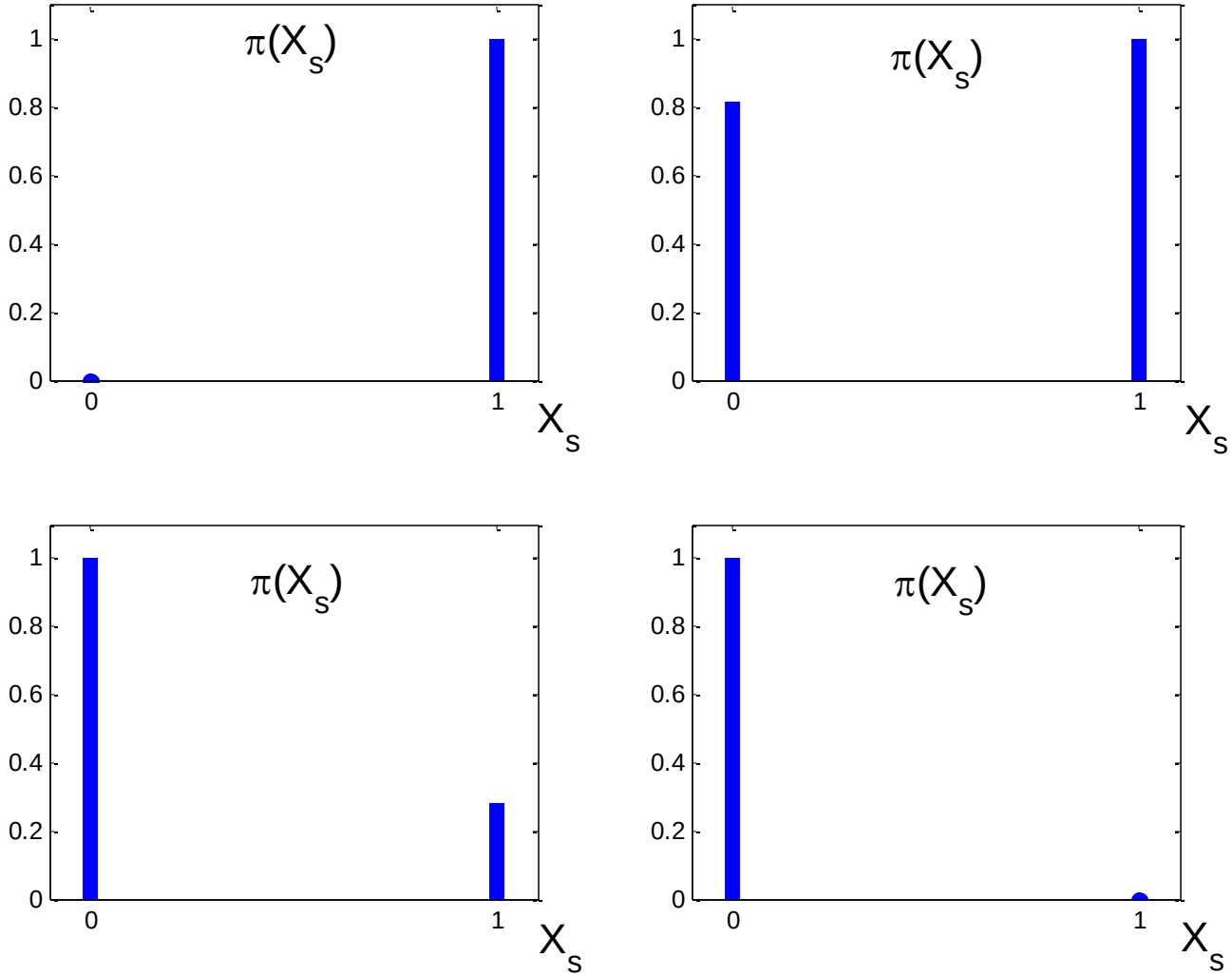
$$0 \leq P\{X_S(T_{miss}) = 0\} \leq 0.8$$

$$0 \leq R(T_{miss}) \leq 0.8$$



output

Example of realizations of Possibility distributions of the component state



Results

•Hybrid Monte Carlo and possibilistic method: the component reliability is between [0.896,0.927]

Conclusions

How to collect the information and input it in the proper mathematical format?

Conclusions

How to collect the information and input it in the proper mathematical format?

- Sufficient information → probability distribution
- Scarce and qualitative information → possibility distribution

does not force information in the epistemic uncertainty representation

Conclusions

How to collect the information and input it in the proper mathematical format?

How to propagate the uncertainty through the model so as to obtain the proper representation of the uncertainty in the output of the analysis?

- Pure Probabilistic Approach
- Extension Principle
- Hybrid Monte Carlo and possibilistic method

Conclusions

How to collect the information and input it in the proper mathematical format?

How to propagate the uncertainty through the model so as to obtain the proper representation of the uncertainty in the output of the analysis?

How to interpret the uncertainty results in a manner that is understandable and useful to decision makers?

- Interval of probabilities, e.g. the component reliability is between [0.896,0.927]

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